EL-MOASSER

In Mathematics

The Main Book

For 2nd Prep.

Second Term

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By
A group of supervisors

The important mathematical symbols used in this book

Symbol	Meaning
$\mathbb{N}$	the set of natural numbers
$\mathbb{Z}$	the set of integers
$\mathbb{Q}$	the set of rational numbers
$\grave{\mathbb{Q}}$	the set of irrational numbers
$\mathbb{R}$	the set of real numbers
$\sqrt{a}$	the positive square root of the number a
$\sqrt[3]{a}$	the cubic root of the number a
$[a , b]$	closed interval
$]a , b[$	open interval
$]a , b]$	half open interval or half closed interval
$[a , b[$	half open interval or half closed interval
$[a , \infty[$	infinite interval
P (A)	the probability of occurence of the event A
$\equiv$	is congruent to
$\sim$	is similar to
$\perp$	is perpendicular to
//	is parallel to
$\overline{AB}$	the line segment AB
$\overrightarrow{AB}$	the ray AB
$\overleftrightarrow{AB}$	the straight line AB
m (∠ A)	the measure of the angle A
>	greater than (more than)
≥	greater than or equal to (more than or equal to)
<	less than (smaller than)
≤	less than or equal to (smaller than or equal to)

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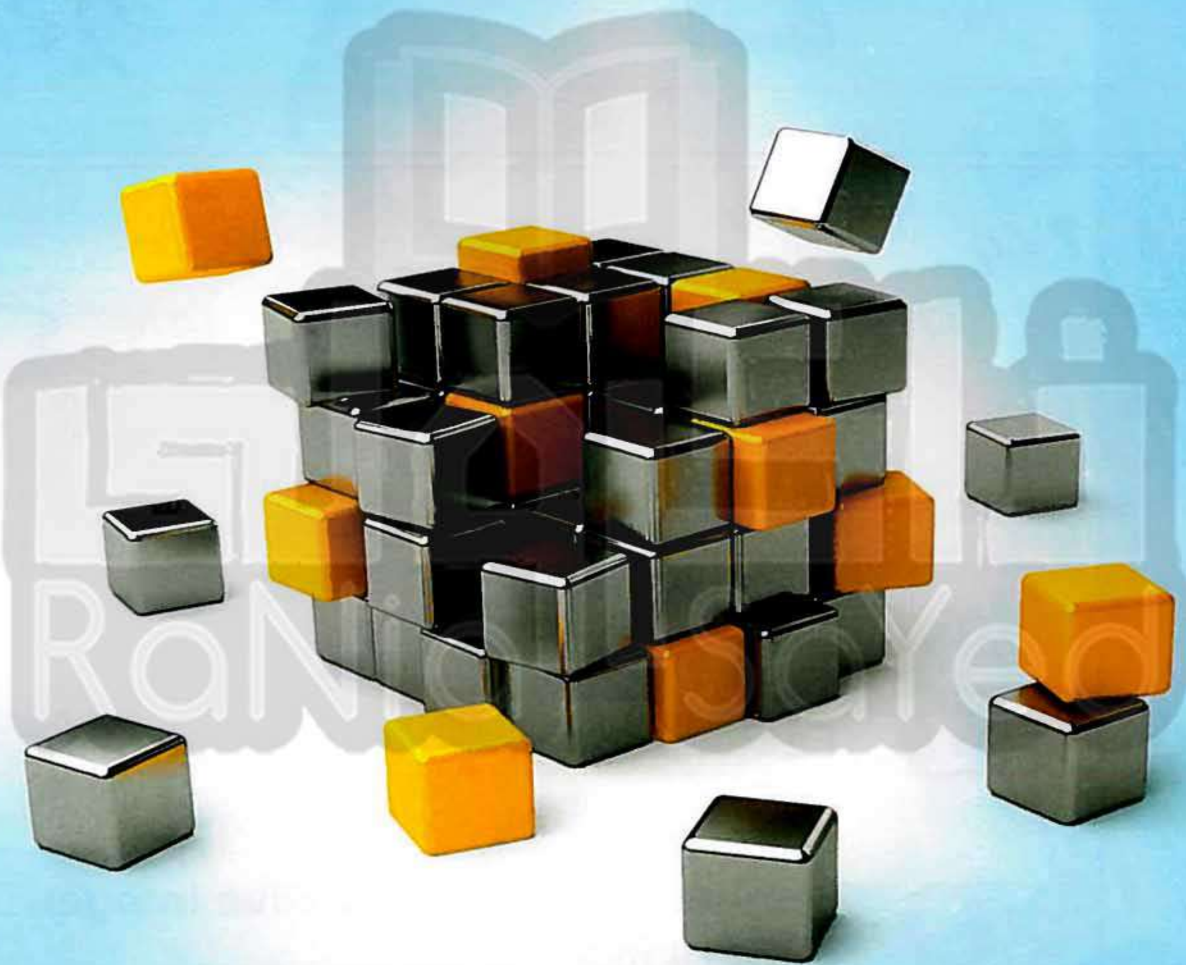
First Algebra and Statistics

UNIT 1

Factorization



Lessons of the unit :

Revision

On factorization by taking out the highest common factor (H.C.F)

1. Factorizing quadratic trinomial in the form : $x^2 + b\,x + c$
2. Factorizing quadratic trinomial in the form : $a\,x^2 + b\,x + c$, where $a \neq \pm 1$
3. Factorizing the perfect square trinomials.
4. Factorizing the difference of two squares.
5. Factorizing the sum and difference of two cubes.
6. Factorizing by grouping.
7. Factorizing by completing the square.
8. Solving quadratic equations in one variable algebraically.
9. Applications on solving quadratic equations in one variable algebraically

Unit Objectives :

By the end of this unit, student should be able to :

- recognize the concept of factorizing the algebraic expression.
- factorize a trinomial perfectly.
- recognize the perfect square trinomial.
- factorize the perfect square trinomial perfectly.
- factorize the difference of two squares perfectly.
- use the difference of two squares to facilitate finding the results of some mathematical operations.
- factorize the sum and difference of two cubes perfectly.
- factorize by grouping.
- factorize by completing the square.
- use factorization to solve a quadratic equation in one variable.
- use equations to solve some word problems.

Revision on factorization by taking out the highest common factor (H.C.F.)

Remember that

- Factorizing any number means to write it as a product of two factors or more.

For example: $16 = 1 \times 16$ or $16 = 2 \times 8$ or $16 = -2 \times -8$

or $16 = 4 \times 4$ or $16 = 2 \times 2 \times 4$ or $16 = 2 \times 2 \times 2 \times 2$

- Also factorizing the algebraic expression means to write it as a product of two factors or more.

How to factorize an expression by taking out the (H.C.F.) :

1. Determine the H.C.F. of the terms of the algebraic expression.
2. Put the H.C.F. out of two arcs.
3. Divide each term of the algebraic expression by the H.C.F. and put the quotients inside the arcs.

Example 1 Factorize each of the following by taking out the highest common factor :

1. $5a + 15b$
2. $10xy - 8xz$
3. $12x^2 - 4xy$
4. $3x^2y + 2xy^2 - xy$
5. $2x(m+3) - 4y(m+3)$
6. $x(z-y) + \ell(y-z)$

Solution

1. $\because$ H.C.F. $= 5$ $\therefore 5a + 15b = 5(a + 3b)$
2. $\because$ H.C.F. $= 2x$ $\therefore 10xy - 8xz = 2x(5y - 4z)$
3. $\because$ H.C.F. $= 4x$ $\therefore 12x^2 - 4xy = 4x(3x - y)$
4. $\because$ H.C.F. $= xy$ $\therefore 3x^2y + 2xy^2 - xy = xy(3x + 2y - 1)$
5. $\because$ H.C.F. $= 2(m+3)$

$\therefore 2x(m+3) - 4y(m+3)$

$= 2(m+3)(x - 2y)$

Notice that : The H.C.F. may be an algebraic expression.

6 $\because y - z = -(z - y)$

, $\because$ H.C.F. $= (z - y)$

$\therefore x(z - y) + \ell(y - z) = x(z - y) - \ell(z - y)$

$= (z - y)(x - \ell)$

Example 2 **If $a(x + y) - b(x + y) = 18$ and $x + y = 3$, then find the value of : $a - b$**

Solution

$\because a(x + y) - b(x + y) = 18$

$\therefore (x + y)(a - b) = 18$ «factorizing by taking out the H.C.F.»

, $\because x + y = 3$

$\therefore 3(a - b) = 18$

$\therefore a - b = \frac{18}{3} = 6$

Another solution :

$\because a(x + y) - b(x + y) = 18$

Substituting by $x + y = 3$:

$\therefore 3a - 3b = 18$

$\therefore 3(a - b) = 18$ "factorizing by taking out the H.C.F."

$\therefore a - b = \frac{18}{3} = 6$

TRY by yourself ***Factorize each of the following by taking out the highest common factor :***

1. $3x + 21y$
2. $2a^3 + 6a^2 - 4a$
3. $3x^2 + 15xz + 21xy^2$
4. $(x - 5)x^2 + (x - 5)y^2$
5. $a(a - b) - b(b - a)$

At the end of each lesson, you will find the final answers of try by yourself questions in the same form.

Answers *of try by yourself*

1. $3(x + 7y)$
2. $2a(a^2 + 3a - 2)$
3. $3x(x + 5z + 7y^2)$
4. $(x - 5)(x^2 + y^2)$
5. $(a - b)(a + b)$

Lesson 1 Factorizing quadratic trinomial in the form : $x^2 + bx + c$

Prelude

The trinomial is an algebraic expression consisting of three terms.

For example: each of the expressions : $x^2 + 6x + 8$ and $x^2 + 2x - 8$ is called a trinomial.

1 You know that : $(x \underline{+ 2})(x \underline{+ 4}) = x^2 \underline{+ 6}\, x \underline{+ 8}$

and we notice from the expression $x^2 + 6x + 8$ that :

$x^2 + 6x + 8$

The last term	The coefficient of x
equals $(+8)$, it is the product of $(+2)$, $(+4)$	equals $(+6)$, it is **the sum** of $(+2)$, $(+4)$

2 You know that : $(x \underline{- 2})(x \underline{+ 4}) = x^2 \underline{+ 2}\, x \underline{- 8}$

and we notice from the expression $x^2 \underline{+ 2}\, x \underline{- 8}$ that :

$x^2 + 2x - 8$

The last term	The coefficient of x
equals (-8) , it is the product of (-2) , $(+4)$	equals $(+2)$, it is **the sum** of (-2) , $(+4)$

Factorizing the trinomial in the form $x^2 + bx + c$

1 To factorize the trinomial $x^2 + 6x + 8$, do as follows :

- Write two pairs of arcs (parentheses) to express the operation of multiplication as follows : (   ) (   )
- Factorize : x^2 to $x \times x$ and write them inside the two parentheses as follows : (x) (x)
- Search for two numbers whose product is 8 and their sum is 6 performing some trials as in the opposite table , you will get that the two numbers are + 2 and + 4 , then write them inside the parentheses as : $(x + 2)(x + 4)$

The product = 8	The sum
+1 , +8	+9
−1 , −8	−9
−2 , −4	−6
+2 , +4	+6

i.e. $x^2 + 6x + 8 = (x + 2)(x + 4)$

2 To factorize the trinomial $x^2 + 2x - 8$, do as follows :

- Factorize x^2 to $x \times x$
- Search for two numbers whose product is (− 8) and their sum is (+ 2) performing some trials as in the opposite table , you will get that the two numbers are (− 2) and (+ 4)

, then $x^2 + 2x - 8 = (x - 2)(x + 4)$

The product = −8	The sum
−1 , +8	+7
+1 , −8	−7
−2 , +4	+2
+2 , −4	−2

Generally

Factorizing the trinomial which is in the form : $x^2 + bx + c$ is to write it as the product of two factors such that :

- The first term in each factor is x
- The two other terms in the two factors are two numbers whose product is c which is the last term in the trinomial , and their sum is b which is the coefficient of x in the trinomial.

Examples for factorizing the trinomial in the form : $x^2 + bx + c$

1 To factorize the expression : $x^2 + 5x + 6$

Search for two numbers :

whose product = +6 and their sum = +5

you will get that the two numbers are +2 and +3

, then $x^2 + 5x + 6 = (x + 2)(x + 3)$

Notice that :

∵ The product is positive and the sum is positive.

∴ The two numbers are positive together.

Unit 1

2 **To factorize the expression : $x^2 - 5x + 6$**

Search for two numbers :

whose product = $+6$ and their sum = -5

you will get that the two numbers are -2 and -3

, then $x^2 - 5x + 6 = (x - 2)(x - 3)$

Notice that :

∵ The product is positive and the sum is negative

∴ The two numbers are negative together.

3 **To factorize the expression : $x^2 + 5x - 6$**

Search for two numbers :

whose product = -6 and their sum = $+5$

you will get that the two numbers are $+6$ and -1

, then $x^2 + 5x - 6 = (x + 6)(x - 1)$

Notice that :

∵ The product is negative

∴ The two numbers have different signs

∵ The sum is positive

∴ The great number numerically is positive and the other is negative

4 **To factorize the expression : $x^2 - 5x - 6$**

Search for two numbers :

whose product = -6 and their sum = -5

you will get that the two numbers are -6 and $+1$

, then $x^2 - 5x - 6 = (x - 6)(x + 1)$

Notice that :

∵ The product is negative

∴ The two numbers have different signs.

∵ The sum is negative

∴ The great number numerically is negative and the other is positive

From the previous examples , notice that :

When you factorize the trinomial : $x^2 + bx + c$ in the form $(x + \ell)(x + m)$, then :

1. If c is positive **i.e.** The product of the two numbers is positive , then ℓ and m have the same sign as b
2. If c is negative **i.e.** The product of the two numbers is negative , then ℓ and m have different signs such that the great one (numerically) has the same sign as b

Remark

Before factorizing the trinomial , you must do the following :

- Arrange the terms of the expression descendingly or ascendingly according to the indices (exponents) of one of the given algebraic symbols. It is better to be descending.
- Take out the H.C.F. of the terms of the expression.
- Perform operations included in arcs and simplify the algebraic expression.

Example 1 **Factorize each of the following :**

1. $x^2 + 56 - 15x$
2. $x^2 + xy - 12y^2$
3. $3a^3 + 9a^2 - 120a$
4. $m(m + 7) - 18$
5. $x^4 - 3x^2y - 10y^2$

Solution

1. Arrange the terms of the expression descendingly according to the powers of x before factorizing.

$\therefore x^2 + 56 - 15x = x^2 - 15x + 56$

$= (x - 7)(x - 8)$

> You can check your solution by multiplying the two factors to get the main expression.

2. $x^2 + xy - 12y^2 = (x - 3y)(x + 4y)$

3. Take out the H.C.F. of the terms of the expression which is 3 a before factorizing

$\therefore 3a^3 + 9a^2 - 120a = 3a(a^2 + 3a - 40) = 3a(a + 8)(a - 5)$

4. At first remove arcs before factorizing.

$\therefore m(m + 7) - 18 = m^2 + 7m - 18 = (m + 9)(m - 2)$

5. $x^4 - 3x^2y - 10y^2 = (x^2 - 5y)(x^2 + 2y)$

(**Notice that :** x^4 is factorized to be $x^2 \times x^2$)

TRY by yourself

Factorize each of the following :

1. $x^2 + 7x + 10$
2. $x^2 - 6x + 8$
3. $-30 + x^2 + 13x$
4. $3x^2 - 48 + 18x$

Example 2 **Find the values of b which make each of the following expressions can be factorized :**

1. $x^2 + bx + 10$
2. $x^2 + bx - 12$

Solution

1. To make the expression : $x^2 + bx + 10$ can be factorized , b should be the sum of two numbers whose product = 10

(Notice that the two numbers have the same sign because their product is positive)

Therefore you search for the pairs of numbers whose product of each = 10

, then you get : 1 , 10 , − 1 , − 10 , 2 , 5 , − 2 , − 5

Then you get the sum of numbers of each pair to get : 11 , – 11 , 7 , – 7 which are the possible values of b

2 To make the expression : $x^2 + b\,x - 12$ can be factorized , b should be the sum of two numbers whose product = – 12

(Notice that the two numbers have different signs because their product is negative)

Therefore you search for the pairs of numbers such that the product of each two numbers = – 12 , then you get :

1 , – 12 , – 1 , 12 ,

2 , – 6 , – 2 , 6 ,

3 , – 4 , – 3 , 4

Then you get the sum of numbers of each pair to get :

– 11 , 11 , – 4 , 4 , – 1 , 1 which are the possible values of b

Example 3 **Find a positive value and a negative value for the number c such that the expression : $x^2 - 6x + c$ can be factorized.**

Solution

- To find a positive value for c , you search for two negative numbers whose sum is – 6 , then c is their product as : – 2 and – 4

  ∴ c = – 2 × – 4 = 8 *(There are other solutions)*

- To find a negative value for c , you search for two numbers having different signs such that their sum is – 6 , then c is their product as : – 8 , 2 , then c = – 8 × 2 = – 16 *(There are other solutions)*

Answers *of try by yourself*

1 $(x + 2)(x + 5)$

2 $(x - 2)(x - 4)$

3 $(x + 15)(x - 2)$

4 $3(x + 8)(x - 2)$

Lesson 2 Factorizing quadratic trinomial in the form : $a x^2 + b x + c$ where $a \neq \pm 1$

To factorize the trinomial : $a x^2 + b x + c$ where ($a \neq \pm 1$) , do as follows :

Step (1) Factorize $a x^2$ into two factors « ℓx , $m x$ » and write them inside two parentheses as shown in the opposite figure.

$(\ell x \quad)$
$(m x \quad)$

Step (2) Factorize the last term in the trinomial (c) into two factors « n , h » and write them as shown in the previous parentheses.

$(\ell x + n)$
$(m x + h)$

Step (3) Find «The product of extremes (outer terms) + the product of means (inner terms)»
If the sum equals the middle term in the trinomial , then the factorization is true.
If not , then the factorization is false hence , you should try again to get the true factorization.

$(\ell x \quad + n)$
$(m x \quad + h)$

The previous method is called the method of scissors.

and here how to apply this method to factorize the expression : $3 x^2 + 13 x + 12$

Step (1) Factorize $3 x^2$ into two factors, say $3 x , x$

Step (2) Factorize 12 (the last term) into two factors, say

1 , 12 **or** 2 , 6 **or** 3 , 4

You ignore the negative factors because the coefficient of x is positive.

Step (3) Perform some trials till you reach :
The product of extremes + the product of means
= the middle term in the trinomial ($13\ x$)

$(3x + 1)$, $(x + 12)$

$(3x \times 12) + (x \times 1) = 37x$
$\neq$ the middle term

(False trial) ✗

$(3x + 12)$, $(x + 1)$

$(3x \times 1) + (x \times 12) = 15x$
$\neq$ the middle term

(False trial) ✗

$(3x + 2)$, $(x + 6)$

$(3x \times 6) + (x \times 2) = 20x$
$\neq$ the middle term

(False trial) ✗

$(3x + 6)$, $(x + 2)$

$(3x \times 2) + (x \times 6) = 12x$
$\neq$ the middle term

(False trial) ✗

$(3x + 3)$, $(x + 4)$

$(3x \times 4) + (x \times 3) = 15x$
$\neq$ the middle term

(False trial) ✗

$(3x + 4)$, $(x + 3)$

$(3x \times 3) + (x \times 4) = 13x$
= the middle term

(True trial) ✓

, then : $3x^2 + 13x + 12 = (3x + 4)(x + 3)$

Example 1 **Factorize :** $14x^2 - 17x + 5$

Solution

1 Factorize $14x^2$ into two factors,

say $x, 14x$ or $2x, 7x$

2 Factorize 5 (the last term) into two negative factors (together)

(because the coefficient of x is negative), say $-1, -5$

The following shows the different trials to factorize the expression :

$14x^2 - 17x + 5$

(a)	(b)	(c)	(d)
$(x \quad -1)$	$(x \quad -5)$	$(2x \quad -1)$	$(2x \quad -5)$
$(14x \quad -5)$	$(14x \quad -1)$	$(7x \quad -5)$	$(7x \quad -1)$

3 Find the sum of the product of extremes and the product of means as you did in the previous example, you will realize that the trial (c) is the true trial.

$\therefore 14x^2 - 17x + 5 = (2x - 1)(7x - 5)$

Remark

- If the sign of the last term of the trinomial is positive, then the sign of the second term in each of the parentheses is the same as the sign of the middle term in the trinomial.
- If the sign of the last term of the trinomial is negative, then the two signs of the second term in each of the parentheses are different.

Example 2 **Factorize each of the following expressions :**

1 $6a - 27 + 5a^2$

2 $14x^2 - 11xy - 15y^2$

3 $48x^3 - 112x^2 - 20x$

4 $(10x + y)(x + y) - 7y^2$

Solution

1 $6a - 27 + 5a^2 = 5a^2 + 6a - 27$

$= (5a - 9)(a + 3)$

$(5a \quad -9)$
$(a \quad +3)$

Unit 1

2 $14x^2 - 11xy - 15y^2$

$= (7x + 5y)(2x - 3y)$

$(7x \quad +5y)$
$(2x \quad -3y)$

3 Notice that there is H.C.F. among the terms of the trinomial which is $4x$

$\therefore 48x^3 - 112x^2 - 20x = 4x(12x^2 - 28x - 5)$

$= 4x(6x + 1)(2x - 5)$

$(6x \quad +1)$
$(2x \quad -5)$

4 Remove the parentheses at first :

$\therefore (10x + y)(x + y) - 7y^2 = 10x^2 + 11xy + y^2 - 7y^2$

$= 10x^2 + 11xy - 6y^2$

$= (5x - 2y)(2x + 3y)$

$(5x \quad -2y)$
$(2x \quad +3y)$

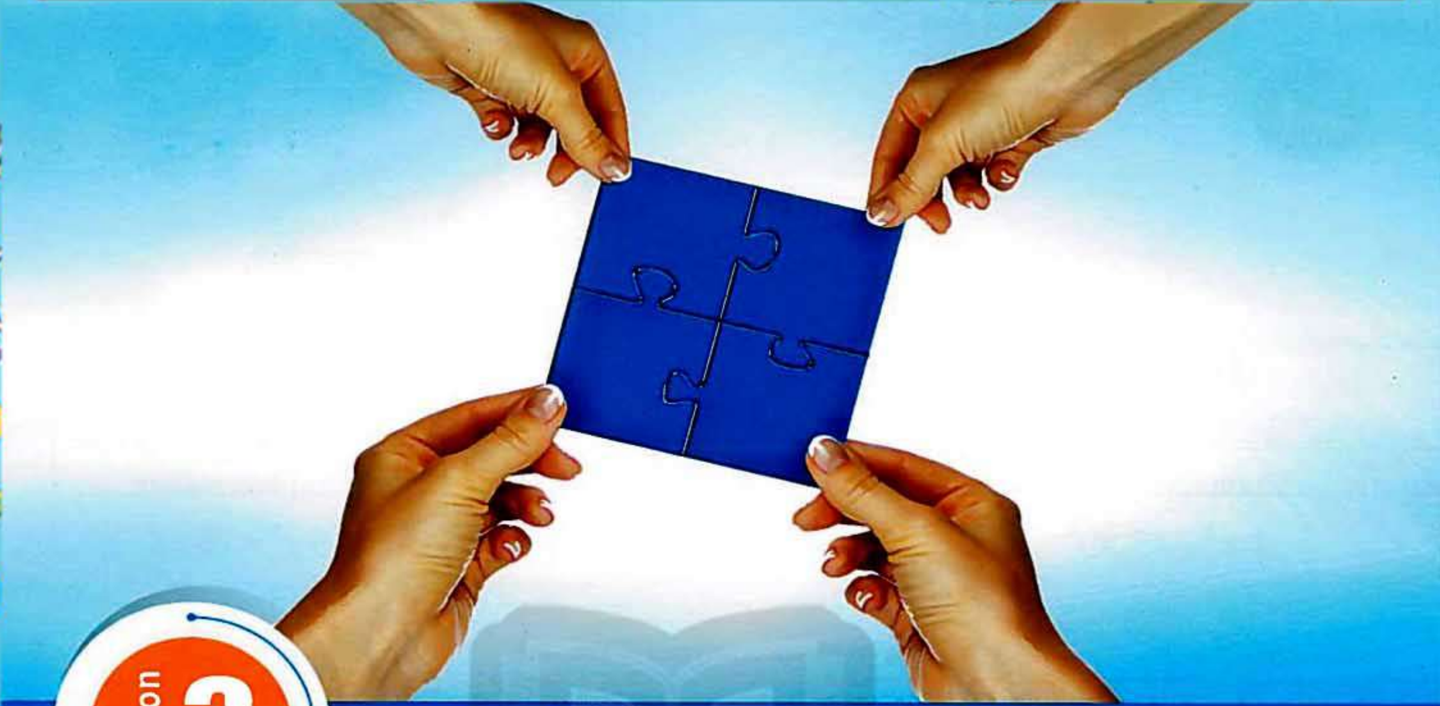
TRY by yourself

Factorize each of the following perfectly :

1. $3b^2 + 7b + 2$
2. $5x^2 - 6x + 1$
3. $12x^2 + 28xy - 5y^2$
4. $6x^2 - x - 12$

Answers of try by yourself

1. $(3b + 1)(b + 2)$
2. $(5x - 1)(x - 1)$
3. $(2x + 5y)(6x - y)$
4. $(2x - 3)(3x + 4)$



Lesson 3

Factorizing the perfect square trinomials

The perfect square trinomial

If the trinomial is arranged ascendingly or descendingly according to the powers of one of its symbols , then this trinomial is called a perfect square trinomial if :

The first term	The middle term	The third term
is a perfect square (and it is always positive)	$= 2 \times \sqrt{\text{first term}} \times \sqrt{\text{third term}}$ (and it may be positive or negative)	is a perfect square (and it is positive also)

For example:

$$4x^2 + 12x + 9$$

First term $4x^2$	Middle term $12x$	Third term 9
Complete square $4x^2 = (2x)^2$	$12x = 2 \times \sqrt{4x^2} \times \sqrt{9}$	Complete square $9 = 3^2$

i.e. $4x^2 + 12x + 9$ **is a perfect square trinomial.**

$$9x^2 - 30xy + 25y^2$$

First term $9x^2$	Middle term $-30xy$	Third term $25y^2$
Complete square $9x^2 = (3x)^2$	$-30xy = -2 \times \sqrt{9x^2} \times \sqrt{25y^2}$	Complete square $25y^2 = (5y)^2$

i.e. $9x^2 - 30xy + 25y^2$ is a perfect square trinomial.

Example 1 **Show which of the following trinomials is a perfect square trinomial and which of them is not :**

1. $36x^2 + 84x + 49$
2. $4x^2 + 44xy + 121y^2$
3. $25x^2 - 5x + 1$
4. $16a^2 - 24a - 9$
5. $12b^2 - 16b + 4$

Solution

1. $\because 36x^2 = (6x)^2$ (a perfect square) , $49 = (7)^2$ (a perfect square)
   , $\because 2 \times 6x \times 7 = 84x =$ the middle term.
   $\therefore$ The trinomial : $36x^2 + 84x + 49$ is a perfect square trinomial.
2. $\because 4x^2 = (2x)^2$ (a perfect square)
   , $121y^2 = (11y)^2$ (a perfect square)
   , $\because 2 \times 2x \times 11y = 44xy =$ the middle term.
   $\therefore$ The trinomial : $4x^2 + 44xy + 121y^2$ is a perfect square trinomial.
3. $\because 25x^2 = (5x)^2$, $1 = (1)^2$
   , $\because 2 \times 5x \times 1 = 10x \neq$ the middle term.
   $\therefore$ The trinomial : $25x^2 - 5x + 1$ is not a perfect square.
4. The trinomial : $16a^2 - 24a - 9$ is not a perfect square because the third term is negative.
5. The trinomial : $12b^2 - 16b + 4$ is not a perfect square because the first term is not a perfect square.

How to find a missing term in a perfect square trinomial

1 Finding the middle term :

The middle term $= \pm 2 \times \sqrt{\text{the first term}} \times \sqrt{\text{the third term}}$

2 Finding the first term :

The first term $= \dfrac{(\text{the middle term})^2}{4 \times \text{the third term}}$

3 Finding the third term :

The third term $= \dfrac{(\text{the middle term})^2}{4 \times \text{the first term}}$

Example 2 **Complete by the missing term in each of the following trinomials to be a perfect square :**

1. $49\,x^2$ .... $+ 25$
2. $25\,x^2 - 60\,x +$ ....
3. .... $+ 12\,x\,y + 9\,y^2$

Solution

1. The middle term $= \pm 2 \times \sqrt{1^{st}\text{term}} \times \sqrt{3^{rd}\text{term}}$

$= \pm 2 \times \sqrt{49\,x^2} \times \sqrt{25} = \pm 2 \times 7\,x \times 5 = \pm 70\,x$

2. The third term $= \dfrac{(\text{the middle term})^2}{4 \times \text{the first term}} = \dfrac{(-60\,x)^2}{4 \times 25\,x^2} = \dfrac{3600\,x^2}{100\,x^2} = 36$

3. The first term $= \dfrac{(\text{the middle term})^2}{4 \times \text{the third term}} = \dfrac{(12\,x\,y)^2}{4 \times 9\,y^2} = \dfrac{144\,x^2\,y^2}{36\,y^2} = 4\,x^2$

Example 3 **Find the positive value of c which makes each of the following trinomials a perfect square :**

1. $x^2 + c\,x + 25$
2. $c^2\,y^2 - 90\,y + 81$

Solution

1. $\because$ The middle term $= \pm 2 \times \sqrt{\text{the first term}} \times \sqrt{\text{the third term}}$

$\therefore c\,x = \pm 2 \times \sqrt{x^2} \times \sqrt{25} = \pm 2 \times x \times 5 = \pm 10\,x$

$\therefore c = \pm 10$, $\because$ c is positive $\therefore c = 10$

2. $\because$ The first term $= \dfrac{(\text{the middle term})^2}{4 \times \text{the third term}}$ $\therefore c^2\,y^2 = \dfrac{(-90\,y)^2}{4 \times 81}$

$\therefore c^2\,y^2 = \dfrac{8100\,y^2}{4 \times 81}$ $\therefore c^2\,y^2 = 25\,y^2$ $\therefore c^2 = 25$

$\therefore c = \pm 5$, $\because$ c is positive $\therefore c = 5$

TRY *by yourself* **1**

Find the missing term in each of the following trinomials to be a perfect square :

1. $4y^2$ .... $+ 25$
2. .... $+ 12x^2 + 36$
3. $25a^2 - 30ab$ ....

Factorizing the perfect square trinomial

- Factorizing the trinomial means to write it as a product of two factors (or more).
- Factorizing the perfect square trinomial means to write it as a product of two equal factors (**i.e.** the square of one of its two equal factors).

If the trinomial is a perfect square arranged descendingly or ascendingly according to the powers of one of its symbols , then we can factorize it to be in the form :

$$\left(\sqrt{\text{the first term}} \pm \sqrt{\text{the third term}}\right)^2$$

Notice that : The sign between the two terms inside the parentheses is the same sign of the middle term in the trinomial.

The following example shows that.

Example 4 **Factorize each of the following trinomials :**

1. $25a^2 + 20a + 4$
2. $16x^2 - 24x + 9$
3. $25a^4 - 90a^2b + 81b^2$
4. $\frac{1}{9}x^2 + \frac{1}{3}x + \frac{1}{4}$
5. $18x^2 - 48x + 32$
6. $28x - 49x^2 - 4$

Solution

After checking that each of the trinomials is a perfect square , we can factorize directly as follows :

1. $25a^2 + 20a + 4 = \left(\sqrt{25a^2} + \sqrt{4}\right)^2 = (5a + 2)^2$
2. $16x^2 - 24x + 9 = \left(\sqrt{16x^2} - \sqrt{9}\right)^2 = (4x - 3)^2$
3. $25a^4 - 90a^2b + 81b^2 = \left(\sqrt{25a^4} - \sqrt{81b^2}\right)^2 = (5a^2 - 9b)^2$
4. $\frac{1}{9}x^2 + \frac{1}{3}x + \frac{1}{4} = \left(\sqrt{\frac{1}{9}x^2} + \sqrt{\frac{1}{4}}\right)^2 = \left(\frac{1}{3}x + \frac{1}{2}\right)^2$

5 $18x^2 - 48x + 32 = 2(9x^2 - 24x + 16)$
$= 2\left(\sqrt{9x^2} - \sqrt{16}\right)^2$
$= 2(3x - 4)^2$

Notice that :
The H.C.F. should be taken out before factorization.

6 $28x - 49x^2 - 4 = -49x^2 + 28x - 4$
$= -(49x^2 - 28x + 4)$
$= -(7x - 2)^2$

Notice that :
$-49x^2 + 28x - 4$ is not a perfect square while $49x^2 - 28x + 4$ is a perfect square.

TRY 2 by yourself

Factorize each of the following :

1 $16m^2 + 56m + 49$
2 $25a^2 - 30a + 9$
3 $2x^2 + 4xy + 2y^2$
4 $50x^2 - 20xy + 2y^2$

Example 5 Use factorization to facilitate getting the value of each of the following :

1 $(55)^2 + 2 \times 55 \times 45 + (45)^2$
2 $(312)^2 - 2 \times 312 \times 311 + (311)^2$

Solution

1 $(55)^2 + 2 \times 55 \times 45 + (45)^2 = \left(\sqrt{(55)^2} + \sqrt{(45)^2}\right)^2$
$= (55 + 45)^2 = (100)^2 = 10000$

2 $(312)^2 - 2 \times 312 \times 311 + (311)^2 = \left(\sqrt{(312)^2} - \sqrt{(311)^2}\right)^2$
$= (312 - 311)^2 = 1^2 = 1$

TRY 3 by yourself

Use factorization to facilitate getting the value of each of the following :

1 $(38)^2 - 2 \times 38 \times 28 + (28)^2$
2 $(14)^2 + 2 \times 14 \times 16 + (16)^2$

Unit 1

Example 6 **Find the value of k which makes each of the following trinomials a perfect square :**

1 $25x^2 - 60x + k$ **2** $kx^2 + 12xy + 9y^2$

Solution

1 $\because 25x^2 - 60x + k$ is a perfect square , then we can factorize it to be in the form $\left(\sqrt{\text{the first term}} - \sqrt{\text{the third term}}\right)^2$

$\therefore 25x^2 - 60x + k = \left(5x - \sqrt{k}\right)^2$

$= 25x^2 - 10x\sqrt{k} + k$

By comparing the terms , you find that :

$-60x = -10x\sqrt{k}$

$\therefore \sqrt{k} = \frac{-60x}{-10x} = 6$

$\therefore k = 6^2$ $\therefore k = 36$

Notice that : The adjacent problem is solved by another method in example (2) in the same lesson.

2 $\because kx^2 + 12xy + 9y^2$ is a perfect square.

$\therefore kx^2 + 12xy + 9y^2 = \left(\sqrt{k}x + 3y\right)^2$

$= kx^2 + 6\sqrt{k}xy + 9y^2$

By comparing the terms , you find that :

$12xy = 6\sqrt{k}xy$ $\therefore \sqrt{k} = \frac{12xy}{6xy} = 2$

$\therefore k = 2^2$ $\therefore k = 4$

TRY 4 by yourself

Find the value of k if : $64x^2 - 32x + k$ is a perfect square.

Answers *of try by yourself*

1 [1] $\mp 20y$ [2] x^4 [3] $9b^2$

2 [1] $(4m + 7)^2$ [2] $(5a - 3)^2$ [3] $2(x + y)^2$ [4] $2(5x - y)^2$

3 [1] 100 [2] 900

4 $k = 4$

Lesson 4 Factorizing the difference of two squares

You know that : $(a + b)(a - b) = a^2 - b^2$, therefore factorizing the expression $a^2 - b^2$

is : $a^2 - b^2 = (a + b)\ (a - b)$

i.e. The difference of two squares of two quantities = the sum of the two quantities × the difference of the two quantities

- **The expression** $a^2 - b^2$ is the difference of the squares a^2 , b^2
- **The expression** $(a + b)$ is the sum of $\sqrt{a^2}$, $\sqrt{b^2}$
- **The expression** $(a - b)$ is the difference of $\sqrt{a^2}$, $\sqrt{b^2}$

Example 1 **Factorize each of the following :**

1. $x^2 - 25$
2. $x^2 - 9y^2$
3. $49x^4 - 1$
4. $\frac{1}{9}a^2 - \frac{1}{4}$

Solution

1. $x^2 - 25 = \left(\sqrt{x^2} + \sqrt{25}\right)\left(\sqrt{x^2} - \sqrt{25}\right) = (x + 5)(x - 5)$
2. $x^2 - 9y^2 = \left(\sqrt{x^2} + \sqrt{9y^2}\right)\left(\sqrt{x^2} - \sqrt{9y^2}\right) = (x + 3y)(x - 3y)$
3. $49x^4 - 1 = \left(\sqrt{49x^4} + \sqrt{1}\right)\left(\sqrt{49x^4} - \sqrt{1}\right) = (7x^2 + 1)(7x^2 - 1)$
4. $\frac{1}{9}a^2 - \frac{1}{4} = \left(\sqrt{\frac{1}{9}a^2} + \sqrt{\frac{1}{4}}\right)\left(\sqrt{\frac{1}{9}a^2} - \sqrt{\frac{1}{4}}\right) = \left(\frac{1}{3}a + \frac{1}{2}\right)\left(\frac{1}{3}a - \frac{1}{2}\right)$

Example 2 Factorize each of the following :

1. $2x^2 - 18$
2. $x^3 - 64x$
3. $\frac{1}{2}x^2 - 2$
4. $16x^4 - 81$

Solution

1. $2x^2 - 18 = 2(x^2 - 9) = 2(x - 3)(x + 3)$
2. $x^3 - 64x = x(x^2 - 64) = x(x - 8)(x + 8)$
3. $\frac{1}{2}x^2 - 2 = \frac{1}{2}(x^2 - 4) = \frac{1}{2}(x - 2)(x + 2)$
4. $16x^4 - 81 = (4x^2 + 9)(4x^2 - 9) = (4x^2 + 9)(2x - 3)(2x + 3)$

Example 3 Use factorization to get the value of each of the following easily :

1. $(25)^2 - (15)^2$
2. $(1.6)^2 - (1.4)^2$
3. $(99)^2 - 1$
4. 52×48

Solution

1. $(25)^2 - (15)^2 = (25 - 15)(25 + 15) = 10 \times 40 = 400$
2. $(1.6)^2 - (1.4)^2 = (1.6 - 1.4)(1.6 + 1.4) = 0.2 \times 3 = 0.6$
3. $(99)^2 - 1 = (99 + 1)(99 - 1) = 100 \times 98 = 9800$
4. $52 \times 48 = (50 + 2)(50 - 2) = (50)^2 - (2)^2 = 2500 - 4 = 2496$

Example 4 Factorize each of the following :

1. $25a^2(2a - b) - 16(2a - b)$
2. $(x + y)^2 - 9$

Solution

1. $25a^2(2a - b) - 16(2a - b) = (2a - b)(25a^2 - 16)$

   $= (2a - b)(5a - 4)(5a + 4)$
2. $(x + y)^2 - 9 = [(x + y) + 3][(x + y) - 3] = (x + y + 3)(x + y - 3)$

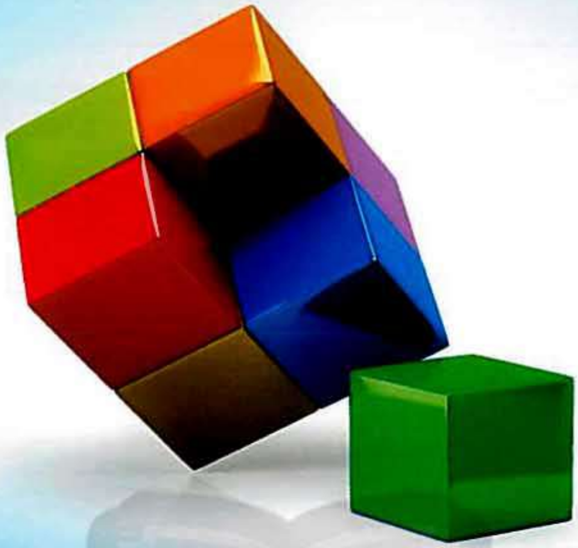
TRY *by yourself*

Factorize each of the following :

1. $x^2 - 16$
2. $4a^2 - 25b^2$
3. $18x^2 - 50y^2$
4. $16x^2(x + y) - y^2(x + y)$

Answers *of try by yourself*

1. $(x - 4)(x + 4)$
2. $(2a - 5b)(2a + 5b)$
3. $2(3x - 5y)(3x + 5y)$
4. $(x + y)(4x - y)(4x + y)$



Lesson 5 Factorizing the sum and difference of two cubes

First Factorizing the sum of two cubes

• You know that :

$$(a + b)(a^2 - ab + b^2) = a(a^2 - ab + b^2) + b(a^2 - ab + b^2)$$

$$= a^3 - a^2 b + ab^2 + a^2 b - ab^2 + b^3 = a^3 + b^3$$

The expression : $a^3 + b^3$ is the sum of the two cubes a^3 and b^3

Generally

$$a^3 \oplus b^3 = (a \overset{\text{same sign}}{\oplus} b)(a^2 \overset{\text{opposite sign}}{\ominus} ab + b^2)$$

i.e. The sum of the two cubes of two quantities = (First ⊕ Second) × (the square of the first ⊖ First × Second ⊕ the square of the second)

For example: $x^3 + 8 = x^3 + 2^3 = (x + 2)(x^2 - x \times 2 + 2^2)$

$$= (x + 2)(x^2 - 2x + 4)$$

Unit 1

Second Factorizing the difference between two cubes

• You know that :

$$(a - b)(a^2 + ab + b^2) = a(a^2 + ab + b^2) - b(a^2 + ab + b^2)$$
$$= a^3 + a^2 b + ab^2 - a^2 b - ab^2 - b^3 = a^3 - b^3$$

The expression : $a^3 - b^3$ is the difference between the two cubes a^3 and b^3

Generally

opposite sign, same sign

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

i.e. The difference between the two cubes of two quantities = (First − Second) × (the square of the first + First × Second + the square of the second)

For example: $x^3 - 27 = x^3 - 3^3$
$$= (x - 3)(x^2 + x \times 3 + 3^2)$$
$$= (x - 3)(x^2 + 3x + 9)$$

Example 1 **Factorize each of the following perfectly :**

1. $8x^3 + 125$
2. $27a^3 - b^3$
3. $8x^3 + \frac{1}{8}$
4. $a^6 - 64b^3$
5. $40x^4 - 5x$
6. $(x + y)^3 + x^3$

Solution

1. $8x^3 + 125 = (2x)^3 + (5)^3 = (2x + 5)\left((2x)^2 - 5 \times 2x + 5^2\right)$
$= (2x + 5)(4x^2 - 10x + 25)$

2. $27a^3 - b^3 = (3a)^3 - b^3 = (3a - b)\left((3a)^2 + 3a \times b + b^2\right)$
$= (3a - b)(9a^2 + 3ab + b^2)$

3. $8x^3 + \frac{1}{8} = (2x)^3 + \left(\frac{1}{2}\right)^3 = \left(2x + \frac{1}{2}\right)\left((2x)^2 - 2x \times \frac{1}{2} + \left(\frac{1}{2}\right)^2\right)$
$= \left(2x + \frac{1}{2}\right)\left(4x^2 - x + \frac{1}{4}\right)$

4. $a^6 - 64b^3 = (a^2)^3 - (4b)^3 = (a^2 - 4b)\left((a^2)^2 + a^2 \times 4b + (4b)^2\right)$
$= (a^2 - 4b)(a^4 + 4a^2 b + 16b^2)$

5 $40x^4 - 5x = 5x(8x^3 - 1)$ (taking out the H.C.F.)

$= 5x(2x - 1)(4x^2 + 2x + 1)$

6 $(x + y)^3 + x^3 = \left((x + y) + x\right)\left((x + y)^2 - x(x + y) + x^2\right)$

$= (2x + y)(x^2 + 2xy + y^2 - x^2 - xy + x^2)$

$= (2x + y)(x^2 + xy + y^2)$

TRY by yourself

Factorize each of the following perfectly :

1. $x^3 + 64$
2. $8x^3 - 27$
3. $2x^3 + 16$
4. $54x^4 - 2xy^3$

Example 2 **Factorize the following perfectly :** $x^6 - 64y^6$

Solution

$x^6 - 64y^6 = (x^3 + 8y^3)(x^3 - 8y^3)$ (difference between two squares)

$= (x + 2y)(x^2 - 2xy + 4y^2)(x - 2y)(x^2 + 2xy + 4y^2)$

(the sum and difference between two cubes)

Remark

"If you factorize the expression : $x^6 - 64y^6$ as a difference between two cubes at first , then it is difficult to carry out the factorization perfectly , therefore you should perform the factorization as a difference between two squares at first."

Example 3 If $x + y = 6$, $x^2 - y^2 = 12$, $x^2 + xy + y^2 = 28$

, find the value of : $x^3 - y^3$

Solution

$\because x^2 - y^2 = 12$ $\therefore (x + y)(x - y) = 12$

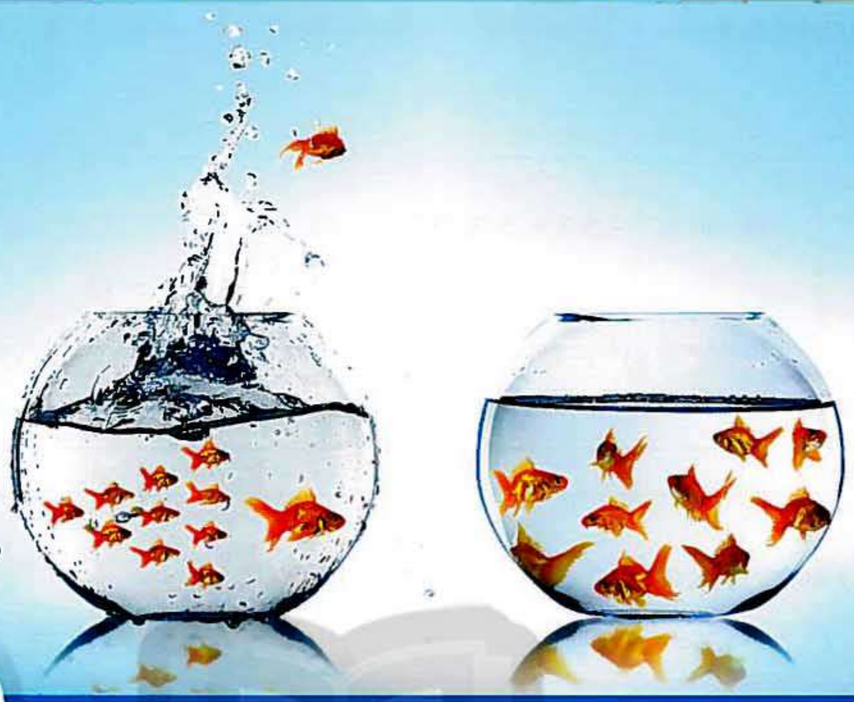
$\because x + y = 6$ $\therefore 6(x - y) = 12$

$\therefore x - y = 2$

$\therefore x^3 - y^3 = (x - y)(x^2 + xy + y^2) = 2 \times 28 = 56$

Answers of try by yourself

1. $(x + 4)(x^2 - 4x + 16)$
2. $(2x - 3)(4x^2 + 6x + 9)$
3. $2(x + 2)(x^2 - 2x + 4)$
4. $2x(3x - y)(9x^2 + 3xy + y^2)$



Lesson 6

Factorizing by grouping

The algebraic expression consisting of four terms can be factorized by one of the following two methods :

The first method

The algebraic expression consisting of four terms is divided into two expressions each of them consisting of two terms , such that you can find a common factor between the two terms , as shown in the following examples.

Example 1 **Factorize :** $a\mathcal{X} + a y + b\mathcal{X} + b y$

Solution

$a\mathcal{X} + a y + b\mathcal{X} + b y = (a\mathcal{X} + a y) + (b\mathcal{X} + b y)$

$= a(\mathcal{X} + y) + b(\mathcal{X} + y) = (\mathcal{X} + y)(a + b)$

Another solution :

$a\mathcal{X} + a y + b\mathcal{X} + b y = (a\mathcal{X} + b\mathcal{X}) + (a y + b y)$ (commutative and associative)

$= \mathcal{X}(a + b) + y(a + b) = (a + b)(\mathcal{X} + y)$

Example 2 **Factorize :** $2a^2 - 2b + ab - 4a$

Solution

If you divide the expression as follows :

$2a^2 - 2b + ab - 4a = (2a^2 - 2b) + (ab - 4a) = 2(a^2 - b) + a(b - 4)$

, then you notice that there is no common factors between $2(a^2 - b)$ and $a(b - 4)$, then you should regroup the main expression by another way as follows :

$2a^2 - 2b + ab - 4a = (2a^2 + ab) + (-2b - 4a)$ (commutative and associative)

$= a(2a + b) - 2(b + 2a)$

$= a(2a + b) - 2(2a + b)$ **Notice that :** $b + 2a = 2a + b$

You notice that there is a common factor which is $(2a + b)$, then you complete factorization by taking out the common factor to be

$2a^2 - 2b + ab - 4a = (2a + b)(a - 2)$

The more you train , the easier you select the proper grouping

Example 3 **Factorize each of the following :**

1. $x^3 - 3x^2 + 27 - 9x$
2. $x^2 - 4y^2 - 5x + 10y$

Solution

1. $x^3 - 3x^2 + 27 - 9x = (x^3 - 3x^2) + (27 - 9x)$

$= x^2(x - 3) + 9(3 - x)$ **Notice that :** $3 - x = -(x - 3)$

$= x^2(x - 3) - 9(x - 3)$

$= (x - 3)(x^2 - 9)$ **Notice that :** $x^2 - 9 = (x - 3)(x + 3)$

$= (x - 3)(x - 3)(x + 3)$

$= (x - 3)^2(x + 3)$

Another solution :

$x^3 - 3x^2 + 27 - 9x = (x^3 + 27) + (-3x^2 - 9x)$

$= (x + 3)(x^2 - 3x + 9) - 3x(x + 3)$

$= (x + 3)(x^2 - 3x + 9 - 3x)$

$= (x + 3)(x^2 - 6x + 9)$ **Notice that :** $x^2 - 6x + 9 = (x - 3)^2$

$= (x + 3)(x - 3)^2$

2. $x^2 - 4y^2 - 5x + 10y = (x^2 - 4y^2) + (-5x + 10y)$

$= (x - 2y)(x + 2y) - 5(x - 2y)$

$= (x - 2y)(x + 2y - 5)$

Unit 1

Example 4 **Factorize :** $12x^3 - 8x^2 + 18x^2y - 12xy$

Solution

Notice that $2x$ is a common factor among the terms of the expression , therefore start firstly by taking out the H.C.F. , then divide the expression as follows :

$12x^3 - 8x^2 + 18x^2y - 12xy$

$= 2x(6x^2 - 4x + 9xy - 6y)$ (taking out the H.C.F.)

$= 2x[(6x^2 - 4x) + (9xy - 6y)]$

$= 2x[2x(3x - 2) + 3y(3x - 2)]$

$= 2x(3x - 2)(2x + 3y)$

TRY by yourself **1**

Factorize each of the following :

1. $5b + xa + 5a + xb$
2. $x^3 - x^2 - 9x + 9$
3. $x^2 - y^2 + 5x - 5y$

The Second method

The algebraic expression consisting of four terms is divided into a trinomial (must be perfect square) and a monomial (must be perfect square also) , such that the main expression should be factorized as a difference between two squares , and the following example shows that.

Example 5 **Factorize each of the following :**

1. $x^2 - 10xy + 25y^2 - 36$
2. $x^2 + 9y^2 - 25 + 6xy$

Solution

1. $x^2 - 10xy + 25y^2 - 36 = (x^2 - 10xy + 25y^2) - 36$

$= (x - 5y)^2 - (6)^2$

$= (x - 5y - 6)(x - 5y + 6)$

2 $x^2 + 9y^2 - 25 + 6xy = (x^2 + 6xy + 9y^2) - 25$

$= (x + 3y)^2 - (5)^2$

$= (x + 3y - 5)(x + 3y + 5)$

TRY 2 *by yourself*

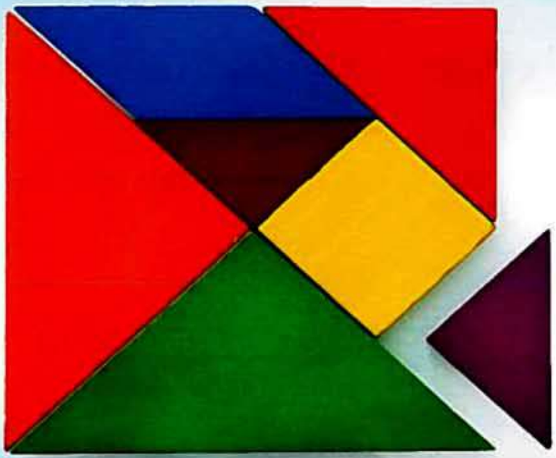
Factorize each of the following :

1. $x^2 - 2xy + y^2 - c^2$
2. $16x^2 - a^2 + 6ab - 9b^2$

Answers *of try by yourself*

1 1. $(a + b)(x + 5)$ 2. $(x - 1)(x - 3)(x + 3)$ 3. $(x - y)(x + y + 5)$

2 1. $(x - y - c)(x - y + c)$ 2. $(4x - a + 3b)(4x + a - 3b)$



Lesson 7

Factorizing by completing the square

• You know that the perfect square trinomial has the following properties :

1 The first term is a perfect square.

2 The third term is a perfect square.

3 The middle term $= \pm 2 \times \sqrt{\text{the first term}} \times \sqrt{\text{the third term}}$

, and it is factorized in the form :

$$\left(\sqrt{\text{the first term}} \pm \sqrt{\text{the third term}}\right)^2$$

* There are some expressions which are not perfect squares but we can complete them to be written in the form of :

a perfect square trinomial $-$ a perfect square monomial

, then we factorize them as a difference between two squares.

This method is called : factorization by completing the square.

* This method is used to factorize the expression that consists of at least two terms each of them is a perfect square and the power of the symbol in each of these two terms (if it exists) is 4 or its multiples.

The method of factorization by completing the square :

1. Add to the given expression twice the product of the two square roots of the two perfect square terms and subtract it again not to change the main expression.
2. Using the commutative and associative properties , rewrite the expression after ordering its terms to get the form :

   a perfect square trinomial − a perfect square monomial

3. Factorize the resultant expression as a difference between two squares.
4. If it is possible , you should factorize the resultant expressions (resultant factors) in order that the factorization is perfect.

The following examples show the previous steps.

Example 1 **Factorize each of the following expressions :**

1 $4x^4 + y^4$ 2 $x^8 - 16$

Solution

1 Add to the given expression : $2 \times \sqrt{4x^4} \times \sqrt{y^4}$ i.e. $4x^2y^2$

, then subtract it again in order not to change the main expression.

$\therefore 4x^4 + y^4 = 4x^4 + y^4 + (4x^2y^2 - 4x^2y^2)$

$= (4x^4 + 4x^2y^2 + y^4) - 4x^2y^2$ (commutative and associative properties)

A perfect square trinomial − A perfect square monomial

$= (2x^2 + y^2)^2 - (2xy)^2$

$= (2x^2 + y^2 - 2xy)(2x^2 + y^2 + 2xy)$ (difference between two squares)

2 $\because x^8 - 16 = (x^4 - 4)(x^4 + 4)$ (difference between two squares) (1)

, $\because (x^4 - 4)$ can be factorized as a difference between two squares as follows : $x^4 - 4 = (x^2 - 2)(x^2 + 2)$ (2)

, $\because (x^4 + 4)$ can be factorized by completing the square as follows :

Add : $2 \times \sqrt{x^4} \times \sqrt{4}$ i.e. $4x^2$, then subtract it again

$\therefore x^4 + 4 = x^4 + 4 + 4x^2 - 4x^2$

$= (x^4 + 4x^2 + 4) - 4x^2$ (commutative and associative properties)

A perfect square trinomial − A perfect square monomial

$= (x^2 + 2)^2 - (2x)^2 = (x^2 + 2 - 2x)(x^2 + 2 + 2x)$

(difference between two squares) (3)

From (1) , (2) and (3) :

$\therefore x^8 - 16 = (x^2 - 2)(x^2 + 2)(x^2 - 2x + 2)(x^2 + 2x + 2)$

Example 2 **Factorize each of the following :**

1. $x^4 + x^2 y^2 + y^4$
2. $x^4 - 19x^2 y^2 + 9y^4$
3. $27x^4 - 30x^2 y^2 + 3y^4$

Solution

1. Add : $2 \times \sqrt{x^4} \times \sqrt{y^4}$ i.e. $2x^2 y^2$, then subtract it again

$\therefore x^4 + x^2 y^2 + y^4 = x^4 + x^2 y^2 + y^4 + 2x^2 y^2 - 2x^2 y^2$

$= (x^4 + 2x^2 y^2 + y^4) + (x^2 y^2 - 2x^2 y^2)$

(commutative and associative properties)

$= (x^4 + 2x^2 y^2 + y^4) - x^2 y^2$

A perfect square trinomial − A perfect square monomial

$= (x^2 + y^2)^2 - (xy)^2 = (x^2 + y^2 - xy)(x^2 + y^2 + xy)$

2. Add : $2 \times \sqrt{x^4} \times \sqrt{9y^4}$ i.e. $6x^2 y^2$, then subtract it again

$\therefore x^4 - 19x^2 y^2 + 9y^4$

$= x^4 - 19x^2 y^2 + 9y^4 + 6x^2 y^2 - 6x^2 y^2$

$= (x^4 + 6x^2 y^2 + 9y^4) + (-19x^2 y^2 - 6x^2 y^2)$

$= (x^4 + 6x^2 y^2 + 9y^4) - 25x^2 y^2$

A perfect square trinomial − A perfect square monomial

$= (x^2 + 3y^2)^2 - (5xy)^2 = (x^2 + 3y^2 - 5xy)(x^2 + 3y^2 + 5xy)$

3 $\because 27x^4 - 30x^2y^2 + 3y^4 = 3(9x^4 - 10x^2y^2 + y^4)$

The expression : $9x^4 - 10x^2y^2 + y^4$ can be factorized by completing the square as follows :

Add : $2 \times \sqrt{9x^4} \times \sqrt{y^4}$ i.e. $6x^2y^2$, then subtract it again

$\therefore 9x^4 - 10x^2y^2 + y^4$

$= 9x^4 - 10x^2y^2 + y^4 + 6x^2y^2 - 6x^2y^2$

$= (9x^4 + 6x^2y^2 + y^4) + (-10x^2y^2 - 6x^2y^2)$

$= (9x^4 + 6x^2y^2 + y^4) - 16x^2y^2$

A perfect square trinomial − A perfect square monomial

$= (3x^2 + y^2)^2 - (4xy)^2 = (3x^2 + y^2 - 4xy)(3x^2 + y^2 + 4xy)$

$= (3x^2 - 4xy + y^2)(3x^2 + 4xy + y^2)$

$= (3x - y)(x - y)(3x + y)(x + y)$

$\therefore 27x^4 - 30x^2y^2 + 3y^4 = 3(3x - y)(x - y)(3x + y)(x + y)$

Another solution :

$27x^4 - 30x^2y^2 + 3y^4 = 3(9x^4 - 10x^2y^2 + y^4)$

$= 3(9x^2 - y^2)(x^2 - y^2)$

$= 3(3x - y)(3x + y)(x - y)(x + y)$

$(9x^2 - y^2)$

$(x^2 - y^2)$

TRY by yourself

Factorize each of the following perfectly :

1. $4x^4 + 1$
2. $x^4 + 64y^4$
3. $36x^4 + 51x^2 + 25$

Answers of try by yourself

1. $(2x^2 + 1 - 2x)(2x^2 + 1 + 2x)$
2. $(x^2 + 8y^2 - 4xy)(x^2 + 8y^2 + 4xy)$
3. $(6x^2 + 5 - 3x)(6x^2 + 5 + 3x)$

SUMMARY

Factorizing the algebraic expressions

To factorize an algebraic expression , do as follows

1. Take out the H.C.F. if it exists.
2. If the algebraic expression is formed from two terms only, then the factorization will be difference between two squares or difference between two cubes or sum of two cubes or by completing the square.
   - The difference between two squares : $x^2 - y^2 = (x - y)(x + y)$
   - The difference between two cubes : $x^3 \ominus y^3 = (x - y)(x^2 \oplus xy + y^2)$
   - The sum of two cubes : $x^3 \oplus y^3 = (x + y)(x^2 \ominus xy + y^2)$
3. If the algebraic expression is formed from three terms (trinomial expression), you should arrange the terms of the expression descendingly or ascendingly according to the powers of any symbol in it (It is better to arrange the expression descendingly).
   - There are two cases :

   First : The trinomial is a perfect square trinomial , that is

   if the middle term $= \pm 2 \times \sqrt{\text{The first term}} \times \sqrt{\text{The third term}}$

   Then the expression is factorized as follows :

   $\left(\sqrt{\text{The first term}} \ \boxed{\text{The sign of the middle term}} \ \sqrt{\text{The third term}}\right)^2$

   Second : The trinomial is not a perfect square , in this case it will be factorized as a trinomial by scissors method or by completing the square.
4. If the algebraic expression is formed from 4 terms , then use the factorizing by grouping.

   i.e. Dividing the expression into two groups according to each problem.

> **Remark**
>
> The factorization should be done perfectly
>
> **i.e.** No factor in the result can be factorized more.

Answer the general exercise on factorizing the algebraic expressions in the free part (1)

Lesson 8 Solving quadratic equations in one variable algebraically

Remember that

- The equation is a mathematical sentence consisting of one variable (or more) and including the equality relation.
- The degree of the equation is the greatest power of the powers of its terms.

For example:

* $5x + 2 = 7$ is an equation of the first degree in one variable x
* $x^2 - 5x - 6 = 0$ is an equation of the second degree in one variable x
* $2x + 3y = 7$ is an equation of the first degree in two variables x and y

- Solving the equation means finding the values of the variable (unknown) which satisfy the equation , and each value of them is called "a root of the equation".

Definition

Any equation can be written at the form : $a x^2 + b x + c = 0$, $a \neq 0$ is an equation of the second degree in one variable and is called "**a quadratic equation**".

For example:

- $x^2 + 4x - 12 = 0$ is a quadratic equation in x
- $2y^2 + 5y = 10$ is a quadratic equation in y
- $-4b^2 - 9 = 0$ is a quadratic equation in b

Notice that : Each of the previous equations is an equation of the second degree in one variable.

Unit 1

Fact

If a and b are two real numbers and a × b = zero , then a = 0 or b = 0

For example:

- If $x(x-3)=0$, then :

either $x=0$

or $x-3=0$, then $x=3$

- If $(x+2)(3x-5)=0$, then :

either $x+2=0$, then $x=-2$

or $3x-5=0$, then $x=\frac{5}{3}$

Solving the quadratic equation in one variable

To solve the quadratic equation in one variable , do as follows :

1. Put the equation in the standard form : $ax^2+bx+c=0$
2. Factorize the expression in the left hand side into two factors.
3. Use the previous fact to get the two roots of the equation.
4. Verify your answer by substituting by each value of x in the main equation.

For example:

To solve the equation $x^2+4x=12$ in $\mathbb{R}$, do as follows :

1. Put the equation in the standard form : $ax^2+bx+c=0$

$\therefore x^2+4x=12$ $\therefore x^2+4x-12=0$

2. Factorize the expression in the left hand side into two factors :

$(x-2)(x+6)=0$ (factorizing a trinomial)

3. Use the pervious fact to get the two roots of the equation :

Either $x-2=0$, then $x=2$

or $x+6=0$, then $x=-6$

4. Verify your answer by substituting by each value of x in the equation $x^2+4x=12$

- At $x=2$: $\therefore x^2+4x=2^2+4\times 2=4+8=12$

$\therefore$ $x=2$ is a right solution ✓ .

- At $x=-6$: $\therefore x^2+4x=(-6)^2+4\times(-6)=36-24=12$

$\therefore$ $x=-6$ is a right solution ✓ .

Example 1 **Find in $\mathbb{R}$ the solution set of each of the following equations :**

1. $x^2 - 5x - 6 = 0$
2. $2x^2 + 7x = 0$
3. $x^2 - 6x = -9$
4. $(x+2)^2 = 25$
5. $x^2 + 4 = 0$

Solution

1 $\because x^2 - 5x - 6 = 0$

$\therefore (x-6)(x+1) = 0$ (factorization of a trinomial)

$\therefore$ Either $x - 6 = 0$, then $x = 6$ or $x + 1 = 0$, then $x = -1$

$\therefore$ The S.S. $= \{6, -1\}$

2 $\because 2x^2 + 7x = 0$ $\therefore x(2x+7) = 0$ (H.C.F.)

$\therefore$ Either $x = 0$ or $2x + 7 = 0$, then $2x = -7$

$\therefore x = -\frac{7}{2}$ $\therefore$ The S.S. $= \left\{0, -\frac{7}{2}\right\}$

3 $\because x^2 - 6x = -9$ $\therefore x^2 - 6x + 9 = 0$

$\therefore (x-3)^2 = 0$ (perfect square trinomial) $\therefore x - 3 = 0$

$\therefore x = 3$ $\therefore$ The S.S. $= \{3\}$

4 $\because (x+2)^2 = 25$ $\therefore x^2 + 4x + 4 = 25$

$\therefore x^2 + 4x - 21 = 0$

$\therefore (x+7)(x-3) = 0$ (factorization of a trinomial)

$\therefore$ Either $x + 7 = 0$, then $x = -7$ or $x - 3 = 0$, then $x = 3$

$\therefore$ The S.S. $= \{-7, 3\}$

Another solution :

$\because (x+2)^2 = 25$ $\therefore (x+2)^2 - 25 = 0$

Using factorization of the difference between two squares.

$\therefore (x+2-5)(x+2+5) = 0$ $\therefore (x-3)(x+7) = 0$

$\therefore$ Either $x - 3 = 0$, then $x = 3$ or $x + 7 = 0$, then $x = -7$

$\therefore$ The S.S. $= \{3, -7\}$

Another solution :

$\because (x+2)^2 = 25$ $\therefore x + 2 = \pm 5$ $\therefore$ Either $x + 2 = 5$, then $x = 3$

or $x + 2 = -5$, then $x = -7$ $\therefore$ The S.S. $= \{3, -7\}$

5 The equation $x^2 + 4 = 0$ (or $x^2 = -4$) has no solution in $\mathbb{R}$ because there is no integer whose square is negative.

$\therefore$ The S.S. $= \varnothing$

Remark

From the previous example , notice that the quadratic equation has at most two solutions (two roots).

Example 2

Find in $\mathbb{R}$ the S.S. of each of the following equations :

1 $(x - 3)(x + 5) = 20$

2 $x - \frac{2}{x} = \frac{7}{2}$

Solution

1 $\because (x - 3)(x + 5) = 20$

$\therefore x^2 + 2x - 15 = 20$

$\therefore x^2 + 2x - 35 = 0$

$\therefore (x - 5)(x + 7) = 0$

$\therefore$ Either $x - 5 = 0$, then $x = 5$

or $x + 7 = 0$, then $x = -7$

$\therefore$ The S.S. $= \{5 , -7\}$

2 Multiplying the two sides by $2x$ (L.C.M.) of denominators

$\therefore x \times 2x - \frac{2}{x} \times 2x = \frac{7}{2} \times 2x$

$\therefore 2x^2 - 4 = 7x$

$\therefore 2x^2 - 7x - 4 = 0$

$(2x + 1)$, $(x - 4)$

$\therefore (2x + 1)(x - 4) = 0$

$\therefore$ Either $2x + 1 = 0$, then $2x = -1$ $\quad \therefore x = -\frac{1}{2}$

or $x - 4 = 0$, then $x = 4$

$\therefore$ The S.S. $= \{-\frac{1}{2} , 4\}$

TRY 1 by yourself

Find in $\mathbb{R}$ the S.S. of each of the following equations :

1 $x^2 - 5x = 0$

2 $4x^2 = 25$

3 $x(x - 1) = 6$

Remark

It is possible in some cases to get a quadratic equation from factorizing a third or fourth degree equation in one variable , in this case it is possible to solve the equation as in the following example.

Example 3 **Find in $\mathbb{R}$ the S.S. of each of the following equations :**

1 $3x^3 = 12x$ 2 $x^4 - 10x^2 + 9 = 0$

Solution

1 $\because 3x^3 = 12x$ $\therefore 3x^3 - 12x = 0$

$\therefore 3x(x^2 - 4) = 0$ (H.C.F.)

$\therefore$ Either $3x = 0$, then $x = 0$

or $x^2 - 4 = 0$ **i.e.** $(x - 2)(x + 2) = 0$ (difference between two squares)

$\therefore$ Either $x - 2 = 0$, then $x = 2$

or $x + 2 = 0$, then $x = -2$

$\therefore$ The S.S. $= \{0 , 2 , -2\}$

Notice that : The equation of the third degree has three solutions at most in $\mathbb{R}$

2 $\because x^4 - 10x^2 + 9 = 0$

$\therefore (x^2 - 1)(x^2 - 9) = 0$ (factorization of a trinomial)

$\therefore$ Either $x^2 - 1 = 0$ or $x^2 - 9 = 0$

$\therefore (x - 1)(x + 1) = 0$ $\therefore (x - 3)(x + 3) = 0$

$\therefore x - 1 = 0$, then $x = 1$ $\therefore x - 3 = 0$, then $x = 3$

or $x + 1 = 0$, then $x = -1$ or $x + 3 = 0$, then $x = -3$

$\therefore$ The S.S. $= \{1 , -1 , 3 , -3\}$

Notice that : The equation of the fourth degree has four solutions at most in $\mathbb{R}$

TRY 2 by yourself

Find in $\mathbb{R}$ *the S.S. of each of the following equations :*

1 $x^3 - 4x = 0$ 2 $x^4 - 13x^2 + 36 = 0$

Answers *of try by yourself*

1 [1] The S.S. $= \{0 , 5\}$ [2] The S.S. $= \{\frac{5}{2} , -\frac{5}{2}\}$ [3] The S.S. $= \{3 , -2\}$

2 [1] The S.S. $= \{0 , 2 , -2\}$ [2] The S.S. $= \{-2 , 2 , -3 , 3\}$

Lesson 9 Applications on solving quadratic equations in one variable algebraically

- To solve word problems in algebra , translate the sentences into symbols and algebraic expressions. The following table shows some examples for that :

The sentence	The algebraic expression
Half of a number	$\frac{x}{2}$ or $\frac{1}{2}x$
Twice of a number	$2x$
Three times of a number	$3x$
Square of a number	x^2
Twice of the square of a number	$2x^2$
Square of the twice of a number	$(2x)^2 = 4x^2$
The additive inverse of a number	$-x$
The multiplicative inverse of (a number) (not equal to zero)	$\frac{1}{x}$
Two numbers , one of them exceeds the other by 5 or one of them is less than the other by 5 or their difference = 5	• The first number = x • The second number = $x + 5$
The sum of two numbers equals 5	• The first number = x • The second number = $5 - x$
Two numbers , one of them is more than twice the other by 5	• The first number = x • The second number = $2x + 5$

Three consecutive integers	• The first number = x • The second number = $x + 1$ • The third number = $x + 2$
Three even (or odd) consecutive numbers	• The first number = x • The second number = $x + 2$ • The third number = $x + 4$
Two numbers , the ratio between them is 2 : 3	• The first number = $2x$ • The second number = $3x$
The age of a man now is x years	• His age after 4 years = $x + 4$ • His age 3 years ago = $x - 3$ • The square of his age 6 years ago = $(x - 6)^2$
A rectangle whose length exceeds its width by 5 cm.	• The width = x cm. , the length = $(x + 5)$ cm. • Its perimeter = $(x + x + 5) \times 2 = (4x + 10)$ cm. • Its area = $x(x + 5) = (x^2 + 5x)$ cm^2.
A square of side length = x cm.	• Its perimeter = $4x$ cm. • Its area = x^2 cm^2.

Example 1 **A positive integer whose square exceeds its double by 8 Find the number.**

Solution Let the number be x

$\therefore$ Its square = x^2 and its double = $2x$

, $\because$ Its square exceeds its double by 8

$\therefore \quad x^2 \quad - \quad 2x \quad = \quad 8$

$\therefore x^2 - 2x - 8 = 0$ $\therefore (x + 2)(x - 4) = 0$

$\therefore$ Either $x + 2 = 0$, then $x = -2$ (refused because the number is positive)

or $x - 4 = 0$, then $x = 4$ $\therefore$ The number is 4

Checking the solution :

$\because$ The number is 4

$\therefore$ Its square = 16 and its double = 8 $\therefore$ Its square – its double = 16 – 8 = 8

Example 2 **The length of a rectangle exceeds its width by 5 cm. If its area = 14 cm², find its length and its width.**

Solution Let the width be x cm.

, ∵ The length exceeds the width by 5 cm. ∴ The length $= (x + 5)$ cm.

, ∵ The area = 14 cm². ∴ $x(x + 5) = 14$

∴ $x^2 + 5x = 14$ ∴ $x^2 + 5x - 14 = 0$

∴ $(x + 7)(x - 2) = 0$

∴ Either $x + 7 = 0$, then $x = -7$ (refused because the lengths are always positive)

or $x - 2 = 0$, then $x = 2$

∴ The width = 2 cm. and the length = 2 + 5 = 7 cm.

Try to check the solution.

Example 3 **Three consecutive positive even numbers, the square of the middle number exceeds the sum of the two other numbers by 8 Find these numbers.**

Solution Let the numbers be x and $x + 2$ and $x + 4$

∵ The square of the middle number exceeds the sum of the others by 8

∴ $(x + 2)^2 - (x + x + 4) = 8$

∴ $x^2 + 4x + 4 - 2x - 4 = 8$

∴ $x^2 + 2x - 8 = 0$

∴ $(x + 4)(x - 2) = 0$

∴ Either $x + 4 = 0$, then $x = -4$ (refused because the numbers are positive)

or $x - 2 = 0$, then $x = 2$

∴ The first number = 2

, the middle number = 4

, the third number = 6

Try to solve the example by assuming that the numbers are $x - 2$, x and $x + 2$

Example 4 **If the age of Nabeel now is twice the age of Nader. 2 years ago, the difference between the squares of their ages was 15 , find the age of each of them now.**

Solution

	Now	**2 years ago**
Nader	x	$x-2$
Nabeel	$2x$	$2x-2$

$\therefore (2x-2)^2-(x-2)^2=15$

Using the factorization of the difference between two squares :

$\therefore (2x-2+x-2)(2x-2-x+2)=15$

$\therefore (3x-4)\,x=15$

$\therefore 3x^2-4x-15=0$

$\therefore (3x+5)(x-3)=0$

$\therefore$ Either $3x+5=0$, then $x=-\frac{5}{3}$ (refused)

or $x-3=0$, then $x=3$

$\therefore$ The age of Nader now = 3 years ,

and the age of Nabeel now = 6 years.

TRY *by yourself*

An integer , if you add it to its square , the result is 56 Find this number.

Answers *of try by yourself*

The number is : – 8 or 7

UNIT 2 Non-negative and negative integer powers in $\mathbb{R}$

Lessons of the unit :

1. Non-negative and negative integer powers in $\mathbb{R}$
2. Solving the exponential equations in $\mathbb{R}$
3. Operations on integer powers

Unit Objectives :

By the end of this unit, student should be able to :

- apply what have been studied before about powers in $\mathbb{Z}$
- recognize the laws of non-negative integer powers in $\mathbb{R}$
- recognize the negative power of a real number not equal to zero
- generalize the laws of non-negative integer powers on the negative integer powers in $\mathbb{R}$
- solve the exponential equations in $\mathbb{R}$
- perform mathematical operations on integer powers.
- use the calculator to verify the results.
- apply the laws of powers to solve some real-life problems

Lesson 1

Non-negative and negative integer powers in $\mathbb{R}$

Non-negative integer powers in $\mathbb{R}$

If $a \in \mathbb{R}$, $n \in \mathbb{Z}^+$, then $a^n = a \times a \times a \times a \times a \times \cdots \times a$ where a is repeated as a factor n times.

- a^n is read as : a to the power n or the $n^{\underline{th}}$ power of the number a and the number a is called the base.

For example:

- $3^3 = (3 \times 3) \times 3 = 9 \times 3 = 27$
- $(-2)^4 = (-2 \times -2) \times (-2 \times -2) = 4 \times 4 = 16$

Remarks

1. If $a \in \mathbb{R}^*$ (The set of non-zero real numbers) , then : $a^0 = 1$

For example:

- $\left(\sqrt{5}\right)^0 = 1$
- $\left(-2\sqrt{3}\right)^0 = 1$

2. From the repeated multiplication , you knew that :

- $(-4)^2 = 16$, $4^2 = 16$ «**Note that :** 2 is an even number»
- $(-4)^3 = -64$, $-(4)^3 = -64$ «**Note that :** 3 is an odd number»

i.e. $(-a)^n = a^n$ if n is an even number

but $(-a)^n = -a^n$ if n is an odd number

Negative integer powers in $\mathbb{R}$

If a is a real number , $a \neq 0$ and n is a positive integer , then :

$$a^{-n} = \frac{1}{a^n} \text{ and } a^n = \frac{1}{a^{-n}}$$

For example:

- $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$
- $\frac{1}{2^{-3}} = 2^3 = 8$

Remarks

1. For every $a \in \mathbb{R}^*$, $n \in \mathbb{Z}^+$, then $a^n \times a^{-n} = a^n \times \frac{1}{a^n} = 1$ (the multiplicative neutral)

i.e. a^n and a^{-n} are the multiplicative inverse of each other.

2. For every $a \in \mathbb{R}^*$, $b \in \mathbb{R}^*$ and $n \in \mathbb{Z}^+$, then $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

For example: $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

Example 1 Find in the simplest form each of the following :

1. $(\sqrt{5})^3$
2. $(\sqrt{3})^{-2}$
3. $\left(\frac{2}{5}\right)^{-2}$
4. $(0.1)^{-2}$
5. $\frac{1}{(-\sqrt{2})^{-3}}$
6. $\frac{2^{-1}}{3^{-1} \times 4}$

Solution

1. $(\sqrt{5})^3 = (\sqrt{5} \times \sqrt{5}) \times \sqrt{5} = 5\sqrt{5}$
2. $(\sqrt{3})^{-2} = \frac{1}{(\sqrt{3})^2} = \frac{1}{3}$
3. $\left(\frac{2}{5}\right)^{-2} = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$
4. $(0.1)^{-2} = \left(\frac{1}{10}\right)^{-2} = (10)^2 = 100$
5. $\frac{1}{(-\sqrt{2})^{-3}} = (-\sqrt{2})^3 = -(\sqrt{2})^3 = -2\sqrt{2}$
6. $\frac{2^{-1}}{3^{-1} \times 4} = \frac{\frac{1}{2}}{\frac{1}{3} \times 4} = \frac{\frac{1}{2}}{\frac{4}{3}} = \frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

Unit 2

Laws of non-negative and negative integer powers in $\mathbb{R}$

If a and b are two real numbers , m and n are two integers , except the cases in which the denominator = 0 and the cases in which both the base and the power = 0 , then :

The law	Example	The explain
1 $a^m \times a^n = a^{m+n}$	$4^3 \times 4^2 = 4^{3+2} = 4^5$	When you multiply numbers of the same base , you add the powers.
2 $\frac{a^m}{a^n} = a^{m-n}$	$\frac{3^6}{3^2} = 3^{6-2} = 3^4$	When you divide numbers of the same base , you subtract the powers.
3 $(ab)^n = a^n b^n$	$(3 \times 4)^2 = 3^2 \times 4^2$	The power of a product of two numbers is distributed over the two numbers.
4 $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3}$	The power on a fraction is distributed over the numerator and the denominator.
5 $(a^m)^n = (a^n)^m = a^{mn}$	$(4^2)^3 = (4^3)^2 = 4^{3 \times 2} = 4^6$	When you raise a number , raised to a power , to another power , you multiply the two powers.

Example 2 Find in the simplest form each of the following :

1 $(\sqrt{3})^7 \times (\sqrt{3})^{-9} \times (\sqrt{3})^4$

2 $\frac{\sqrt{2}}{(\sqrt{2})^{-2}}$

3 $(2\sqrt{5} \times \sqrt{3})^{-2}$

4 $\left(\frac{3\sqrt{2}}{\sqrt{3}}\right)^4$

5 $((\sqrt{3})^{-2})^2$

Solution

1 $(\sqrt{3})^7 \times (\sqrt{3})^{-9} \times (\sqrt{3})^4 = (\sqrt{3})^{7+(-9)+4}$

$= (\sqrt{3})^2 = 3$

2 $\frac{\sqrt{2}}{(\sqrt{2})^{-2}} = (\sqrt{2})^{1-(-2)}$

$= (\sqrt{2})^3 = 2\sqrt{2}$

Another solution by using the definition of the negative power :

$$\frac{\sqrt{2}}{(\sqrt{2})^{-2}} = \sqrt{2} \times (\sqrt{2})^2 = (\sqrt{2})^3 = 2\sqrt{2}$$

3 $\left(2\sqrt{5}\times\sqrt{3}\right)^{-2}=2^{-2}\times\left(\sqrt{5}\right)^{-2}\times\left(\sqrt{3}\right)^{-2}$

$$=\frac{1}{2^2}\times\frac{1}{\left(\sqrt{5}\right)^2}\times\frac{1}{\left(\sqrt{3}\right)^2}$$

$$=\frac{1}{4}\times\frac{1}{5}\times\frac{1}{3}=\frac{1}{60}$$

4 $\left(\frac{3\sqrt{2}}{\sqrt{3}}\right)^4=\frac{\left(3\sqrt{2}\right)^4}{\left(\sqrt{3}\right)^4}=\frac{3^4\times\left(\sqrt{2}\right)^4}{\left(\sqrt{3}\right)^4}$

$$=\frac{81\times 4}{9}=36$$

Another solution :

$\because 3=\sqrt{3}\times\sqrt{3}$

$\therefore \left(\frac{3\sqrt{2}}{\sqrt{3}}\right)^4=\left(\frac{\sqrt{3}\times\sqrt{3}\times\sqrt{2}}{\sqrt{3}}\right)^4=\left(\sqrt{3}\right)^4\times\left(\sqrt{2}\right)^4$

$$=9\times 4=36$$

5 $\left(\left(\sqrt{3}\right)^{-2}\right)^2=\left(\sqrt{3}\right)^{-2\times 2}=\left(\sqrt{3}\right)^{-4}$

$$=\frac{1}{\left(\sqrt{3}\right)^4}=\frac{1}{9}$$

Example 3 **Simplify each of the following to the simplest form :**

1 $\frac{\left(\sqrt{5}\right)^7\times\left(\sqrt{5}\right)^{-5}}{\left(\sqrt{5}\right)^{-2}}$

2 $\frac{\sqrt{3}\times\left(2\sqrt{3}\right)^2\times\left(-\sqrt{3}\right)^5}{\left(2\sqrt{3}\right)^4}$

3 $\frac{\left(\sqrt{18}\right)^5\times\left(\sqrt{2}\right)^3}{\left(\sqrt{12}\right)^4}$

4 $\frac{(10)^{-3}\times 0.01}{(10)^{-9}\times(10)^3}$

Solution

1 $\frac{\left(\sqrt{5}\right)^7\times\left(\sqrt{5}\right)^{-5}}{\left(\sqrt{5}\right)^{-2}}=\frac{\left(\sqrt{5}\right)^{7+(-5)}}{\left(\sqrt{5}\right)^{-2}}=\frac{\left(\sqrt{5}\right)^2}{\left(\sqrt{5}\right)^{-2}}$

$$=\left(\sqrt{5}\right)^{2-(-2)}=\left(\sqrt{5}\right)^4=25$$

2 $\dfrac{\sqrt{3}\times\left(2\sqrt{3}\right)^2\times\left(-\sqrt{3}\right)^5}{\left(2\sqrt{3}\right)^4}=\dfrac{\sqrt{3}\times 2^2\times\left(\sqrt{3}\right)^2\times-\left(\sqrt{3}\right)^5}{2^4\times\left(\sqrt{3}\right)^4}$

$=-\left(\sqrt{3}\right)^{1+2+5-4}\times 2^{2-4}$

$=-\left(\sqrt{3}\right)^4\times 2^{-2}=-9\times\dfrac{1}{2^2}=-\dfrac{9}{4}$

3 $\dfrac{\left(\sqrt{18}\right)^5\times\left(\sqrt{2}\right)^3}{\left(\sqrt{12}\right)^4}=\dfrac{\left(3\sqrt{2}\right)^5\times\left(\sqrt{2}\right)^3}{\left(2\sqrt{3}\right)^4}$

$=\dfrac{3^5\times\left(\sqrt{2}\right)^5\times\left(\sqrt{2}\right)^3}{2^4\times\left(\sqrt{3}\right)^4}$

$=\dfrac{3^5\times\left(\sqrt{2}\right)^{5+3}}{2^4\times 3^2}$

$=\dfrac{3^5\times\left(\sqrt{2}\right)^8}{2^4\times 3^2}$

$=\dfrac{3^5\times 2^4}{2^4\times 3^2}=3^{5-2}\times 2^{4-4}$

$=3^3\times 2^0=27\times 1=27$

Remember that

- $\sqrt{18}=\sqrt{9\times 2}=\sqrt{9}\times\sqrt{2}=3\sqrt{2}$
- $\sqrt{12}=\sqrt{4\times 3}=\sqrt{4}\times\sqrt{3}=2\sqrt{3}$

Remember that

- $\left(\sqrt{3}\right)^4=\sqrt{3^4}=3^2$
- $\left(\sqrt{2}\right)^8=\sqrt{2^8}=2^4$

4 $\dfrac{(10)^{-3}\times 0.01}{(10)^{-9}\times(10)^3}=\dfrac{(10)^{-3}\times(10)^{-2}}{10^{-9}\times 10^3}$

$=\dfrac{(10)^{-3-2}}{(10)^{-9+3}}=\dfrac{(10)^{-5}}{(10)^{-6}}=(10)^{-5+6}=(10)^1=10$

Remember that

$0.01=(10)^{-2}$

TRY 1 by yourself

Simplify each of the following to the simplest form :

1 $\left(\sqrt{3}\right)^3\times\left(\sqrt{3}\right)^{-2}\times\sqrt{3}$

2 $\dfrac{4\times 6^3\times 3^{-5}}{2^5\times 3^3}$

3 $\left(\dfrac{2\sqrt{5}}{5\sqrt{2}}\right)^4$

4 $\dfrac{\left(\left(\sqrt{2}\right)^3\right)^2\times\left(\sqrt{8}\right)^3}{\left(\left(\sqrt{2}\right)^5\right)^3}$

Example 4 **Simplify to the simplest form :**

1. $\frac{4^{x} \times 2^{x-3}}{8^{x-2}}$

2. $\frac{2^{5n} \times 3^{2n+1}}{4^{-n} \times 6^{2n+1}}$, then find the value of the result when n = 1

Solution

1. $\frac{4^{x} \times 2^{x-3}}{8^{x-2}} = \frac{2^{2x} \times 2^{x-3}}{2^{3(x-2)}} = \frac{2^{2x} \times 2^{x-3}}{2^{3x-6}}$

$= 2^{2x+x-3-(3x-6)}$

$= 2^{2x+x-3-3x+6} = 2^3 = 8$

2. $\frac{2^{5n} \times 3^{2n+1}}{4^{-n} \times 6^{2n+1}} = \frac{2^{5n} \times 3^{2n+1}}{(2^2)^{-n} \times (2 \times 3)^{2n+1}}$

$= \frac{2^{5n} \times 3^{2n+1}}{2^{-2n} \times 2^{2n+1} \times 3^{2n+1}}$

$= 2^{5n+2n-2n-1} \times 3^{2n+1-2n-1} = 2^{5n-1} \times 3^0 = 2^{5n-1}$

When n = 1 :

$\therefore$ The expression $= 2^{5-1} = 2^4 = 16$

Example 5 **Prove that :** $\frac{(\sqrt{3})^{2-n} \times (15)^{n+2}}{(\sqrt{3})^{-n} \times 3^{n} \times 5^{n+2}} = 27$

Solution

L.H.S. $= \frac{(\sqrt{3})^{2-n} \times (3 \times 5)^{n+2}}{(\sqrt{3})^{-n} \times 3^{n} \times 5^{n+2}}$

$= \frac{(\sqrt{3})^{2-n} \times 3^{n+2} \times 5^{n+2}}{(\sqrt{3})^{-n} \times 3^{n} \times 5^{n+2}}$

$= (\sqrt{3})^{2-n+n} \times 3^{n+2-n} \times 5^{n+2-n-2}$

$= (\sqrt{3})^{2} \times 3^2 \times 5^0 = 3 \times 3^2 \times 1$

$= 3^3 = 27 =$ R.H.S.

Unit 2

TRY 2 by yourself

1 ***Simplify to the simplest form :*** $\frac{3^{2-x} \times (81)^{x}}{3^{x+1} \times 3^{1-x}}$

, then find the numerical value of the result when $x = -1$

2 ***Prove that :*** $\frac{9^{x+1} \times 4^{x}}{6^{2x}} = 9$

Example 6 **If $x = 3$, $y = \sqrt{3}$ and $z = \frac{1}{\sqrt{3}}$, find the value of each of the following in the simplest form :**

1 $(x\,y)^2$ **2** $(x + y)^2$ **3** $\frac{x^{-3}}{y^{-3}}$

4 $(x^{-2}\,y^{4})^{-2}$ **5** $x^2 + (x\,y)^2\,z^2$

Solution

1 $(x\,y)^2 = x^2\,y^2 = 3^2 \times \left(\sqrt{3}\right)^2 = 9 \times 3 = 27$

2 $(x + y)^2 = x^2 + 2\,x\,y + y^2$

$= 3^2 + 2 \times 3 \times \sqrt{3} + \left(\sqrt{3}\right)^2$

$= 9 + 6\sqrt{3} + 3 = 12 + 6\sqrt{3}$

Remember that

- $(a + b)^n \neq a^n + b^n$
- $(a - b)^n \neq a^n - b^n$

3 $\frac{x^{-3}}{y^{-3}} = \left(\frac{x}{y}\right)^{-3} = \left(\frac{3}{\sqrt{3}}\right)^{-3} = \left(\frac{\sqrt{3}}{3}\right)^{3}$

$= \frac{\left(\sqrt{3}\right)^3}{3^3} = \frac{3\sqrt{3}}{27} = \frac{\sqrt{3}}{9}$

4 $(x^{-2}\,y^{4})^{-2} = (x^{-2})^{-2} \times (y^{4})^{-2}$

$= x^{4} \times y^{-8} = \frac{x^4}{y^8} = \frac{3^4}{\left(\sqrt{3}\right)^8} = \frac{3^4}{\sqrt{3^8}} = \frac{3^4}{3^4} = 1$

5 $x^2 + (x\,y)^2\,z^2 = x^2 + (x\,y\,z)^2 = 3^2 + \left(3 \times \sqrt{3} \times \frac{1}{\sqrt{3}}\right)^2$

$= 3^2 + 3^2 = 9 + 9 = 18$

Example 7

1. If $5^{x} = 2$, find the value of : 5^{-x}
2. If $3^{x} = 2$, find the value of : $(27)^{x}$
3. If $2^{x} = 5$, find the value of : 2^{x+2}

Solution

1. $\because 5^{-x} = \frac{1}{5^{x}}$, $\because 5^{x} = 2$ $\therefore 5^{-x} = \frac{1}{2}$
2. $\because (27)^{x} = (3^3)^{x} = (3^{x})^3$, $\because 3^{x} = 2$ $\therefore (27)^{x} = (2)^3 = 8$
3. $\because 2^{x+2} = 2^{x} \times 2^2$, $\because 2^{x} = 5$ $\therefore 2^{x+2} = 5 \times 4 = 20$

TRY 3 *by yourself*

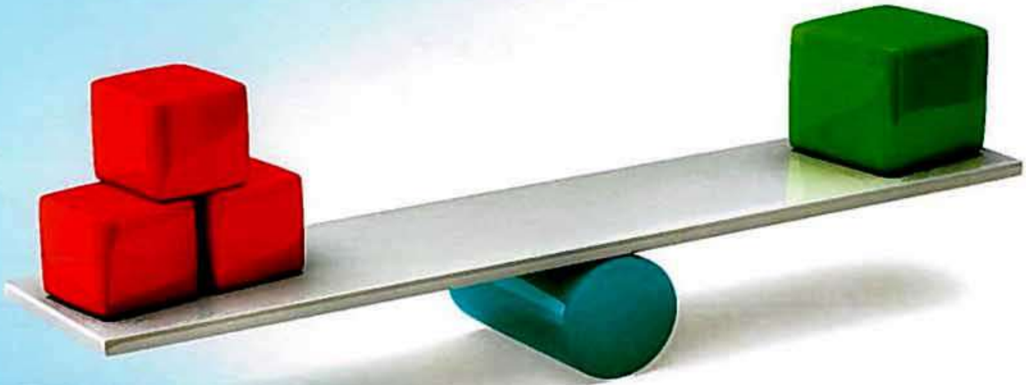
1. If $x = \sqrt{6}$ and $y = \sqrt{3}$, find in the simplest form : $x^4 y^{-4}$
2. If $7^{x} = 5$, find the value of : 7^{-x}
3. If $5^{y} = 9$, find the value of : $(125)^{y}$
4. If $3^{n+1} = 6$, find the value of : 3^{n}

Answers *of try by yourself*

1 (1) 3 (2) $\frac{1}{243}$ (3) $\frac{4}{25}$ (4) 1

2 (1) The expression = 3^{3x} , the numerical value = $\frac{1}{27}$
(2) Prove by yourself.

3 (1) 4 (2) $\frac{1}{5}$ (3) 729 (4) 2



Lesson 2 Solving the exponential equations in $\mathbb{R}$

The exponential equations

The exponential equations are the equations in which the unknown is a power.

Examples for the exponential equations : • $5^{n} = 125$ • $3^{x+4} = 27$

You can solve some of the equations and the exponential equations by using one of the following methods :

The first method

Make the base = the base , then : the power = the power such that : the base ≠ 0 or ± 1

i.e. If a is a real number , m and n are two integers

and $a^m = a^n$, then $m = n$ where : $a \neq 0$, $a \neq \pm 1$

For example:

If $3^n = 9$, then : $3^n = 3^2$

, ∵ The base = the base ∴ The power = the power ∴ $n = 2$

The second method

Make the power = the power , then :

Either the base = the base , if the power is an odd number

or the base = ± the base , if the power is an even number

or the power = zero , if the base ≠ ± the base

i.e. If a and b are two real numbers , m is an integer and $a^m = b^m$, then :

- $a = b$ if m is an odd number. **For example:** If $n^5 = 3^5$, then : $n = 3$
- $a = \pm b$ if m is an even number. **For example:** If $n^2 = 3^2$, then : $n = \pm 3$
- $m = \text{zero}$ if $a \neq \pm b$

For example: If $7^{n-2} = 5^{n-2}$, then : $n - 2 = 0$ $\therefore n = 2$

Example 1 **Find the value of n in each of the following :**

1. $2^{n+5} = 8$
2. $9^{n-1} = \frac{1}{81}$
3. $\left(\frac{3}{5}\right)^{n+2} = \left(2\frac{7}{9}\right)^{-2}$
4. $3^{3n-6} = 5^{3n-6}$
5. $7^{n(n-3)} = 1$
6. $3^{n+2} = n^{n+2}$

Solution

1. $\because 2^{n+5} = 8$ $\therefore 2^{n+5} = 2^3$

$\because$ The base = the base $\therefore$ The power = the power

$\therefore n + 5 = 3$ $\therefore n = -2$

2. $\because 9^{n-1} = \frac{1}{81}$ $\therefore 9^{n-1} = \frac{1}{9^2} = 9^{-2}$

$\because$ The base = the base $\therefore$ The power = the power

$\therefore n - 1 = -2$ $\therefore n = -1$

3. $\because \left(\frac{3}{5}\right)^{n+2} = \left(2\frac{7}{9}\right)^{-2}$ $\therefore \left(\frac{3}{5}\right)^{n+2} = \left(\frac{25}{9}\right)^{-2}$

$\therefore \left(\frac{3}{5}\right)^{n+2} = \left(\frac{9}{25}\right)^{2}$ $\therefore \left(\frac{3}{5}\right)^{n+2} = \left(\left(\frac{3}{5}\right)^{2}\right)^{2}$

$\therefore \left(\frac{3}{5}\right)^{n+2} = \left(\frac{3}{5}\right)^{4}$

, $\because$ The base = the base $\therefore$ The power = the power

$\therefore n + 2 = 4$ $\therefore n = 2$

4. $\because 3^{3n-6} = 5^{3n-6}$, $\because$ The power = the power

$\therefore$ Either the base = the base or the power = 0

, $\because 3 \neq 5$ $\therefore 3n - 6 = 0$

$\therefore 3n = 6$ $\therefore n = 2$

5 $\because 7^{n(n-3)} = 1$ $\therefore 7^{n(n-3)} = 7^0$

, $\because$ The base = the base

$\therefore$ The power = the power

$\therefore n(n-3) = 0$

$\therefore$ Either $n = 0$ or $n - 3 = 0$, then $n = 3$

Notice that :

If $a^n = 1$, then : $n = 0$

where : $a \neq 0$, $a \neq \pm 1$

6 $\because 3^{n+2} = n^{n+2}$

$\because$ The power = the power

$\therefore$ Either the base = the base, then $n = 3$

or the power = 0, then $n + 2 = 0$ $\therefore n = -2$

TRY 1 by yourself

Find the value of n in each of the following :

1 $2^{n-2} = 16$ **2** $4^{n+1} = \frac{1}{64}$ **3** $3^{n-5} = 7^{n-5}$

Example 2 **Find the S.S. of each of the following equations in $\mathbb{R}$:**

1 $\frac{(18)^n}{8^n \times 9^n} = 16$ **2** $3^{|x|} = 81$

3 $\left(\frac{3}{2}\right)^{x^2 - x} = 2\frac{1}{4}$ **4** $\frac{1}{(x+3)^2} = 0.01$

Solution

1 $\because \frac{(18)^n}{8^n \times 9^n} = 16$ $\therefore \frac{(3^2 \times 2)^n}{(2^3)^n \times (3^2)^n} = 2^4$

$\therefore \frac{3^{2n} \times 2^n}{2^{3n} \times 3^{2n}} = 2^4$

$\therefore 2^{n-3n} = 2^4$ $\therefore n - 3n = 4$

$\therefore -2n = 4$ $\therefore n = -2$

$\therefore$ The S.S. $= \{-2\}$

2 $\because 3^{|x|} = 81$ $\therefore 3^{|x|} = 3^4$

$\therefore |x| = 4$ $\therefore x = \pm 4$

$\therefore$ The S.S. $= \{4, -4\}$

3 $\because \left(\frac{3}{2}\right)^{x^2-x} = 2\frac{1}{4}$ $\therefore \left(\frac{3}{2}\right)^{x^2-x} = \frac{9}{4}$

$\therefore \left(\frac{3}{2}\right)^{x^2-x} = \left(\frac{3}{2}\right)^2$

$\therefore x^2 - x = 2$ $\therefore x^2 - x - 2 = 0$

By factorizing : $\therefore (x-2)(x+1) = 0$

$\therefore$ Either $x - 2 = 0$, then $x = 2$

or $x + 1 = 0$, then $x = -1$ $\therefore$ The S.S. $= \{2 , -1\}$

4 $\because \frac{1}{(x+3)^2} = 0.01$ $\therefore \frac{1}{(x+3)^2} = \frac{1}{100}$

$\therefore \frac{1}{(x+3)^2} = \frac{1}{(10)^2}$ $\therefore (x+3)^2 = (10)^2$

, $\because$ The power is an even number $\therefore x + 3 = 10$, then $x = 7$

or $x + 3 = -10$, then $x = -13$ $\therefore$ The S.S. $= \{7 , -13\}$

Another solution :

$\because \frac{1}{(x+3)^2} = 0.01$ $\therefore \frac{1}{(x+3)^2} = \frac{1}{100}$

$\therefore (x+3)^2 = 100$ $\therefore x + 3 = \pm\sqrt{100}$

$\therefore x + 3 = \pm 10$ $\therefore x + 3 = 10$, then $x = 7$

or $x + 3 = -10$, then $x = -13$

$\therefore$ The S.S. $= \{7 , -13\}$

TRY 2 by yourself

Find the S.S. of each of the following equations in ℝ :

1. $2^{x^2-9} = 1$
2. $\left(\sqrt{3}\right)^{|x|} = 9$
3. $\frac{4^{x} \times 9^{x+1}}{(36)^{x}} = 3^{x}$

Answers *of try by yourself*

1 [1] 6 [2] −4 [3] 5

2 [1] The S.S. = {3 , −3} [2] The S.S. = {4 , −4} [3] The S.S. = {2}

Lesson 3

Operations on integer powers

- You had studied the order of operating the mathematical operations as follows :

1	2	3	4
Do the operations inside the parenthesis (the interior, then the exterior).	Calculate the powers of the numbers (indices).	Do the multiplication and the division in order from left to right.	Do the addition and the subtraction in order from left to right.

- Also the scientific calculators follow the same previous order in operating the mathematical operations.

In the following, some examples as an application for the previous order.

Example 1 **Find the result of each of the following :**

1 $20 \div (12-2) \times 3^2 - 2$

2 $(\sqrt{3})^7 \div 3\sqrt{3} + 2\sqrt{5} \times (\sqrt{5})^{-1}$

Solution

1 $20 \div (12-2) \times 3^2 - 2 = 20 \div 10 \times 3^2 - 2$ (parenthesis)

$= 20 \div 10 \times 9 - 2$ (the powers of the numbers)

$= 2 \times 9 - 2$ (division)

$= 18 - 2$ (multiplication)

$= 16$ (subtraction)

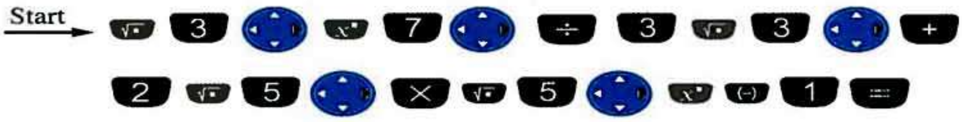
- You can verify the solution using the scientific calculator fx-991 ES PLUS , by clicking the keys in the following sequence from left to right :

2 $(\sqrt{3})^7 \div 3\sqrt{3} + 2\sqrt{5} \times (\sqrt{5})^{-1} = (\sqrt{3})^7 \div (\sqrt{3})^3 + 2\,(\sqrt{5})^{1-1}$

$= (\sqrt{3})^{7-3} + 2\,(\sqrt{5})^0$

$= (\sqrt{3})^4 + 2 \times 1 = 9 + 2 = 11$

• You can verify your solution using the calculator as follows :



Example 2 **Find the result of the following in the simplest form :** $\dfrac{(\sqrt{8})^3 \div 2\,(\sqrt{2})^3}{(\sqrt{2}+2)^2 - 4\sqrt{2}}$

Solution

$$\frac{(\sqrt{8})^3 \div 2\,(\sqrt{2})^3}{(\sqrt{2}+2)^2 - 4\sqrt{2}} = \frac{(2\sqrt{2})^3 \div 2\,(\sqrt{2})^3}{(2+4+4\sqrt{2}) - 4\sqrt{2}}$$

$$= \frac{2^3\,\cancel{(\sqrt{2})^3} \div 2\,\cancel{(\sqrt{2})^3}}{6}$$

$$= \frac{8 \div 2}{6} = \frac{4}{6} = \frac{2}{3}$$

TRY 1 *by yourself*

Find the result of the following in the simplest form , then check your solution by using the calculator : $\dfrac{(2\sqrt{5})^5 \div 2\sqrt{20}}{2\sqrt{15} + (\sqrt{5}-\sqrt{3})^2}$

Example 3 **If $x = \sqrt{5}$ and $y = \sqrt{7}$, find the numerical value of each of :**

1 $\dfrac{x^4 - y^4}{x^2 + y^2}$

2 $\dfrac{x^3 - y^3}{x - y}$

Solution

1 $\dfrac{x^4 - y^4}{x^2 + y^2} = \dfrac{(x^2 - y^2)\,\cancel{(x^2 + y^2)}}{\cancel{(x^2 + y^2)}}$ (difference between two squares)

$= x^2 - y^2 = (\sqrt{5})^2 - (\sqrt{7})^2 = 5 - 7 = -2$

2 $\dfrac{x^3 - y^3}{x - y} = \dfrac{\cancel{(x - y)}\,(x^2 + xy + y^2)}{\cancel{(x - y)}}$ (difference between two cubes)

$= x^2 + xy + y^2 = (\sqrt{5})^2 + \sqrt{5} \times \sqrt{7} + (\sqrt{7})^2$

$= 5 + \sqrt{35} + 7 = 12 + \sqrt{35}$

Unit 2

TRY 2 by yourself

If $x = \sqrt{5}$ and $y = \sqrt{3}$, find the numerical value of each of :

1. $\frac{x^4 - y^4}{x^2 - y^2}$
2. $\frac{x^3 + y^3}{x + y}$

Example 4 **(Connecting with geometry)**

Find to the nearest cm. the radius length of the circle whose area is 5.2×10^3 cm^2. given $\pi = 314 \times 10^{-2}$ **(The area of the circle = πr^2)**

Solution

$\because$ The area of the circle = πr^2

$\therefore 5.2 \times 10^3 = 314 \times 10^{-2} \times r^2$

$\therefore r^2 = \frac{5.2 \times 10^3}{314 \times 10^{-2}}$

$\therefore r = \sqrt{\frac{5.2 \times 10^3}{314 \times 10^{-2}}}$

Using the calculator as in the following sequence :

Start →

You find r = 40.69460597 cm. **i.e.** $r \simeq 41$ cm. to the nearest cm.

TRY 3 by yourself

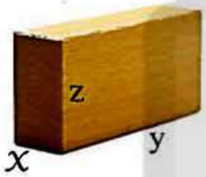
If the volume of the sphere = $\frac{4}{3}\pi r^3$, find to the nearest cm. the radius length of a sphere whose volume is 3.5×10^5 cm^3.

given $\pi = 3141 \times 10^{-3}$

Notice that :

To find $\sqrt[3]{\ }$ using the calculator , click : shift then (key)

Remember the rules of calculating the areas and volumes of some solids

The solid		Lateral area	Total area	Volume
The cube		$4\ell^2$	$6\ell^2$	ℓ^3
The cuboid		$2(x+y)\times z$	$2(xy+yz+zx)$	xyz
The right circular cylinder		$2\pi rh$	$2\pi rh+2\pi r^2$ $=2\pi r(h+r)$	$\pi r^2 h$
The sphere		—	$4\pi r^2$	$\frac{4}{3}\pi r^3$

Answers of try by yourself

1 25

2 (1) 8 (2) $8-\sqrt{15}$

3 r = 44 cm.

UNIT 3

Probability

Lessons of the unit :

1. Probability

Unit Objectives :

By the end of this unit, student should be able to :

- apply what have been studied before about the concept of a sample and how to choose it.
- carry out a random experiment and write the sample space.
- calculate the probability of an event.
- recognize the impossible event.
- recognize the certain event.

Lesson 1 Probability

You had studied before in the previous year the concept of a sample , its importance and its types , and you knew that :

The sample

- The sample is a small part from a large society that looks like this society and represents it well and is selected randomly.
- The sample should wholly represent the society (the object of study) and it shouldn't be based on a certain group and neglect the other , so that the results of the study can be near reality and we can make decisions according to these results , so we can generalize these results on the society as a whole.

For example:

When we hold a survey to know which TV programmes are the most effective on the public opinion , the survey cannot be applied on all people but we select a sample representing the society and we perform our investigations on it , then we generalize the results on all society.

Statistical inference

It's a kind of statistical studies that depends on the idea of choosing a sample from the society it represents, performing a survey for this sample , then generalizing the results on the society as a whole. This means that we recognize the existence of these results in the society through their existence in the taken sample.

For example:

If we took a sample from a farm for producing oranges in order to know the possibility of exporting the production of this farm due to certain conditions and we found that 3% of this sample is not well for exporting , this does not mean that for each 100 oranges , there are 3 bad oranges not well for exporting , but we may find one orange or 2 or 3 or 4

oranges not well for exporting or we may not find any oranges well for exporting but this ratio means that

the average of bad oranges of the production of the farm that are not well for exporting represents 3% of the total production of the farm

We use probability (as we studied before) to express this as we say :

The probability of bad oranges which are not well for exporting from the production of the farm is 3% (It can be written $\frac{3}{100}$ or 0.03)

Example 1 **A pupil carried out a survey on a sample of pupils in his school to know how much they liked mathematics. The sample consists of 30 pupils. The following table shows the results of the survey :**

The degree of liking maths	High degree	Middle degree	Weak degree
The number of pupils	15	10	5

According to this survey , if a pupil is chosen randomly from this school :

1. What is the probability that (The pupil likes maths with high degree) ?
2. What is the probability that (The pupil likes maths with middle degree) ?
3. What is the probability that (The pupil likes maths with weak degree) ?
4. If the number of pupils in this school is 1200 pupils , then what is the expected number of pupils who like maths with high degree ?

Solution

1. The probability that (The pupil likes maths with high degree)

$= \frac{\text{the number of pupils who like maths with high degree}}{\text{the number of all pupils in the sample}} = \frac{15}{30} = \frac{1}{2}$

2. The probability that (The pupil likes maths with middle degree)

$= \frac{\text{the number of pupils who like maths with middle degree}}{\text{the number of all pupils in the sample}} = \frac{10}{30} = \frac{1}{3}$

3. The probability that (The pupil likes maths with weak degree)

$= \frac{\text{the number of pupils who like maths with weak degree}}{\text{the number of all pupils in the sample}} = \frac{5}{30} = \frac{1}{6}$

4. In the selected sample , we obtained that the probability that the pupil likes maths with high degree = $\frac{1}{2}$

Then it is expected that half the number of the pupils in the school like maths in high degree.

i.e. The expected number of pupils who like maths in the school with high degree = $\frac{1}{2} \times 1200 = 600$ pupils.

Unit 3

Probability

You had studied before the experimental probability and the theoretical probability and you knew that :

- **The experimental probability :**

Depends on performing an experiment , then we register the outcomes , then we use these outcomes to calculate the probability of occurrence of one of these outcomes using the relation :

$$\text{The probability of occurrence of a certain event} = \frac{\text{the number of times of repeating this outcome}}{\text{the number of all possible outcomes}}$$

It is noticed that : The more we carry out the experiment , the more we obtain an accurate value for the probability.

The expected number for occurrence of a certain event
= the probability of its occurrence × the total number of the given individuals

- **The theoretical probability :**

It depends on equivalent chances i.e. all individuals have the same chance to occur.

For example:

- When we toss a fair coin and observe the apparent face , then we find one chance of two chances will occur (either head or tail).
- When we roll a fair die and observe the number on the upper face , then the chance of appearance of each face is the same.

The random experiment

The random experiment is an experiment , where all its possible outcomes are known before doing it but we can't determine the actual outcome.

The sample space

The sample space is the set of all possible outcomes of a random experiment and it is denoted by S. The number of its elements is denoted by n (S)

For example:

- As tossing a fair coin once , then S = {**Head** , **Tail**}
- As flipping a fair die once and observing the apparent number on the upper face , then S = {**1** , **2** , **3** , **4** , **5** , **6**}

The event

It is a subset of the sample space.

For example:

If A is the event of appearance of an odd number when we throw a fair die once and observe the apparent number on the upper face, then :

$A = \{1, 3, 5\}$, $A \subset S$ and it is said that A is an event in S

Generally

The probability of occurrence of an event $A \subset S$ is denoted by P (A)

It is found by using the relation :

$$P(A) = \frac{\text{the number of elements of A}}{\text{the number of elements of the sample space}} = \frac{n(A)}{n(S)}$$

Example 2 **If a fair die is thrown once and we observe the number on the upper face, find the probability of each of the following events :**

1. A is the event of appearance of a number greater than 4
2. B is the event of appearance of an even number.
3. C is the event of appearance of the number 5
4. D is the event of appearance of the number 7
5. E is the event of appearance of a number less than 7

Solution $S = \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$

1. $A = \{5, 6\}$, $n(A) = 2$ $\therefore P(A) = \frac{2}{6} = \frac{1}{3}$
2. $B = \{2, 4, 6\}$, $n(B) = 3$ $\therefore P(B) = \frac{3}{6} = \frac{1}{2}$
3. $C = \{5\}$, $n(C) = 1$ $\therefore P(C) = \frac{1}{6}$
4. $D = \{\ \}$ or $\varnothing$, $n(D) =$ zero

   $\therefore P(D) = \frac{0}{6} =$ zero (impossible event)
5. $E = \{1, 2, 3, 4, 5, 6\}$, $n(E) = 6$

   $\therefore P(E) = \frac{6}{6} = 1$ (certain event) or (sure event)

Unit 3

Remarks

1. **The impossible event :** is the event which has no chance for occurring.
   i.e. The probability of the impossible event equals zero.
2. **The certain event :** is the event whose outcomes are all possible.
   i.e. The probability of the certain event = 1
3. The probability of any event is not less than zero and it is not more than 1
   i.e. For any event A , $0 \leq P(A) \leq 1$ **i.e.** $P(A) \in [0 , 1]$

Example 3 **A bag contains an amount of marbles of the same size and softness. If 2 marbles are red , 3 marbles are blue and 5 marbles are white. A marble is drawn randomly. Calculate :**

1. The probability that the drawn marble is red.
2. The probability that the drawn marble is blue.
3. The probability that the drawn marble is white.
4. The probability that the drawn marble is not blue.

Solution The probability of getting a certain occurrent

$$= \frac{\text{the number of chances of getting this occurrent}}{\text{the total number of chances}}$$

$\because$ The total number of marbles = 2 + 3 + 5 = 10 marbles.

1. The probability that the drawn marble is red

$$= \frac{\text{number of red marbles}}{\text{total number of marbles}} = \frac{2}{10} = \frac{1}{5}$$

2. The probability that the drawn marble is blue

$$= \frac{\text{number of blue marbles}}{\text{total number of marbles}} = \frac{3}{10}$$

3. The probability that the drawn marble is white

$$= \frac{\text{number of white marbles}}{\text{total number of marbles}} = \frac{5}{10} = \frac{1}{2}$$

4. The probability that the drawn marble is not blue

$$= \frac{\text{number of the marbles which are not blue}}{\text{total number of marbles}} = \frac{10 - 3}{10} = \frac{7}{10}$$

Remark

In the previous example , notice that :

P (The marble is red) = $\frac{2}{10}$, P (The marble is blue) = $\frac{3}{10}$, P (The marble is white) = $\frac{5}{10}$

, $\because \frac{2}{10} + \frac{3}{10} + \frac{5}{10} = 1$

i.e. The sum of probabilities of all outcomes of the sample space equals 1 , then the probability that an event A does not occur is 1 – P (A)

According to this , we can find the probability that the drawn marble is not blue as follows :

The probability that the drawn marble is not blue

= 1 – the probability that it is blue = $1 - \frac{3}{10} = \frac{7}{10}$

Example 4 **In a class , some pupils wear glasses and others don't wear glasses. If one pupil is chosen randomly from this class and the probability that this pupil wears glasses is 0.1**

1. Find the probability that the pupil doesn't wear glasses.
2. If the number of pupils in this class is 30 pupils , find the expected number of pupils who wear glasses.

Solution

1. The probability that the pupil doesn't wear glasses = 1 – the probability that the pupil wears glasses = 1 – 0.1 = 0.9
2. The expected number of pupils who wear glasses = 0.1 × 30 = 3 pupils.

Example 5 **A factory of electric sets produces two kinds of televisions. In order to change the amount of productions due to the requests of shopping market , a sample is formed from 50 TV sets from 5 shops randomly. Its data was as follows :**

Index of shop	1	2	3	4	5
Number of sold TV sets from the 1st kind	30	42	24	15	40
Number of sold TV sets from the 2nd kind	20	8	26	35	10

1. Which kind is more requested? And what is your advice to the factory ?
2. If the total production of this factory is 3000 TV sets , what is the expected number from the first kind?

Solution

1 • The total sold number of TV sets by the five shops from the first kind
$= 30 + 42 + 24 + 15 + 40 = 151$ TV sets.

• The total sold number of TV sets by the five shops from the second kind
$= 20 + 8 + 26 + 35 + 10 = 99$ TV sets.

$\therefore$ The first kind is more requested , we advise the factory to increase its production from the first kind.

2 The probability of production from the first kind

$$= \frac{\text{number of sold sets from the } 1^{st} \text{ kind}}{\text{the total number of sold sets from the two kinds}} = \frac{151}{250} = 0.604$$

$\therefore$ The expected number of TV sets produced from the first kind
= the probability of production from the 1^{st} kind $\times$ the total production from the two kinds $= 0.604 \times 3000 = 1812$ TV sets.

TRY by yourself

1 A box contains cards numbered from 1 to 15. If a card is drawn randomly from the box , what is the probability that the number on it is divisible by 5 ?

2 A pupil carried out a survey on a sample formed from 30 pupils in his school to know which sports are preferable to them. He registered his results in the following table :

Sport	Football	Basketball	Volleyball	Total
Number of pupils	20	6	4	30

By using the previous table , complete the following :

- If a pupil is chosen randomly , then the probability that he practises basketball = ...
- The expected number of pupils who prefer football from the pupils of the school whose number is 450 pupils = ...

3 An experiment has 3 outcomes. If the probability of the first outcome is 0.3 and the probability of the second outcome is 0.45 , calculate the probability of the third outcome.

4 A farm has 2000 cows. If the probability that they get infected with cow-madness in this farm is 0.17 , what is the number of cows expected to be infected with this disease ?

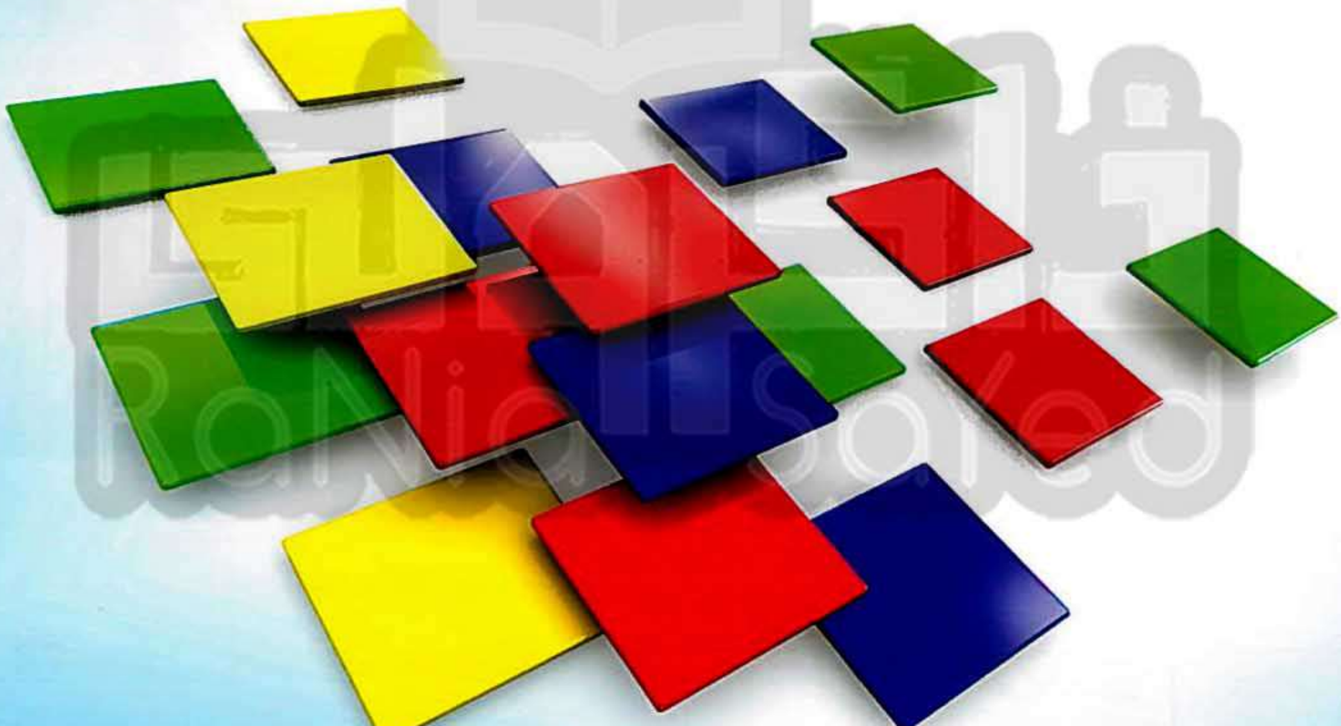
Answers *of try by yourself*

1 $\frac{1}{5}$ **2** • $\frac{1}{5}$ • 300 pupils

3 0.25 **4** 340 cows

Second Geometry

UNIT 4 Areas



Lessons of the unit :

1. Equality of the areas of two parallelograms (Theorem (1) and its corollaries).
2. Follow : Corollaries on theorem (1).
3. Equality of the areas of two triangles (Theorem (2) and its corollaries).
4. Follow : Equality of the areas of two triangles (Theorem (3)).
5. Areas of some geometric figures.

Unit Objectives :

By the end of this unit, student should be able to :

- recognize the altitude of the parallelogram.
- recognize the relation between the areas of two parallelograms with a common base and between two parallel straight lines, one is carrying this base.
- recognize the relation between the areas of a parallelogram and a rectangle with a common base and between two parallel straight lines.
- recognize the relation between the areas of a triangle and a parallelogram with a common base lying on one of two parallel straight lines including them.
- recognize the relation between the areas of two triangles with a common base and their vertices opposite to this base are on a straight line parallel to it.
- calculate the area of a parallelogram.
- calculate the area of a triangle.
- know that the median of a triangle divides its surface into two triangular surfaces equal in area.
- recognize the properties of the rhombus and calculate its area.
- recognize the properties of the isosceles trapezium.
- calculate the area of the trapezium.
- use deductive proof to solve geometrical problems.

Lesson 1

Equality of the areas of two parallelograms

Studing the area of the parallelogram needs firstly to know the concept of the altitude of the parallelogram and its base.

The altitude of the parallelogram

- You can consider any side of the parallelogram as a base.
- The altitude (height) of the parallelogram is the length of a line segment perpendicular to its base from any point of the opposite side to this base.

For example:

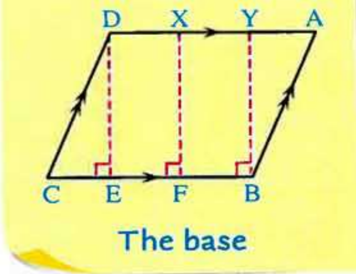
In the opposite figure :

Taking $\overline{BC}$ as a base of the parallelogram ABCD

, then the length of each of $\overline{DE}$, $\overline{XF}$, $\overline{YB}$ is an altitude (height) of the parallelogram ABCD

Since the perpendicular distance between two parallel straight lines is constant ,

then DE = XF = YB



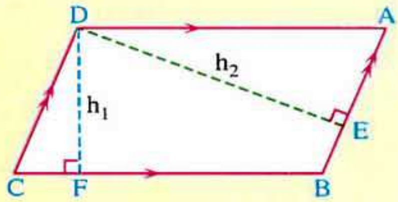
The base

Remark

The parallelogram has two different altitudes (heights). The smaller altitude (height) is corresponding to the longer base , and the greater altitude (height) is corresponding to the shorter base.

For example:

In the opposite figure :



ABCD is a parallelogram whose two different altitudes (heights) are :

- h_1 (the length of $\overline{DF}$) is the altitude (height) corresponding to the base $\overline{BC}$, also it is corresponding to $\overline{AD}$
- h_2 (the length of $\overline{DE}$) is the altitude (height) corresponding to the base $\overline{AB}$, also it is corresponding to $\overline{DC}$

Notice that :

$BC > AB$, $h_1 < h_2$

Theorem 1

Surfaces of two parallelograms with common base and between two parallel straight lines , one is carrying this base , are equal in area.

Given ABCD and EBCF are two parallelograms with a common base $\overline{BC}$ and $\overleftrightarrow{BC} \parallel \overleftrightarrow{AF}$

R.T.P. The area of ▱ ABCD = the area of ▱ EBCF

Proof

$\because$ Δ DCF is the image of Δ ABE by a translation of magnitude BC in the direction of $\overrightarrow{BC}$

$\therefore$ Δ DCF ≡ Δ ABE

(because translation is isometry)

Remember that

The congruent polygons are equal in area.

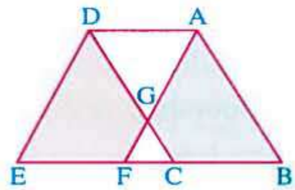
The area of Δ DCF = the area of Δ ABE

$\therefore$ The area of the figure ABCF – the area of Δ DCF

= the area of the figure ABCF – the area of Δ ABE

$\therefore$ The area of ▱ ABCD = the area of ▱ EBCF (Q.E.D.)

Example 1 **In the opposite figure :**



ABCD and AFED are two parallelograms ,

$C \in \overline{BE}$, $F \in \overline{BE}$ and $\overline{AF} \cap \overline{DC} = \{G\}$

Prove that :

The area of the figure ABCG = the area of the figure DEFG

Solution

Given ABCD and AFED are two parallelograms , $C \in \overline{BE}$ and $F \in \overline{BE}$

R.T.P. The area of the figure ABCG = the area of the figure DEFG

Proof $\because$ ABCD and AFED are two parallelograms with a common base $\overline{AD}$

, $\because$ $\overline{AD} // \overline{BE}$

$\therefore$ The area of ▱ ABCD = the area of ▱ AFED

Subtracting the area of Δ AGD from the two sides :

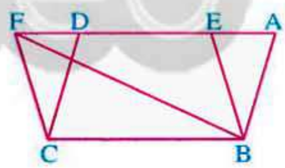
$\therefore$ The area of ▱ ABCD – the area of Δ AGD

= the area of ▱ AFED – the area of Δ AGD

$\therefore$ The area of the figure ABCG = the area of the figure DEFG (Q.E.D.)

TRY 1 by yourself

In the opposite figure :



ABCD and EBCF are two parallelograms ,

$E \in \overleftrightarrow{AD}$ and $F \in \overleftrightarrow{AD}$

If the area of ▱ ABCD = 60 cm^2.

Find : The area of Δ BFC

Important corollaries

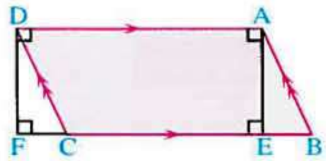
Corollary 1

The parallelogram and the rectangle with common base and between two parallel straight lines are equal in area.

In the opposite figure :

The area of the parallelogram ABCD

= the area of the rectangle AEFD



(They have a common base $\overline{AD}$

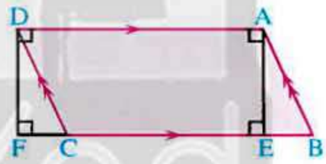
and they are between the two parallel straight lines $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$)

You can deduce that according to the previous theorem where the rectangle is a special case of the parallelogram.

Corollary 2

The area of the parallelogram = the length of the base × its corresponding height.

You can deduce that from the opposite figure as follows :



$\because$ The area of the rectangle = length × width

$\therefore$ The area of the rectangle AEFD = AD × AE

$\because$ The area of the rectangle AEFD = the area of the parallelogram ABCD (Corollary)

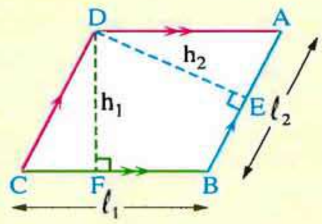
$\therefore$ The area of ▱ ABCD = AD × AE = BC × AE

Remark

In the opposite figure :

If ABCD is a parallelogram , DF is the corresponding height of the base $\overline{BC}$ and DE is the corresponding height of the base $\overline{AB}$, then :

the area of the parallelogram ABCD = BC × DF = AB × DE

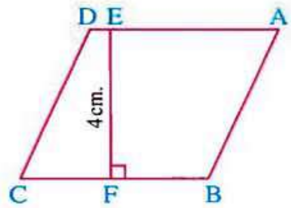


i.e. $\ell_1 \times h_1 = \ell_2 \times h_2$

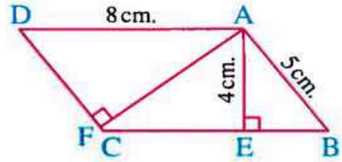
Unit 4

Example 2 **Complete the required beside each figure :**

1 If the area of ▱ ABCD = 400 cm², then BC = ……… cm.



2 If ABCD is a parallelogram, then AF = ……… cm.



Solution

1 ∵ The area of the parallelogram = the length of the base × its corresponding height

∴ 400 = BC × EF ∴ 400 = BC × 4

∴ BC = $\frac{400}{4}$ = 100 cm.

2 ∵ AD × AE = AB × AF

∴ 8 × 4 = 5 × AF ∴ AF = $\frac{8 \times 4}{5}$ = 6.4 cm.

Example 3

1 A parallelogram in which the lengths of two adjacent sides are 4 cm. and 6 cm. and its smaller height is 2 cm. Find its area.

2 A parallelogram in which the lengths of two adjacent sides are 6 cm. and 8 cm. If its greater height is 4 cm., find its smaller height.

Solution

1 ∵ The smaller height corresponds to the longer base.

∴ The area of the parallelogram = 6 × 2 = 12 cm².

2 ∵ The area of the parallelogram

= the length of the smaller base × the greater height

= the length of the greater base × the smaller height

∴ 6 × 4 = 8 × the smaller height.

∴ The smaller height = $\frac{6 \times 4}{8}$ = 3 cm.

TRY 2 *by yourself*

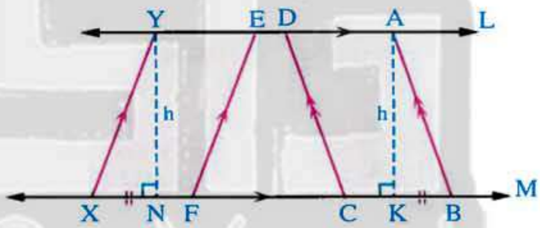
Complete the following :

1. A parallelogram whose base length is 12 cm. and its corresponding height is 5 cm. , then its area = ... cm^2.
2. The area of a parallelogram is 63 cm^2 and the length of its base is 7 cm. , then the corresponding height of it = ... cm.
3. ABCD is a parallelogram in which : AB = 6 cm. , BC = 12 cm. and its greater height is 4 cm. , then its area = ... cm^2.

Corollary 3

The parallelograms with bases equal in length and lying on a straight line , while the opposite sides to these bases are on another straight line , are equal in area.

You can deduce that from the opposite figure as follows :



$\because$ The straight line L // the straight line M

$\therefore$ AK = YN = h

$\therefore$ The area of ▱ ABCD = BC × h and the area of ▱ EFXY = FX × h

If BC = FX , then the area of ▱ ABCD = the area of ▱ EFXY

Answers of try by yourself

1. 30 cm^2
2. 1 60 2 9 3 24

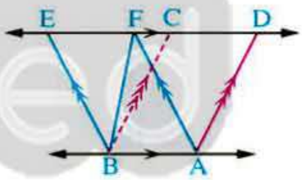
Lesson 2

Follow : Corollaries on theorem (1)

Corollary 4

Area of a triangle is equal to half of area of a parallelogram if they have a common base lying on one of two parallel straight lines including them.

You can deduce that from the opposite figure as follows :



$\overleftrightarrow{DE} \parallel \overleftrightarrow{AB}$, ABCD and ABEF are two parallelograms ,

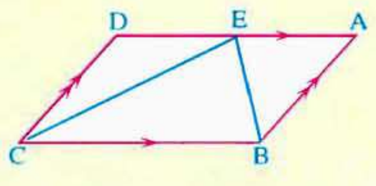
$\overline{BF}$ is a diagonal in ▱ ABEF

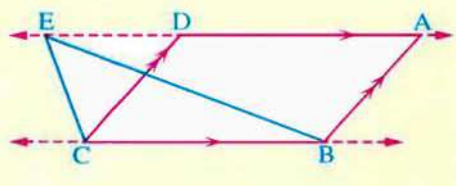
$\therefore$ The area of Δ ABF = $\frac{1}{2}$ the area of ▱ ABEF

, $\because$ The area of ▱ ABCD = the area of ▱ ABEF (Theorem)

$\therefore$ The area of Δ ABF = $\frac{1}{2}$ the area of ▱ ABCD

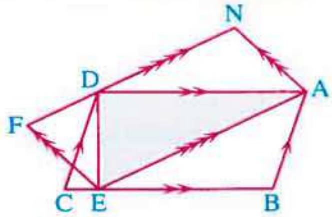
Remark





In each of the previous figures , the area of Δ BCE = $\frac{1}{2}$ of the area of ▱ ABCD

Example 1 **In the opposite figure :**



ABCD and AEFN are two parallelograms ,
$E \in \overline{BC}$, $D \in \overline{NF}$

Prove that : The area of ▱ ABCD = the area of ▱ AEFN

Solution

Given ABCD and AEFN are two parallelograms.

R.T.P. The area of ▱ ABCD = the area of ▱ AEFN

Proof ∵ Δ AED has a common base $\overline{AD}$ with ▱ ABCD and $E \in \overline{BC}$

∴ The area of Δ AED = $\frac{1}{2}$ of the area of ▱ ABCD (1)

, ∵ Δ AED has a common base $\overline{AE}$ with ▱ AEFN and $D \in \overline{FN}$

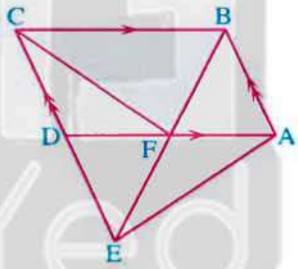
∴ The area of Δ AED = $\frac{1}{2}$ of the area of ▱ AEFN (2)

From (1) and (2) , we deduce that :

The area of ▱ ABCD = the area of ▱ AEFN (Q.E.D.)

TRY 1 by yourself

In the opposite figure :



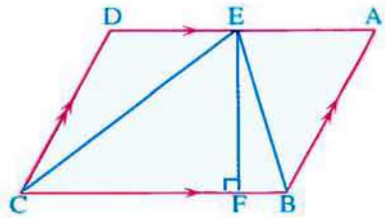
ABCD is a parallelogram , $E \in \overrightarrow{CD}$ and $F \in \overline{AD}$

Prove that : The area of Δ ABE = the area of Δ BFC

Corollary 5

Area of the triangle = $\frac{1}{2}$ of the length of the base × its corresponding height

You can deduce that from the opposite figure as follows:



∵ The area of Δ BCE = $\frac{1}{2}$ of the area of ▱ ABCD

, ∵ The area of ▱ ABCD = BC × EF

∴ The area of Δ BCE = $\frac{1}{2}$ BC × EF

Since BC is the length of the base of the triangle ,

EF is the height of the triangle

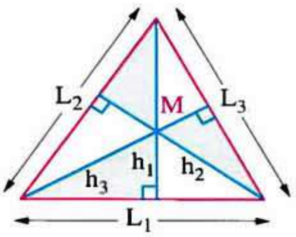
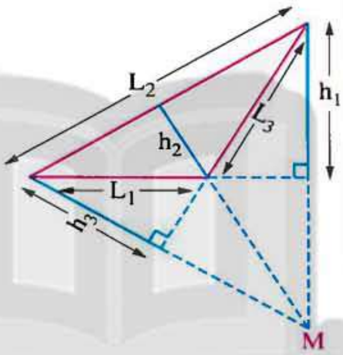
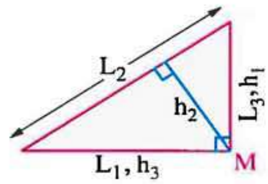
corresponding to the base $\overline{BC}$

Notice that :
The height of a triangle is the length of the perpendicular line segment drawn from a vertex to the opposite side.

Unit 4

Remark

You can consider any side of the triangle as a base , so any triangle has three bases and each base has a corresponding altitude , the straight lines carrying these altitudes intersect at one point as shown in the following figures :

The acute-angled triangle	The obtuse-angled triangle	The right-angled triangle
		
They intersect at a point inside the triangle.	They intersect at a point outside the triangle.	They intersect at the vertex of the right angle.

Example 2

1. A triangle has a base of length = 8 cm. and its corresponding height = 5 cm. Find its area.
2. A triangle of area = 24 cm^2 and one of its heights = 4 cm. Find the length of the corresponding base of this height.
3. ABC is a right-angled triangle at B , BC = 10 cm. and AB = 8 cm. Find its area.

Solution

1. $\because$ The area of the triangle = $\frac{1}{2}$ of the base length × its corresponding height.

   $\therefore$ The area of the triangle = $\frac{1}{2} \times 8 \times 5 = 20$ cm^2.

2. $\because$ The area of the triangle = $\frac{1}{2}$ of the base length × its corresponding height.

   $\therefore 24 = \frac{1}{2}$ × the base length × 4

   $\therefore 24 = 2$ × the base length.

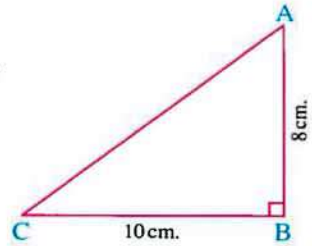
   $\therefore$ The base length = $\frac{24}{2}$ = 12 cm.

3 $\because$ Δ ABC is right-angled at B

$\therefore$ AB is the corresponding height of the base $\overline{BC}$

$\therefore$ The area of Δ ABC $= \frac{1}{2}$ BC $\times$ AB

$= \frac{1}{2} \times 10 \times 8 = 40$ cm^2.



TRY 2 by yourself

Complete the following :

1 If the base length of a triangle is 4 cm. and its corresponding height is 3 cm. , then its area = ...

2 If the area of a triangle is 36 cm^2. and its base length is 9 cm. , then the corresponding height to this base = ...

3 The area of the right-angled triangle in which the lengths of the sides of the right angle are 5 cm. and 6 cm. = ...

Remark

In the opposite figure :

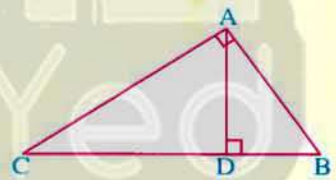
If Δ ABC is right-angled at A and D $\in \overline{BC}$

such that : $\overline{AD} \perp \overline{BC}$

Then the area of Δ ABC $= \frac{1}{2}$ BC $\times$ AD $= \frac{1}{2}$ AB $\times$ AC

$\therefore \frac{1}{2}$ BC $\times$ AD $= \frac{1}{2}$ AB $\times$ AC

$\therefore$ BC $\times$ AD = AB $\times$ AC

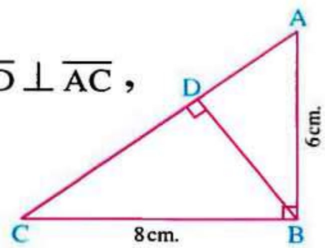


Example 3 **In the opposite figure :**

Δ ABC is right-angled at B , D $\in \overline{AC}$ such that : $\overline{BD} \perp \overline{AC}$,

if AB = 6 cm. and BC = 8 cm.

Find : The length of $\overline{BD}$



Solution

Given Δ ABC is right-angled at B , $\overline{BD} \perp \overline{AC}$, AB = 6 cm. and BC = 8 cm.

R.T.F. The length of $\overline{BD}$

Proof

$\because \triangle ABC$ is right-angled at B

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 36 + 64 = 100$

$\therefore AC = 10$ cm.

$, \because \overline{BD} \perp \overline{AC}$

$\therefore BD \times AC = AB \times BC$

$\therefore BD \times 10 = 6 \times 8$

$\therefore BD = \frac{6 \times 8}{10} = 4.8$ cm. (The req.)

TRY 3 by yourself

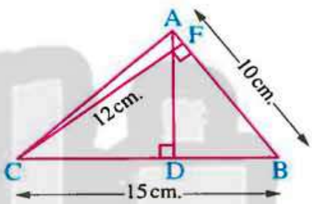
In the opposite figure :

ABC is a triangle in which AB = 10 cm. , BC = 15 cm. ,

$\overrightarrow{AD} \perp \overline{BC}$, $\overrightarrow{AD} \cap \overline{BC} = \{D\}$,

$\overrightarrow{CF} \perp \overline{AB}$ and $\overrightarrow{CF} \cap \overline{AB} = \{F\}$, if CF = 12 cm. ,

Find : The length of $\overline{AD}$



Answers *of try by yourself*

1 Prove by yourself

Hint : The area of $\triangle ABE = \frac{1}{2}$ the area of parallelogram ABCD

, the area of $\triangle BFC = \frac{1}{2}$ the area of parallelogram ABCD

2 [1] 6 cm^2 [2] 8 cm. [3] 15 cm^2

3 8 cm.

Lesson 3 Equality of the areas of two triangles

• **You knew in the previous lesson that :**

The area of the triangle $= \frac{1}{2}$ of the base length × its corresponding height.

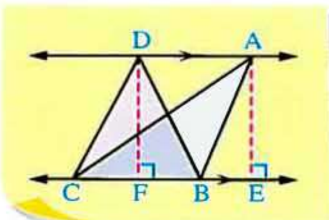
According to this , you can say :

If the lengths of the two bases of two triangles are equal and their corresponding heights are equal , then the areas of the two triangles are equal.

• In this lesson , you shall study some different cases of the equality of the areas of two triangles.

Theorem 2

Two triangles which have the same base and the vertices opposite to this base on a straight line parallel to the base have the same area.



Given $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$,
Δ ABC and Δ DBC have the common base $\overline{BC}$

R.T.P. The area of Δ ABC = the area of Δ DBC

Construction Draw $\overline{AE} \perp \overleftrightarrow{BC}$ and $\overline{DF} \perp \overleftrightarrow{BC}$

Proof $\because \overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$, $\overline{AE} \perp \overleftrightarrow{BC}$ and $\overline{DF} \perp \overleftrightarrow{BC}$

$\therefore$ AEFD is a rectangle. $\therefore$ AE = DF

, $\because$ The area of Δ ABC $= \frac{1}{2}$ BC × AE (1)

and the area of Δ DBC $= \frac{1}{2}$ BC × DF $= \frac{1}{2}$ BC × AE (2)

From (1) and (2), we deduce that :

The area of Δ ABC = the area of Δ DBC (Q.E.D.)

Unit 4

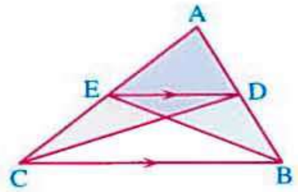
Example 1 **In the opposite figure :**

ABC is a triangle in which :

D ∈ $\overline{AB}$ and E ∈ $\overline{AC}$ where $\overline{DE}$ // $\overline{BC}$

Prove that :

The area of Δ ABE = the area of Δ ACD



Solution

Given ABC is a triangle , $\overline{DE}$ // $\overline{BC}$

R.T.P. The area of Δ ABE = the area of Δ ACD

Proof

∵ ΔΔ DBE and DCE have the common base $\overline{DE}$ and $\overline{BC}$ // $\overline{DE}$

∴ The area of Δ DBE = the area of Δ DCE

Adding the area of Δ ADE to both sides :

∴ The area of Δ DBE + the area of Δ ADE

= the area of Δ DCE + the area of Δ ADE

∴ The area of Δ ABE = the area of Δ ACD (Q.E.D.)

TRY 1 by yourself

In the opposite figure :

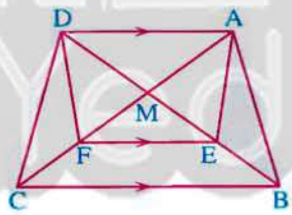
ABCD is a quadrilateral.

Its diagonals intersect at M

, $\overline{AD}$ // $\overline{EF}$ // $\overline{BC}$

Prove that :

The area of Δ ABE = the area of Δ DFC

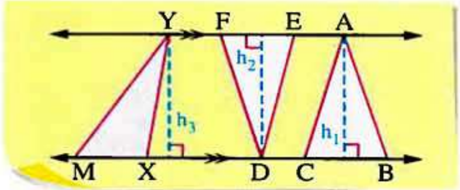


Important corollaries

Corollary 1

Triangles of bases equal in length and lying between two parallel straight lines are equal in area.

In the opposite figure :

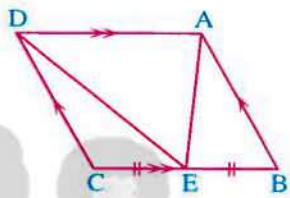


If $\overleftrightarrow{AE} // \overleftrightarrow{BC}$

and BC = EF = XM ,

then the area of Δ ABC = the area of Δ DEF = the area of Δ YXM **Notice that:** $h_1 = h_2 = h_3$

Example 2 **In the opposite figure :**



ABCD is a parallelogram, its area equals 32 cm^2.

and E is the midpoint of $\overline{BC}$

Find : The area of Δ ABE

Solution

Given ABCD is a parallelogram whose area = 32 cm^2.

and E is the midpoint of $\overline{BC}$

R.T.F. The area of Δ ABE

Proof ∵ Δ AED has a common base $\overline{AD}$ with ▱ ABCD and E ∈ $\overline{BC}$

∴ The area of Δ AED = $\frac{1}{2}$ of the area of ▱ ABCD

∴ The area of Δ ABE + the area of Δ DEC

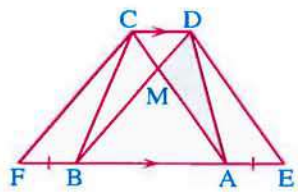
= $\frac{1}{2}$ the area of ▱ ABCD = $\frac{32}{2}$ = 16 cm^2.

, ∵ BE = EC , $\overline{AD} // \overline{BC}$

∴ The area of Δ ABE = the area of Δ DEC = $\frac{16}{2}$ = 8 cm^2. (The req.)

TRY 2 by yourself

In the opposite figure :



ABCD is a quadrilateral. Its diagonals intersect at M

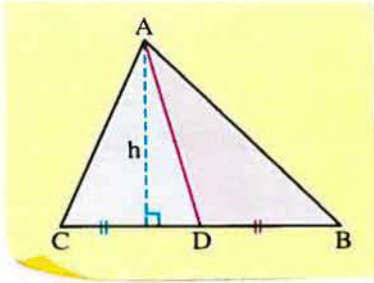
, $\overline{AB} // \overline{CD}$, E ∈ $\overrightarrow{BA}$ and F ∈ $\overrightarrow{AB}$ such that AE = BF

Prove that :

The area of the figure EAMD = the area of the figure FBMC

Corollary 2

The median of a triangle divides its surface into two triangular surfaces equal in area.



In the opposite figure :

If $\overline{AD}$ is a median in ΔABC ,

then the area of ΔABD = the area of ΔADC

Notice that : The two triangles have the same height h and BD = DC

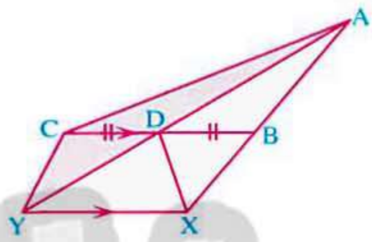
Example 3

In the opposite figure :

$\overline{XY} \parallel \overline{BC}$ and D is the midpoint of $\overline{BC}$

Prove that :

The area of ΔAXD = the area of ΔACY



Solution

Given $\overline{XY} \parallel \overline{BC}$ and D is the midpoint of $\overline{BC}$

R.T.P. The area of ΔAXD = the area of ΔACY

Proof

$\because$ BD = CD and $\overline{BC} \parallel \overline{XY}$

$\therefore$ The area of ΔBXD = the area of ΔCYD (1)

$\because$ D is the midpoint of $\overline{BC}$

$\therefore$ $\overline{AD}$ is a median in ΔABC

$\therefore$ The area of ΔABD = the area of ΔACD (2)

Adding (1) and (2) :

$\therefore$ The area of ΔBXD + the area of ΔABD

= the area of ΔCYD + the area of ΔACD

$\therefore$ The area of ΔAXD = the area of ΔACY (Q.E.D.)

TRY 3 by yourself

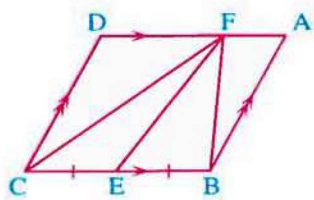
In the opposite figure :

ABCD is a parallelogram , F $\in$ $\overline{AD}$

and E is the midpoint of $\overline{BC}$

Prove that :

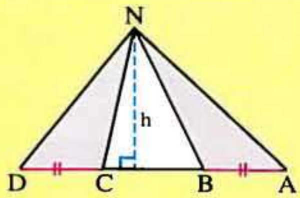
The area of $\Delta BEF = \frac{1}{4}$ the area of ▱ ABCD



Corollary 3

Triangles with congruent bases on one straight line and have a common vertex are equal in area.

In the opposite figure :



The area of Δ NAB = the area of Δ NCD

Notice that :

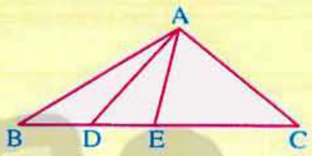
The two triangles have the same height (h) and AB = CD

Remark

In the opposite figure :

If BD = $\frac{1}{2}$ EC ,

then the area of Δ ABD = $\frac{1}{2}$ the area of Δ AEC



Example 4

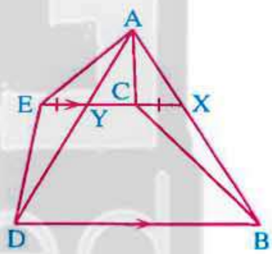
In the opposite figure :

$\overline{XE}$ // $\overline{BD}$, C ∈ $\overline{XE}$ and Y ∈ $\overline{XE}$

such that XC = YE

Prove that :

The area of Δ ABC = the area of Δ ADE



Solution

Given $\overline{XE}$ // $\overline{BD}$ and XC = YE

R.T.P. The area of Δ ABC = the area of Δ ADE

Proof ∵ ΔΔ XCB and EYD have equal bases in length

and $\overleftrightarrow{XE}$ // $\overleftrightarrow{BD}$

∴ The area of Δ XCB = the area of Δ EYD (1)

∵ ΔΔ AXC and AEY have a common vertex A , $\overline{XC} \equiv \overline{YE}$

and are lying on the same straight line.

∴ The area of Δ AXC = the area of Δ AEY (2)

Adding (1) and (2) :

$\therefore$ The area of Δ XCB + the area of Δ AXC

= the area of Δ EYD + the area of Δ AEY

$\therefore$ The area of Δ ABC = the area of Δ ADE (Q.E.D.)

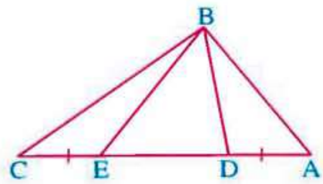
TRY 4 by yourself

In the opposite figure :

ABC is a triangle in which D $\in \overline{AC}$ and E $\in \overline{AC}$ such that : AD = CE

Prove that :

The area of Δ ABE = the area of Δ CBD



Answers *of try by yourself*

1 Prove by yourself

(Hint : The area of Δ ABD = The area of Δ ACD , the area of Δ AED = the area of Δ AFD)

2 Prove by yourself

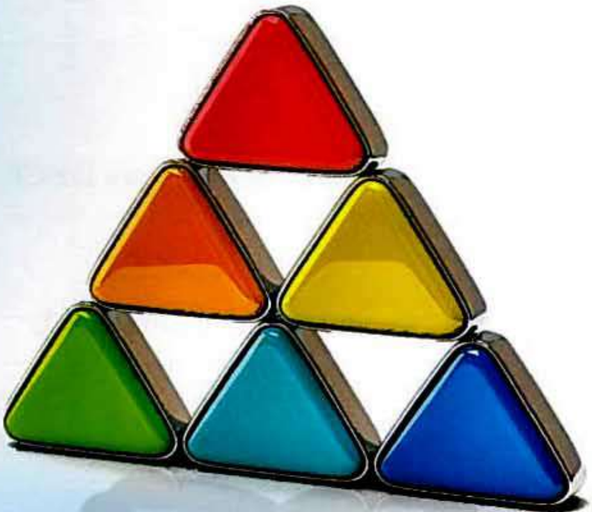
(Hint : The area of Δ DAE = The area of Δ CBF , the area of Δ DMA = area of Δ CMB)

3 Prove by yourself

(Hint : The area of Δ BEF = $\frac{1}{2}$ the area of Δ BCF , the area of Δ BCF = $\frac{1}{2}$ the area of ▱ ABCD)

4 Prove by yourself

(Hint : The area of Δ ABD = The area of Δ BCE , then add the area of Δ BDE to both sides)

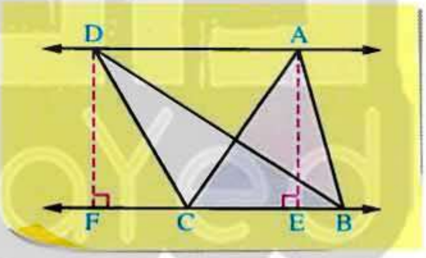


Lesson 4

Follow : Equality of the areas of two triangles

Theorem 3

If two triangles are equal in area and drawn on the same base and on one side of it , then their vertices lie on a straight line parallel to this base.



Given Area of Δ ABC = area of Δ DBC ,

$\overline{BC}$ is a common base.

R.T.P. $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$

Construction Draw $\overline{AE} \perp \overleftrightarrow{BC}$ to cut it at E , $\overline{DF} \perp \overleftrightarrow{BC}$ to cut it at F

Proof

$\because$ Area of Δ ABC = area of Δ DBC

$\therefore \frac{1}{2}$ BC $\times$ AE $= \frac{1}{2}$ BC $\times$ DF

$\therefore$ AE = DF

, $\because \overline{AE} \perp \overleftrightarrow{BC}$, $\overline{DF} \perp \overleftrightarrow{BC}$

$\therefore \overline{AE} \parallel \overline{DF}$

$\therefore$ The figure AEFD is a rectangle.

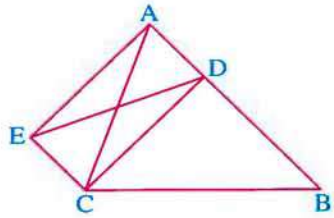
$\therefore \overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ (Q.E.D.)

Unit 4

Example 1 **In the opposite figure :**

The area of Δ ABC = the area of the figure DBCE

Prove that : $\overline{AE}$ // $\overline{DC}$



Solution

Given The area of Δ ABC = the area of the figure DBCE

R.T.P. $\overline{AE}$ // $\overline{DC}$

Proof $\because$ The area of Δ ABC = the area of the figure DBCE

, subtracting the area of Δ DBC from both sides

$\therefore$ The area of Δ ABC – the area of Δ DBC

= the area of the figure DBCE – the area of Δ DBC

$\therefore$ The area of Δ ADC = the area of Δ EDC

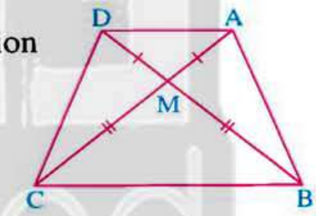
$\because$ $\overline{DC}$ is a common base for them and they are on the same side of it

$\therefore$ $\overline{AE}$ // $\overline{DC}$ (Q.E.D.)

Example 2 **In the opposite figure :**

ABCD is a quadrilateral , M is the point of intersection of its diagonals , MA = MD and MB = MC

Prove that : $\overline{AD}$ // $\overline{BC}$



Solution

Given MA = MD , MB = MC

R.T.P. $\overline{AD}$ // $\overline{BC}$

Proof In $\Delta\Delta$ ABM and DCM : $\because$ $\begin{cases} MA = MD \text{ (given)} \\ MB = MC \text{ (given)} \\ m(\angle AMB) = m(\angle DMC) \text{ (V.O.A.)} \end{cases}$

$\therefore$ Δ ABM $\equiv$ Δ DCM ,

then we deduce that :

The area of Δ ABM = the area of Δ DCM

Adding the area of Δ AMD to both sides :

$\therefore$ The area of Δ ABM + the area of Δ AMD

= the area of Δ DCM + the area of Δ AMD

$\therefore$ The area of Δ BAD = the area of Δ CAD ,

but they have a common base $\overline{AD}$ and on one side of it.

$\therefore$ $\overline{AD}$ // $\overline{BC}$ (Q.E.D.)

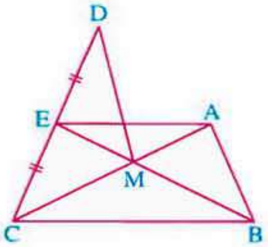
TRY 1 by yourself

In the opposite figure :

E is the midpoint of $\overline{CD}$, $\overline{AC} \cap \overline{BE} = \{M\}$

and the area of Δ MAB = the area of Δ MDE

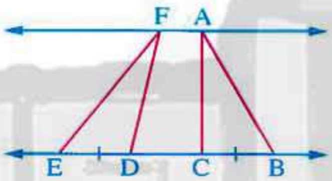
Prove that : $\overline{AE} // \overline{BC}$



Remark

" If two triangles have the same area and they are included between two straight lines and their bases on these two straight lines are equal in length , then the two straight lines are parallel. "

1 In the following figure :



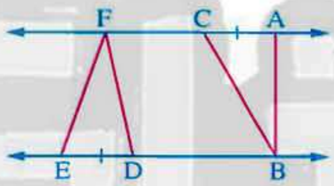
If B , C , D and E are on a straight line ,

BC = DE ,

the area of Δ ABC = the area of Δ FDE

, then $\overleftrightarrow{AF} // \overleftrightarrow{BE}$

2 In the following figure :



If C $\in \overline{AF}$, D $\in \overline{BE}$,

AC = DE ,

the area of Δ ABC = the area of Δ FDE

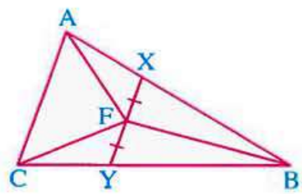
, then $\overleftrightarrow{AF} // \overleftrightarrow{BE}$

Example 3 **In the opposite figure :**

F is the midpoint of $\overline{XY}$,

the area of Δ ABF = the area of Δ CFB

Prove that : $\overline{AC} // \overline{XY}$



Solution

Given F is the midpoint of $\overline{XY}$,

the area of Δ ABF = the area of Δ CFB

R.T.P. $\overline{AC} // \overline{XY}$

Proof

$\because$ F is the midpoint of $\overline{XY}$

$\therefore$ The area of Δ BXF = the area of Δ BFY (1)

, $\because$ The area of Δ ABF = the area of Δ CFB (2)

Subtracting (1) from (2) :

$\therefore$ The area of Δ ABF – the area of Δ BXF

= the area of Δ CFB – the area of Δ BFY

$\therefore$ The area of Δ AXF = the area of Δ CYF

$\because$ F is the midpoint of $\overline{XY}$ $\therefore$ XF = FY

$\therefore$ $\overline{AC}$ // $\overline{XY}$ (Q.E.D.)

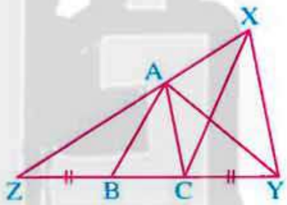
TRY 2 by yourself

In the opposite figure :

B $\in \overline{ZY}$, C $\in \overline{ZY}$ such that ZB = YC

, the area of Δ AZB = the area of Δ ACX

Prove that : $\overline{AC}$ // $\overline{XY}$



Answers *of try by yourself*

1 Prove by yourself

(Hint : The area of Δ AMB = The area of Δ CME , then add the area of Δ AME to both sides)

2 Prove by yourself

(Hint : The area of Δ AZB = The area of Δ ACX , the area of Δ AZB = The area of Δ ACY)

Lesson 5

Areas of some geometric figures

1 The rhombus

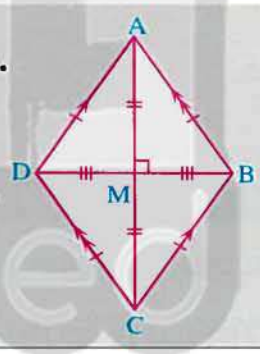
Remember that

- **The rhombus is a parallelogram whose sides are equal in length.**

i.e. • $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$ • AB = BC = CD = DA

- **The two diagonals of the rhombus are perpendicular and bisect each other.**

i.e. • $\overline{AC} \perp \overline{BD}$ • AM = CM , BM = DM



And now you shall study how to find the area of the rhombus by two methods :

1. Knowing the length of its side and its height.
2. Knowing the lengths of its diagonals.

First The area of the rhombus knowing the length of its side and its height :

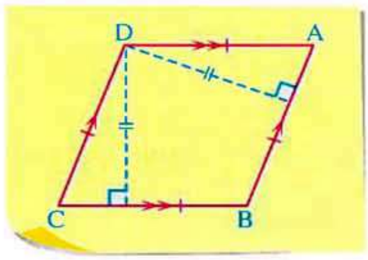
∵ The rhombus is a parallelogram.

∴ The area of the rhombus

= the base length × its corresponding height

and since the side lengths of the rhombus are equal

, then the heights of the rhombus are equal.



i.e. The area of the rhombus = its side length × its height.

For example : A rhombus whose side length is 5 cm. and its height is 3 cm.,
then its area = $5 \times 3 = 15$ cm^2.

Example 1

1. A rhombus whose perimeter is 20 cm. and its height is 4 cm. Find its area.
2. A rhombus whose perimeter is 24 cm. and its area is 30 cm^2. Find its height.

Solution

1. $\because$ The perimeter of the rhombus = the side length $\times$ 4

$\therefore$ 20 = the side length $\times$ 4

$\therefore$ The side length = $\frac{20}{4} = 5$ cm.

$\therefore$ The area of the rhombus = the side length $\times$ the height

$= 5 \times 4 = 20$ cm^2.

2. $\because$ The perimeter of the rhombus = 24 cm.

$\therefore$ The side length of the rhombus = $\frac{24}{4} = 6$ cm.

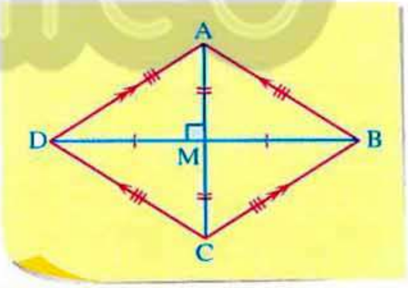
, $\because$ The area of the rhombus = the side length $\times$ the height

$\therefore$ 30 = 6 $\times$ the height

$\therefore$ The height = $\frac{30}{6} = 5$ cm.

Second The area of the rhombus knowing the lengths of its two diagonals :

The opposite figure represents a rhombus ABCD whose diagonals intersect at M



$\therefore$ The area of the rhombus ABCD

= The area of Δ ABD + the area of Δ CBD

$= \frac{1}{2} \times BD \times AM + \frac{1}{2} \times BD \times CM$

$= \frac{1}{2} BD (AM + CM) = \frac{1}{2} \times BD \times AC$

i.e. The area of the rhombus = $\frac{1}{2}$ of the product of the lengths of its two diagonals.

Remark

$\because$ The square is a rhombus with two equal diagonals in length.

$\therefore$ The area of the square = $\frac{1}{2}$ of the square of the length of its diagonal.

Example 2

1. A rhombus whose diagonals lengths are 8 cm. and 6 cm. Find its area.
2. A square whose diagonal length is 8 cm. Find its area.
3. The area of a rhombus is 24 cm^2, the length of one of its diagonals is 4 cm. Find the length of the other diagonal.
4. The perimeter of a rhombus is 40 cm. and the length of one of its diagonals is 12 cm. Find its area.
5. A rhombus is of side length 8 cm. and the measure of one of its angles is 60° Find its area.

Solution

1. The area of the rhombus = $\frac{1}{2}$ of the product of the lengths of its diagonals

$$= \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$$

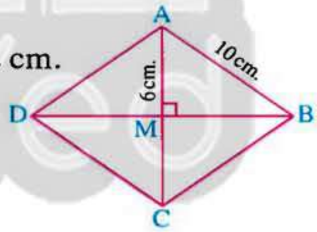
2. The area of the square = $\frac{1}{2}$ of the square of the length of its diagonal

$$= \frac{1}{2} \times (8)^2 = \frac{1}{2} \times 64 = 32 \text{ cm}^2$$

3. $\because$ The area of the rhombus = $\frac{1}{2}$ of the product of the lengths of its diagonals

$\therefore 24 = \frac{1}{2} \times 4 \times$ the length of the other diagonal.

$\therefore$ The length of the other diagonal = $\frac{24 \times 2}{4} = 12$ cm.



4. $\because$ The perimeter of the rhombus = 40 cm.

$\therefore$ Its side length = $\frac{40}{4} = 10$ cm.

and by drawing the rhombus as the figure where AC = 12 cm.

$\therefore AM = \frac{12}{2} = 6$ cm.

$\because \overline{AC} \perp \overline{BD}$ $\qquad \therefore (BM)^2 = (AB)^2 - (AM)^2$

$\therefore (BM)^2 = 100 - 36 = 64$

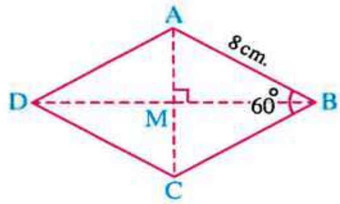
$\therefore BM = \sqrt{64} = 8$ cm.

$\therefore BD = 16$ cm.

$\therefore$ The area of the rhombus ABCD = $\frac{1}{2}$ AC × BD

$$= \frac{1}{2} \times 12 \times 16 = 96 \text{ cm}^2$$

5 By drawing the rhombus ABCD in which the side length = 8 cm. and m (∠ B) = 60°



, ∵ The diagonal of the rhombus bisects the two angles joining their vertices.

∴ m (∠ ABM) = 30°

, ∵ The two diagonals of the rhombus are perpendicular.

∴ m (∠ AMB) = 90°

, ∵ In the right-angled triangle the length of the side opposite to the angle of measure 30° equals $\frac{1}{2}$ the length of the hypotenuse

∴ AM = $\frac{1}{2} \times 8 = 4$ cm.

∴ AC = 2 AM = 8 cm. (1)

, ∵ m (∠ AMB) = 90°

∴ $(BM)^2 = (AB)^2 - (AM)^2$

∴ $(BM)^2 = 64 - 16 = 48$

∴ BM = $\sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3}$ cm.

∴ BD = 2 BM = $8\sqrt{3}$ cm. (2)

From (1) and (2) :

∴ The area of the rhombus ABCD = $\frac{1}{2}$ AC × BD = $\frac{1}{2} \times 8 \times 8\sqrt{3}$

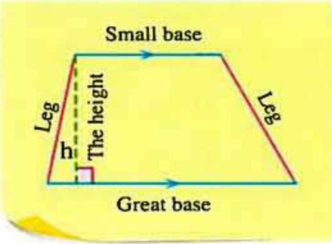
= $32\sqrt{3}$ cm².

TRY 1 by yourself

Complete the following :

1. The rhombus whose base length is 7 cm. and its height is 5 cm. , its area = ...
2. The rhombus in which the lengths of its diagonals are 4 cm. and 6 cm. , its area = ...
3. The square whose diagonal length is 6 cm. , its area = ...
4. The rhombus whose area is 21 cm² and the length of one of its diagonals is 7 cm. , then the length of the other diagonal = ...
5. The square whose area is 32 cm² , its diagonal length = ...

2 The trapezium (The trapezoid)



- It is a quadrilateral in which two sides are parallel.
- The two parallel sides are called the **bases** of the trapezium.
- The other two sides are called the **two legs** of the trapezium.
- The trapezium has one height (h) which is the perpendicular distance between its two bases.

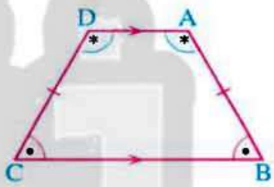
The isosceles trapezium

It is a trapezium whose two legs are equal in length.

The following are the properties of the isosceles trapezium :

1 The two base angles of the isosceles trapezium are equal in measure.

In the opposite figure :



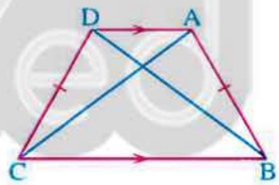
If $\overline{AD} // \overline{BC}$ and AB = DC,

then :

m (∠ B) = m (∠ C) and m (∠ A) = m (∠ D)

2 The two diagonals of the isosceles trapezium are equal in length.

In the opposite figure :

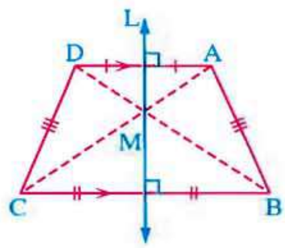


If $\overline{AD} // \overline{BC}$ and AB = DC,

then AC = BD

3 The isosceles trapezium has only one axis of symmetry which is the perpendicular bisector of its bases.

In the opposite figure :



If $\overline{AD} // \overline{BC}$ and AB = DC,

then the straight line L which is the perpendicular bisector of each of its two bases $\overline{AD}$ and $\overline{BC}$

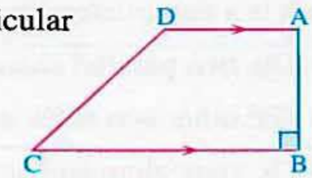
is the axis of symmetry of the isosceles trapezium ABCD

Notice that : The axis of symmetry of the isosceles trapezium passes through the point of intersection of its two diagonals.

Unit 4

The right trapezium

- A right trapezium is a trapezium whose one of its legs is perpendicular to its two parallel bases.
- In this case , the length of this perpendicular leg is the height of the trapezium.



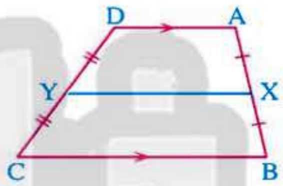
The middle base of the trapezium

- It is the line segment joining the two midpoints of the two legs of the trapezium.
- The middle base of the trapezium is parallel to each of its two parallel bases and its length equals half of the sum of lengths of the two parallel bases.

In the opposite figure :

If $\overline{AD} // \overline{BC}$, X is the midpoint of $\overline{AB}$ and Y is the midpoint of $\overline{CD}$,

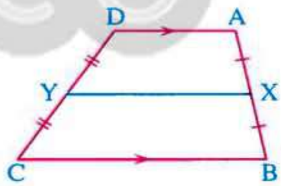
then :



1. $\overline{XY}$ is the middle base of the trapezium ABCD
2. $\overline{XY} // \overline{BC} // \overline{AD}$
3. $XY = \frac{1}{2}(AD + BC)$

For example:

If ABCD is a trapezium in which the lengths of the two parallel bases are 5 cm. and 13 cm. ,



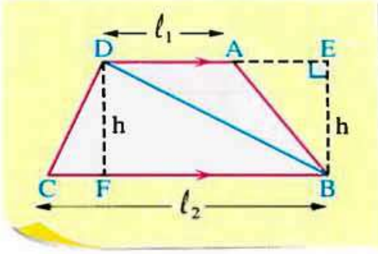
then the length of the middle base $\overline{XY} = \frac{5+13}{2} = 9$ cm.

And now you shall study how to find the area of the trapezium by two methods :

1. Knowing the lengths of its two parallel bases and its height.
2. Knowing the length of its middle base and its height.

First — The area of the trapezium knowing the lengths of its two parallel bases and its height :

In the opposite figure :



The area of the trapezium ABCD

= the area of Δ ABD + the area of Δ DBC

$= \frac{1}{2}$ AD × BE + $\frac{1}{2}$ BC × DF

$= \frac{1}{2} \ell_1 \times h + \frac{1}{2} \ell_2 \times h$

$= \frac{1}{2} h (\ell_1 + \ell_2) = \frac{1}{2} (\ell_1 + \ell_2) \times h$

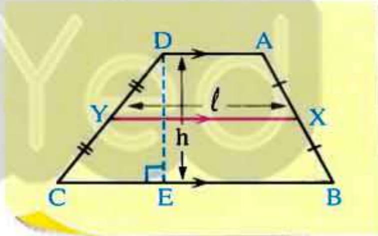
i.e. The area of the trapezium = half of the sum of lengths of the two parallel bases × height

Second — The area of the trapezium knowing the length of the middle base and its height :

∵ The length of the middle base = $\frac{1}{2}$ the sum of the two lengths of the two parallel bases.

∴ The area of the trapezium = the length of the middle base × height

In the opposite figure :



If ABCD is a trapezium in which $\overline{AD} \parallel \overline{BC}$,

X is the midpoint of $\overline{AB}$,

Y is the midpoint of $\overline{CD}$ and E ∈ $\overline{BC}$ such that $\overline{DE} \perp \overline{BC}$,

then the area of the trapezium ABCD = $\ell \times h$

Example 3

1. A trapezium in which the lengths of the two parallel bases are 7 cm. and 9 cm. and its height is 5 cm. Find its area.
2. A trapezium in which the length of its middle base is 8 cm. and its height is 12 cm. Find its area.
3. A trapezium whose area is 126 cm^2, and the length of its middle base is 21 cm. Find its height.
4. A trapezium whose area is 63 cm^2 and the length of one of its parallel bases is 8 cm. and its height is 9 cm. Find the length of its other base.

Solution

1 The area of the trapezium

= half the sum of lengths of the two parallel bases × height

$= \frac{1}{2}(7+9) \times 5 = \frac{1}{2} \times 16 \times 5 = 8 \times 5 = 40 \text{ cm}^2.$

2 The area of the trapezium = the length of the middle base × height

$= 8 \times 12 = 96 \text{ cm}^2.$

3 $\because$ The area of the trapezium = the length of the middle base × height.

$\therefore 126 = 21 \times$ height. $\therefore$ height $= \frac{126}{21} = 6$ cm.

4 $\because$ The area of the trapezium $= \frac{1}{2}(\ell_1 + \ell_2) \times h$

$\therefore 63 = \frac{1}{2}(8 + \ell_2) \times 9$ $\therefore (8 + \ell_2) \times 9 = 63 \times 2$

$\therefore 8 + \ell_2 = \frac{63 \times 2}{9} = 14$ cm. $\therefore \ell_2 = 14 - 8 = 6$ cm.

TRY 2 by yourself

Complete the following :

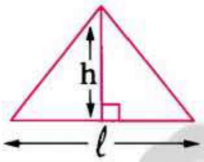
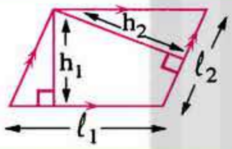
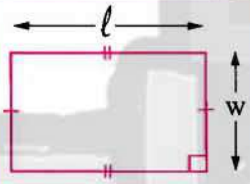
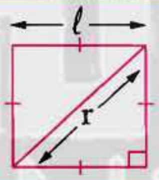
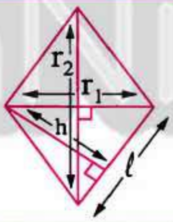
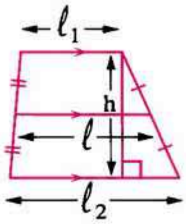
1 The trapezium in which the lengths of the two parallel bases are 5 cm. and 7 cm. and its height is 4 cm., then its area = ...

2 The trapezium in which the length of the middle base is 10 cm. and its height is 5 cm., its area = ...

3 The trapezium whose area = 60 cm^2. and its height = 6 cm., then the length of its middle base = ...

4 The trapezium whose area = 100 cm^2. and the lengths of the two parallel bases are 5 cm. and 15 cm., then its height = ...

SUMMARY

The figure		The perimeter	The area
The triangle		The sum of the lengths of its three sides	$\frac{1}{2}$ of the base length × height $= \frac{1}{2}\ \ell \times h$
The parallelogram		The sum of lengths of two adjacent sides × 2 $= 2\ (\ell_1 + \ell_2)$	The base length × height $= \ell_1 \times h_1 = \ell_2 \times h_2$
The rectangle		2 (Length + Width) $= 2\ (\ell + w)$	Length × Width $= \ell \times w$
The square		Side length × 4 = 4 ℓ	Square of side length = ℓ^2 or $\frac{1}{2}$ of the square of its diagonal length = $\frac{1}{2}\ r^2$
The rhombus		Side length × 4 = 4 ℓ	Side length × height = $\ell \times h$ or $\frac{1}{2}$ the product of the lengths of the two diagonals = $\frac{1}{2}\ r_1 \times r_2$
The trapezium		The sum of lengths of its sides	$\frac{1}{2}$ the sum of lengths of the two parallel bases × height $= \frac{1}{2}\ (\ell_1 + \ell_2) \times h$ or the length of the middle base × height $= \ell \times h$

Answers of try by yourself

1 (1) 35 cm² (2) 12 cm² (3) 18 cm² (4) 6 cm. (5) 8 cm.

2 (1) 24 cm² (2) 50 cm² (3) 10 cm. (4) 10 cm.

UNIT 5

Similarity , converse of Pythagoras' theorem and Euclidean theorem.

Lessons of the unit :

1. Similarity.
2. Converse of Pythagoras' theorem.
3. Projections.
4. Euclidean theorem.
5. Classifying triangles according to their angles.

Unit Objectives :

By the end of this unit, student should be able to :

- recognize the two conditions of similarity of two polygons.
- recognize when two triangles be similar.
- use similarity to solve some real life problems in geometry.
- revise what have been studied before about Pythagoras' theorem.
- apply the converse of Pythagoras' theorem to determine whether a triangle is right - angled or not.
- recognize the projection of a line segment on a straight line.
- determine the relation between the length of a line segment and the length of its projection on a straight line.
- recognize Euclidean theorem.
- use Euclidean theorem to find the unknown side lengths of a triangle.
- classify the triangles according to their angles.
- determine the type of an angle of a triangle given its side lengths.
- appreciate the role of geometry in real life.

Lesson 1

Similarity

* The concept of similarity is used many times in our daily life.

For example:

- When you take a photo for you with a camera , then your photo appears on the screen as a minimizing. In this case , it is clear that your photo is similar to you.
- The data show set converts an image from the computer into an enlarged image on the screen. In this case , it is said that the image on the screen and the image on the computer are similar.

Similarity of two polygons

Definition

It is said that two polygons (of the same number of sides) are similar if the following two conditions are verified together :

1. Their corresponding angles are equal in measure.
2. Their corresponding side lengths are proportional.

The symbol ~ is used to express similarity , where we write

the polygon ABCD ~ the polygon XYZL

and it is read as the polygon ABCD is similar to the polygon XYZL

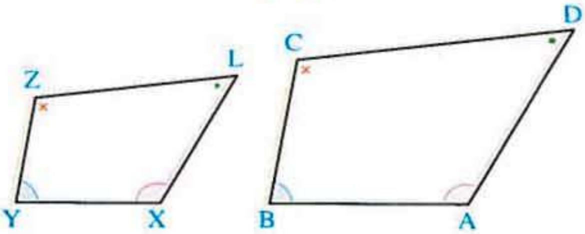
According to the previous definition , if ABCD and XYZL are two polygons where :

1 m (∠ A) = m (∠ X)

, m (∠ B) = m (∠ Y)

, m (∠ C) = m (∠ Z)

, m (∠ D) = m (∠ L)



i.e. The measures of the corresponding angles are equal.

2 $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZL} = \frac{DA}{LX}$ = constant.

i.e. The lengths of the corresponding sides are proportional

, then from 1 and 2 , we deduce that : the polygon ABCD ~ the polygon XYZL

Remark 1

In the two similar polygons P_1 and P_2 , the constant ratio among the lengths of the corresponding sides of P_1 and P_2 is called the ratio of enlargement or the drawing scale.

If the constant ratio is :

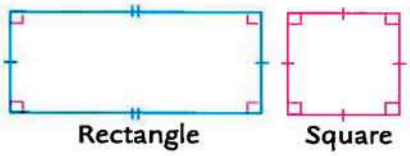
- Greater than 1 , then the polygon P_1 is an enlargement to the polygon P_2
- Less than 1 , then the polygon P_1 is a minimizing of the polygon P_2
- Equal to 1 , then the polygon P_1 is congruent to the polygon P_2

Remark 2

In order that two polygons are similar , the two conditions should be verified together and verifying one of them only is not enough to be similar.

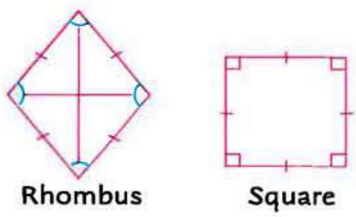
For example:

- The square and the rectangle are not similar polygons although the measures of their corresponding angles are equal (each of them is a right angle) but their corresponding side lengths are not proportional.



- So the square and the rhombus are not similar polygons although their corresponding side lengths are proportional but the measures of their corresponding angles are not equal.

  In the square , each angle is a right angle but in the rhombus that doesn't exist.



Unit 5

" Remark 3

The congruent polygons are similar but it is not necessary that the similar polygons are congruent. "

" Remark 4

All regular polygons of the same number of sides are similar. "

For example: All squares are similar.

" Remark 5

If each of two polygons is similar to a third polygon , then they are similar. "

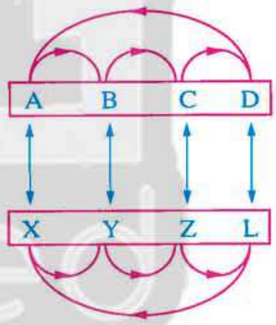
" Remark 6

The order of corresponding vertices should be kept in giving names of similar polygons that to help us finding the proportional sides lengths and the equal angles in measures. "

For example:

If we write P_1 (ABCD) is similar to P_2 (XYZL) ,

then we deduce directly that :

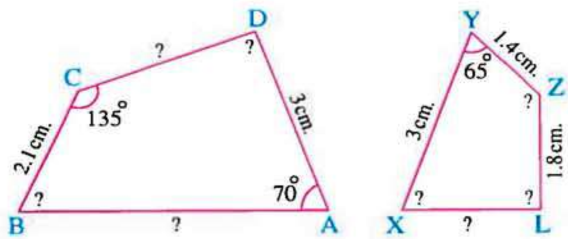


1. $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZL} = \frac{DA}{LX}$
2. $m(\angle A) = m(\angle X)$, $m(\angle B) = m(\angle Y)$, $m(\angle C) = m(\angle Z)$, $m(\angle D) = m(\angle L)$

Example 1 In the opposite figure :

The polygon ABCD ~ the polygon XYZL find the measures and lengths of the unknown elements in the two polygons.

Also find the ratio of enlargement.



Solution

Given

The polygon ABCD ~ the polygon XYZL , $m(\angle A) = 70°$, $m(\angle C) = 135°$,

$m(\angle Y) = 65°$, AD = XY = 3 cm. , BC = 2.1 cm. ,

YZ = 1.4 cm. , ZL = 1.8 cm.

R.T.F.

1 $m(\angle B)$, $m(\angle D)$, $m(\angle X)$, $m(\angle Z)$, $m(\angle L)$

2 The length of each of : $\overline{AB}$, $\overline{CD}$ and $\overline{LX}$

Proof

$\because$ The polygon ABCD ~ the polygon XYZL (given)

$\therefore m(\angle A) = m(\angle X)$, $m(\angle B) = m(\angle Y)$,

$m(\angle C) = m(\angle Z)$, $m(\angle D) = m(\angle L)$

$\therefore m(\angle X) = 70°$, $m(\angle B) = 65°$, $m(\angle Z) = 135°$

, $\because$ The sum of measures of the angles of the quadrilateral = 360°

$\therefore m(\angle D) = m(\angle L) = 360° - (70° + 65° + 135°) = 90°$ (First req.)

, $\because$ The polygon ABCD ~ the polygon XYZL (given)

$$\therefore \frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZL} = \frac{DA}{LX}$$

$$\therefore \frac{AB}{3} = \frac{2.1}{1.4} = \frac{CD}{1.8} = \frac{3}{LX}$$

$$\therefore \frac{AB}{3} = \frac{CD}{1.8} = \frac{3}{LX} = \frac{3}{2}$$

$$\therefore AB = \frac{3 \times 3}{2} = 4.5 \text{ cm.} , CD = \frac{1.8 \times 3}{2} = 2.7 \text{ cm.} , LX = 2 \text{ cm.}$$

The ratio of enlargement (The constant ratio among the lengths of the corresponding sides) $= \frac{BC}{YZ} = \frac{3}{2}$ (Second req.)

Remark

In the previous example , we notice that :

The perimeter of the polygon ABCD = 4.5 + 2.1 + 2.7 + 3 = 12.3 cm.

The perimeter of the polygon XYZL = 3 + 1.4 + 1.8 + 2 = 8.2 cm.

$$\frac{\text{The perimeter of polygon ABCD}}{\text{The perimeter of polygon XYZL}} = \frac{12.3}{8.2} = \frac{3}{2} = \text{the ratio of enlargement.}$$

i.e. The ratio between the perimeters of two similar polygons = the ratio between the lengths of two corresponding sides.

Unit 5

Example 2 Two similar polygons whose lengths of the sides of one of them are 3 , 5 , 6 , 8 and 10 cm. and the perimeter of the other = 48 cm.

Find : The lengths of the sides of the second polygon.

Solution

Given Let the first polygon be ABCDE and the second be ÀB̀C̀D̀È

ABCDE is a polygon whose lengths of its sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$ and $\overline{EA}$ are 3 , 5 , 6 , 8 and 10 cm. respectively , the perimeter of the polygon ÀB̀C̀D̀È equals 48 cm. , the polygon ABCDE is similar to ÀB̀C̀D̀È

R.T.F. The lengths of the sides of the polygon ÀB̀C̀D̀È

Proof

$\because$ The polygon ABCDE ~ The polygon ÀB̀C̀D̀È

$\therefore \dfrac{\text{The perimeter of } \grave{A}\grave{B}\grave{C}\grave{D}\grave{E}}{\text{The perimeter of ABCDE}}$ = the ratio of enlargement.

$\therefore \dfrac{48}{3+5+6+8+10} = \dfrac{48}{32} = \dfrac{3}{2}$

$, \because \dfrac{\grave{A}\grave{B}}{AB} = \dfrac{\grave{B}\grave{C}}{BC} = \dfrac{\grave{C}\grave{D}}{CD} = \dfrac{\grave{D}\grave{E}}{DE} = \dfrac{\grave{E}\grave{A}}{EA}$

$\therefore \dfrac{\grave{A}\grave{B}}{3} = \dfrac{\grave{B}\grave{C}}{5} = \dfrac{\grave{C}\grave{D}}{6} = \dfrac{\grave{D}\grave{E}}{8} = \dfrac{\grave{E}\grave{A}}{10} = \dfrac{3}{2}$

$\therefore$ ÀB̀ = 4.5 cm. , B̀C̀ = 7.5 cm. ,

C̀D̀ = 9 cm. , D̀È = 12 cm. , ÈÀ = 15 cm. (The req.)

TRY 1 by yourself

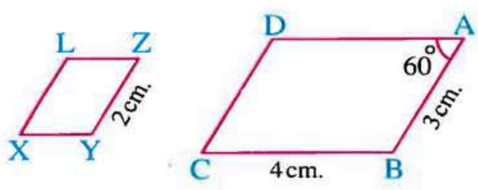
In the opposite figure :

▱ ABCD ~ ▱ XYZL

Find :

1 m (∠ Y)

2 The length of $\overline{XY}$



Similarity of two triangles

You knew that for two polygons in order to be similar, two conditions should be verified together, one of them is not enough to say that the two polygons are similar.

But in triangles, one condition is enough to say that the two triangles are similar.

A geometric fact :

The two triangles are similar if one of the two following conditions is verified :

1. The measures of their corresponding angles are equal.
2. The lengths of their corresponding sides are proportional.

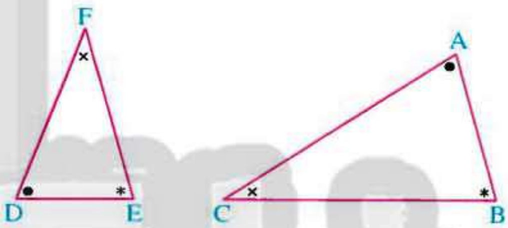
According to the previous fact :

1. If ABC and DEF are two triangles where :

   $m(\angle A) = m(\angle D)$,

   $m(\angle B) = m(\angle E)$, $m(\angle C) = m(\angle F)$

   , then $\Delta ABC \sim \Delta DEF$



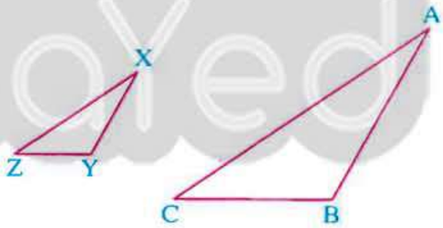
   As a result for their similarity , we find that :

   $$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

2. If ABC and XYZ are two triangles where :

   $$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$$

   , then $\Delta ABC \sim \Delta XYZ$



   As a result for their similarity , we find that :

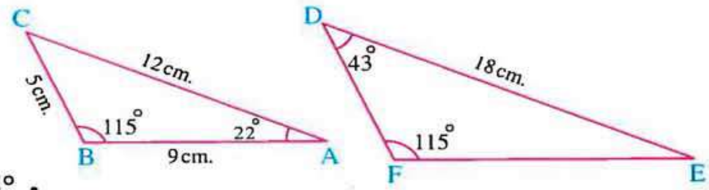
   $m(\angle A) = m(\angle X)$, $m(\angle B) = m(\angle Y)$

   , $m(\angle C) = m(\angle Z)$

Remarks

1. The two right-angled triangles are similar if the measure of an acute angle in one of them is equal to the measure of an acute angle in the other.
2. The two equilateral triangles are similar.
3. The two isosceles triangles are similar if the measure of an angle in one of them equals the measure of the corresponding angle in the other.

Example 3 **In the opposite figure :**



ABC and EFD are two triangles. In them ,

m (∠ B) = m (∠ F) = 115° ,

m (∠ A) = 22° , m (∠ D) = 43°,

AB = 9 cm. , BC = 5 cm. , AC = 12 cm. and ED = 18 cm.

Find : The length of each of $\overline{EF}$ and $\overline{FD}$

Solution

Given m (∠ B) = m (∠ F) = 115°,

m (∠ A) = 22° , m (∠ D) = 43°,

AB = 9 cm. , BC = 5 cm. , AC = 12 cm. , ED = 18 cm.

R.T.F. The length of each of : $\overline{EF}$ and $\overline{FD}$

Proof ∵ The sum of the measures of the interior angles of a triangle = 180°

∴ In Δ ABC : m (∠ C) = 180° – (115° + 22°) = 43°

In Δ EFD : m (∠ E) = 180° – (43° + 115°) = 22°

∵ m (∠ B) = m (∠ F) = 115° , m (∠ C) = m (∠ D) = 43°,

m (∠ A) = m (∠ E) = 22°

∴ Δ ABC ~ Δ EFD

$$\therefore \frac{AB}{EF} = \frac{BC}{FD} = \frac{AC}{ED}$$

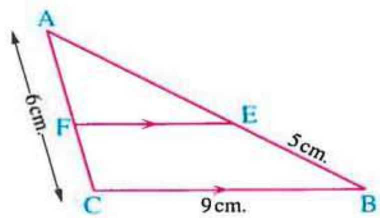
$$\therefore \frac{9}{EF} = \frac{5}{FD} = \frac{12}{18} = \frac{2}{3}$$

$$\therefore EF = \frac{9 \times 3}{2} = 13.5 \text{ cm.} ,$$

$$FD = \frac{5 \times 3}{2} = 7.5 \text{ cm.}$$

(The req.)

Example 4 **In the opposite figure :**



ABC is a triangle in which :

$AC = \frac{1}{2} AB = 6$ cm. , BC = 9 cm.

, E ∈ $\overline{AB}$, where EB = 5 cm. and $\overline{EF}$ // $\overline{BC}$

Find : The length of each of $\overline{EF}$ and $\overline{CF}$

Solution

Given $AC = \frac{1}{2} AB = 6$ cm. , $BC = 9$ cm. , $BE = 5$ cm. and $\overline{EF} \parallel \overline{BC}$

R.T.P. The length of each of : $\overline{EF}$ and $\overline{CF}$

Proof $\because \frac{1}{2} AB = 6$

$\therefore AB = 12$ cm.

$\therefore AE = 12 - 5 = 7$ cm.

$\because \overline{EF} \parallel \overline{BC}$

$\therefore m(\angle AEF) = m(\angle B)$ (Corresponding angles)

, $m(\angle AFE) = m(\angle C)$ (Corresponding angles)

,$\because \angle A$ is a common angle in $\Delta\Delta$ AEF and ABC

$\therefore \Delta AEF \sim \Delta ABC$, then we deduce that : $\frac{AE}{AB} = \frac{EF}{BC} = \frac{AF}{AC}$

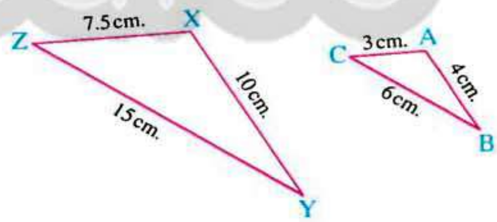
$\therefore \frac{7}{12} = \frac{EF}{9} = \frac{AF}{6}$

$\therefore EF = \frac{9 \times 7}{12} = 5\frac{1}{4}$ cm. ,

$AF = \frac{6 \times 7}{12} = 3\frac{1}{2}$ cm.

$\therefore CF = 6 - 3\frac{1}{2} = 2\frac{1}{2}$ cm. (The req.)

Example 5 **In the opposite figure :**



ABC and XYZ are two triangles.

In them , AB = 4 cm. , BC = 6 cm.

, AC = 3 cm. , XY = 10 cm.

, YZ = 15 cm. and XZ = 7.5 cm.

1 **Prove that :** $\Delta ABC \sim \Delta XYZ$

2 **Find :** $m(\angle A) + m(\angle Y) + m(\angle Z)$

Solution

Given AB = 4 cm. , BC = 6 cm. , AC = 3 cm. ,

XY = 10 cm. , YZ = 15 cm. and XZ = 7.5 cm.

R.T.P. $\Delta ABC \sim \Delta XYZ$

R.T.F. $m(\angle A) + m(\angle Y) + m(\angle Z)$

Proof In $\Delta\Delta$ ABC and XYZ ,

$\frac{AB}{XY} = \frac{4}{10} = \frac{2}{5}$, $\frac{BC}{YZ} = \frac{6}{15} = \frac{2}{5}$, $\frac{AC}{XZ} = \frac{3}{7.5} = \frac{2}{5}$

$\therefore \frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$

$\therefore \Delta ABC \sim \Delta XYZ$ (First req.)

$\therefore m(\angle A) = m(\angle X)$ (1)

$\because$ The sum of the measures of the interior angles of the triangle = 180°

$\therefore$ From ΔXYZ :

$\therefore m(\angle X) + m(\angle Y) + m(\angle Z) = 180°$

Substituting from (1)

$\therefore m(\angle A) + m(\angle Y) + m(\angle Z) = 180°$ (Second req.)

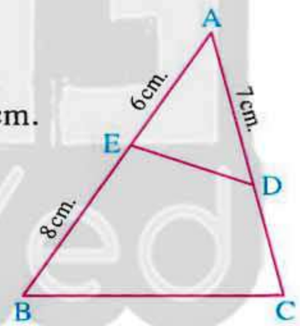
TRY 2 by yourself

In the opposite figure :

$\Delta ADE \sim \Delta ABC$, AE = 6 cm. , AD = 7 cm. and BE = 8 cm.

Find :

1. The length of $\overline{DC}$
2. The ratio $\frac{DE}{BC}$



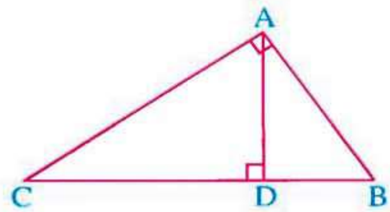
Example 6 **In the opposite figure :**

ABC is a right-angled triangle at A

, D $\in \overline{BC}$ where $\overline{AD} \perp \overline{BC}$

Prove that :

1. $\Delta ABD \sim \Delta CAD$
2. $\Delta ABD \sim \Delta CBA$



Solution

Given ABC is a triangle in which : m (∠ A) = 90° , $\overline{AD} \perp \overline{BC}$

R.T.P. 1 Δ ABD ~ Δ CAD 2 Δ ABD ~ Δ CBA

Proof

In Δ ABD : ∵ m (∠ ADB) = 90°

∴ m (∠ B) + m (∠ BAD) = 90°

, ∵ m (∠ BAD) + m (∠ DAC) = 90°

∴ m (∠ B) = m (∠ DAC)

In ΔΔ ABD , CAD :

∵ m (∠ B) = m (∠ DAC) (Proof)

, m (∠ ADB) = m (∠ CDA) = 90°

∴ m (∠ BAD) = m (∠ ACD)

∴ Δ ABD ~ Δ CAD (Q.E.D. 1)

In ΔΔ ABD , CBA : ∵ m (∠ BDA) = m (∠ BAC) = 90° , ∠ B is common

∴ m (∠ BAD) = m (∠ C)

∴ Δ ABD ~ Δ CBA (Q.E.D. 2)

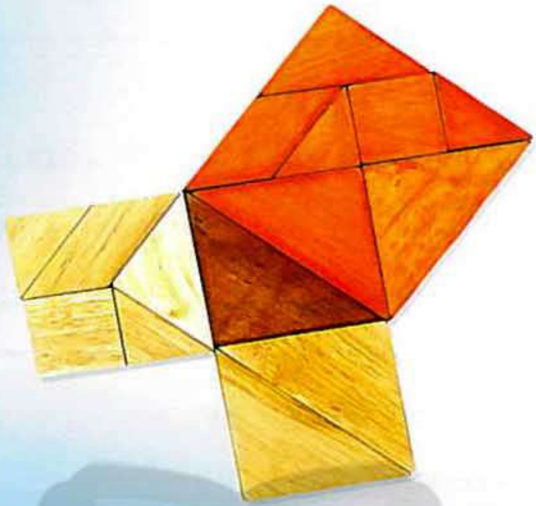
From the previous example , we deduce that :

In the right-angled triangle , the perpendicular from the vertex of the right angle to the hypotenuse divides the triangle into two similar triangles and each of them is similar to the original triangle.

Answers of try by yourself

1 [1] 120° [2] $1\frac{1}{2}$ cm.

2 [1] 5 cm. [2] $\frac{1}{2}$



Lesson 2 Converse of Pythagoras' theorem

You have studied last year how to find a side length of a right-angled triangle , knowing the lengths of the other sides , using Pythagoras' theorem which show the relation among the squares of the side lengths of the right-angled triangle.

If ABC is a right-angled triangle at A

, then $(BC)^2 = (AB)^2 + (AC)^2$

In this lesson , we present to you how to determine whether the triangle is right-angled or not , using the converse of Pythagoras' theorem.

The converse of Pythagoras' theorem

In a triangle , if the sum of the areas of two squares on two sides is equal to the area of the square on the third side, then the angle opposite to this side is a right angle.

Pythaghoras

i.e. If ABC is a triangle in which $(AB)^2 + (AC)^2 = (BC)^2$,
then $m(\angle A) = 90°$

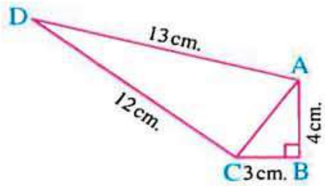
We can state this theorem as follows :

In a triangle, if the square of the length of a side is equal to the sum of the squares of the lengths of the other two sides, then the angle opposite to this side is a right angle.

Corollary

In Δ ABC , if $\overline{AC}$ is the longest side and if $(AC)^2 \neq (AB)^2 + (BC)^2$, then m ($\angle$ B) $\neq 90°$ and the triangle is not right-angled.

Example 1 **In the opposite figure :**



ABCD is a quadrilateral in which :
m ($\angle$ B) = 90° , AB = 4 cm.,
BC = 3 cm. , CD = 12 cm. and DA = 13 cm.

Prove that : m ($\angle$ ACD) = 90°

Solution

Given m ($\angle$ B) = 90° , AB = 4 cm. , BC = 3 cm. , CD = 12 cm. , DA = 13 cm.

R.T.P. m ($\angle$ ACD) = 90°

Proof $\because$ ABC is a triangle in which : m ($\angle$ B) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2$ (Pythagoras' theorem)

$\therefore (AC)^2 = 16 + 9 = 25$ $\therefore$ AC = 5 cm.

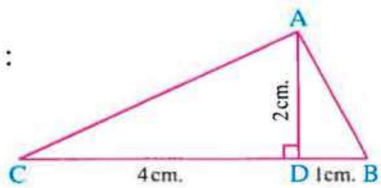
In Δ ACD :

$\because (AD)^2 = (13)^2 = 169$, $(CD)^2 = (12)^2 = 144$, $(AC)^2 = (5)^2 = 25$

$\therefore (AD)^2 = (AC)^2 + (CD)^2$

$\therefore$ m ($\angle$ ACD) = 90° (Converse of Pythagoras' theorem) (Q.E.D.)

Example 2 **In the opposite figure :**



ABC is a triangle , $\overline{AD} \perp \overline{BC}$, D $\in \overline{BC}$ where :

BD = 1 cm. , DC = 4 cm. and AD = 2 cm.

Prove that : m ($\angle$ BAC) = 90°

Solution

Given ABC is a triangle in which : $\overline{AD} \perp \overline{BC}$

, AD = 2 cm. , DB = 1 cm. and DC = 4 cm.

R.T.P. m (∠ BAC) = 90°

Proof In Δ ADB :

$\because$ m (∠ ADB) = 90°

$\therefore (AB)^2 = (AD)^2 + (DB)^2$ (Pythagoras' theorem)

$\therefore (AB)^2 = 4 + 1 = 5$ (1)

In Δ ADC : $\because$ m (∠ ADC) = 90°

$\therefore (AC)^2 = (AD)^2 + (DC)^2$ (Pythagoras' theorem)

$\therefore (AC)^2 = 4 + 16 = 20$ (2)

Adding (1) and (2) : $\therefore (AB)^2 + (AC)^2 = 5 + 20 = 25$

, $\because$ BC = BD + DC = 1 + 4 = 5 cm.

$\therefore (BC)^2 = 25$ $\qquad$ $\therefore (AB)^2 + (AC)^2 = (BC)^2$

$\therefore$ m (∠ BAC) = 90° (Converse of Pythagoras' theorem) (Q.E.D.)

TRY by yourself

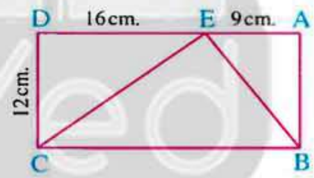
In the opposite figure :

ABCD is a rectangle , E ∈ $\overline{AD}$

such that : AE = 9 cm. , ED = 16 cm.

and DC = 12 cm.

Prove that : m (∠ BEC) = 90°



Answers *of try by yourself*

Prove by yourself

Hint : prove that $(BC)^2 = (BE)^2 + (CE)^2$

Lesson 3

Projections

1 The projection of a point on a straight line

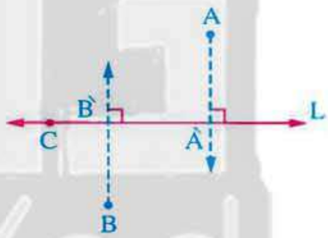
- **In the opposite figure :**

  L is a straight line , the two points

  A and B are not belonging to the straight line L

  Draw from A the ray $\overrightarrow{AÀ} \perp$ L to cut L at À

  Then draw from B the ray $\overrightarrow{BB̀} \perp$ L to cut L at B̀



- The point À is the position of the perpendicular segment drawn from A to the straight line L and it is called the projection of the point A on the straight line L
- Also the point B̀ is the position of the perpendicular segment drawn from B to the straight line L and it is called the projection of the point B on the straight line L

Special Case :

If the point C $\in$ L , then its perpendicular projection on the straight line L is the same point C

Generally

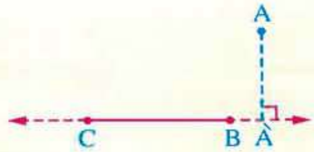
1. The projection of a point on a straight line is the point of intersection of the perpendicular segment from this point to the straight line.
2. If the point lies on the straight line , its projection on it is the same point.

Unit 5

Remark

In the opposite figure :

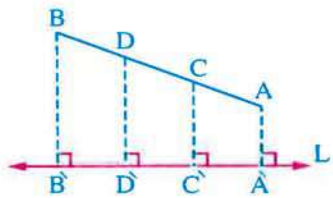
The point $\grave{A}$ is the projection of the point A on the straight line $\overleftrightarrow{BC}$



2 The projection of a line segment on a straight line

- **In the opposite figure :**

$\overline{AB}$ is a given line segment, L is a given straight line in the same plane.



Through our study of the projection of a point on a straight line , we can get the projection of the point A on the straight line L to be $\grave{A}$, also we can get the projection of the point B on the straight line L to be $\grave{B}$

Similarly we can get the projection of any point belonging to $\overline{AB}$ on the straight line L , then we will find that this projection belongs to $\overline{\grave{A}\grave{B}}$

For example:

If the point $C \in \overline{AB}$, then $\grave{C}$ (the projection of C on L) $\in \overline{\grave{A}\grave{B}}$

and if the point $D \in \overline{AB}$, then $\grave{D}$ (the projection of D on L) $\in \overline{\grave{A}\grave{B}}$ and so on ...

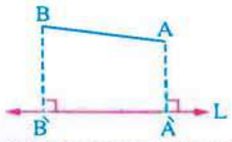
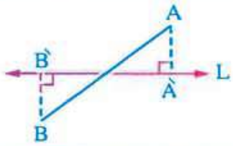
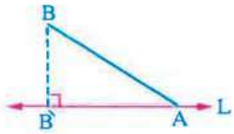
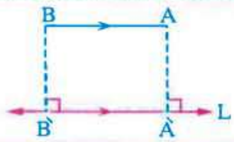
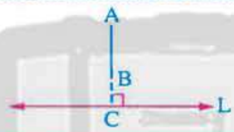
Therefore :

The line segment $\overline{\grave{A}\grave{B}}$ is the projection of $\overline{AB}$ on the straight line L

Generally

The projection of a line segment on a given straight line is the line segment whose two endpoints are the projections of the two endpoints of the main line segment on this straight line.

- **The following table shows the relation between the length of a line segment and the length of its projection on a given straight line :**

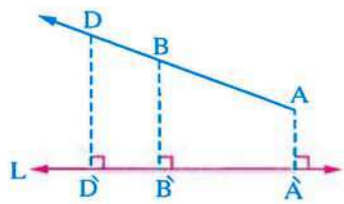
The shape	The line segment	Its projection on L	The relation
	$\overline{AB}$	$\overline{\grave{A}\grave{B}}$	$\grave{A}\grave{B} < AB$
	$\overline{AB}$	$\overline{\grave{A}\grave{B}}$	$\grave{A}\grave{B} < AB$
	$\overline{AB}$	$\overline{A\grave{B}}$	$A\grave{B} < AB$
	$\overline{AB}$	$\overline{\grave{A}\grave{B}}$	$\grave{A}\grave{B} = AB$
	$\overline{AB}$	The point C	$\grave{A}\grave{B}$ = zero

From the table, we notice that :

The length of the projection of a line segment on a given straight line $\leq$ the length of the line segment.

3 The projection of a ray on a straight line

1 In the opposite figure :



$\overrightarrow{AB}$ is a given ray , L is a given straight line in the same plane. If $\grave{A}$ is the projection of A on the straight line L , $\grave{B}$ is the projection of B on the straight line L,

then the ray $\overrightarrow{\grave{A}\grave{B}}$ is the projection of the ray $\overrightarrow{AB}$ on the straight line L

If $D \in \overrightarrow{AB}$, $D \notin \overline{AB}$ and if $\grave{D}$ is the projection of D on the straight line L, then $\grave{D} \in \overrightarrow{\grave{A}\grave{B}}$, $\grave{D} \notin \overline{\grave{A}\grave{B}}$

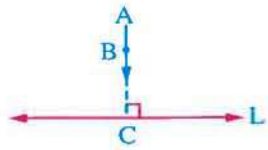
Generally

The projection of a ray on a straight line not perpendicular to it is a ray $\subset$ this straight line.

Unit 5

2 **In the opposite figure :**

If $\overrightarrow{AB} \perp$ the straight line L, then the projection of $\overrightarrow{AB}$ on the straight line L is the point C



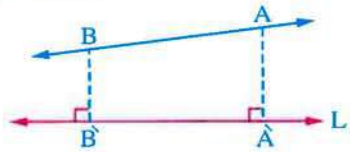
Generally

The projection of a ray on a straight line perpendicular to it is a point belonging to the straight line.

4 The projection of a straight line on another straight line

1 **In the opposite figure :**

The projection of $\overleftrightarrow{AB}$ on the straight line L is the straight line $\overleftrightarrow{A'B'}$ which is the straight line L itself.

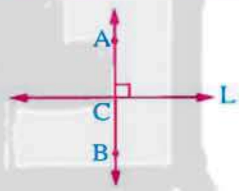


Generally

The projection of a straight line on another straight line not perpendicular to it is that another straight line.

2 **In the opposite figure :**

If $\overleftrightarrow{AB} \perp$ the straight line L , then the projection of $\overleftrightarrow{AB}$ on the straight line L is the point C

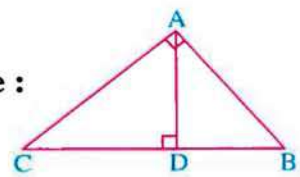


Generally

The projection of a straight line on another straight line perpendicular to it is the point of intersection of the two straight lines.

Example 1 **In the opposite figure :**

Δ ABC is right-angled at A and $\overline{AD} \perp \overline{BC}$ **, complete :**



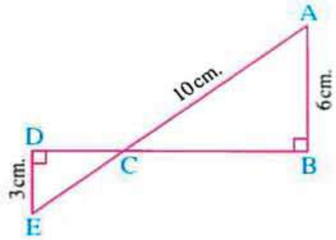
1. The projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is ....
2. The projection of $\overline{AC}$ on $\overleftrightarrow{BC}$ is ....
3. The projection of $\overline{BC}$ on $\overleftrightarrow{AC}$ is ....
4. The projection of $\overline{BC}$ on $\overleftrightarrow{AB}$ is ....
5. The projection of $\overline{AC}$ on $\overleftrightarrow{AD}$ is ....
6. The projection of $\overline{AD}$ on $\overleftrightarrow{BC}$ is ....
7. The projection of $\overline{AB}$ on $\overleftrightarrow{AD}$ is ....

Solution

1. $\overline{DB}$ 2. $\overline{DC}$ 3. $\overline{AC}$ 4. $\overline{BA}$
5. $\overline{AD}$ 6. The point D 7. $\overline{AD}$

Example 2 **In the opposite figure :**

$\overline{BD} \cap \overline{AE} = \{C\}$, m (∠ B) = m (∠ D) = 90°,
AB = 6 cm. , AC = 10 cm. and DE = 3 cm.
Find : The length of the projection of $\overline{AE}$ on $\overleftrightarrow{BD}$



Solution

$\because \overline{AB} \perp \overleftrightarrow{BD}$ $\therefore$ B is the projection of A on $\overleftrightarrow{BD}$

, $\because \overline{ED} \perp \overleftrightarrow{BD}$ $\therefore$ D is the projection of E on $\overleftrightarrow{BD}$

$\therefore \overline{BD}$ is the projection of $\overline{AE}$ on $\overleftrightarrow{BD}$, $\because \Delta$ ABC is right-angled at B

$\therefore (BC)^2 = (AC)^2 - (AB)^2 = (10)^2 - (6)^2 = 64$ (Pythagoras' theorem)

$\therefore$ BC = 8 cm. , $\because$ In ΔΔ ABC and EDC :

m (∠ B) = m (∠ D) = 90° and m (∠ ACB) = m (∠ ECD) (V.O.A.)

$\therefore$ m (∠ A) = m (∠ E)

$\therefore \Delta ABC \sim \Delta EDC$ and we deduce that :

$\frac{AB}{ED} = \frac{BC}{DC} = \frac{AC}{EC}$ $\therefore \frac{6}{3} = \frac{8}{DC} = \frac{10}{EC}$

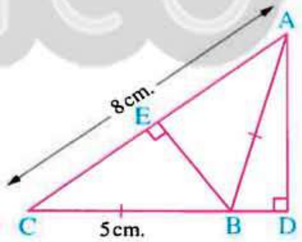
$\therefore DC = \frac{3 \times 8}{6} = 4$ cm. $\therefore$ BD = BC + DC = 8 + 4 = 12 cm.

$\therefore$ The length of the projection of $\overline{AE}$ on $\overleftrightarrow{BD}$ = 12 cm. (The req.)

TRY by yourself

In the opposite figure :

ABC is a triangle in which AB = BC = 5 cm.,
AC = 8 cm. , $\overrightarrow{AD} \perp \overrightarrow{CB}$ and $\overline{BE} \perp \overline{AC}$
Complete the following :



1. The projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is ....
2. The length of the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$ = ...
3. The projection of $\overline{AB}$ on $\overleftrightarrow{AD} \equiv$ the projection of .... on $\overleftrightarrow{AD}$
4. The length of the projection of $\overline{BE}$ on $\overleftrightarrow{AC}$ = ...
5. The area of Δ ABC = ...

Answers of try by yourself

1. $\overline{BD}$ 2. AE = 4 cm. 3. $\overline{AC}$ 4. zero 5. 12 cm²

Lesson 4 Euclidean theorem

Euclidean theorem

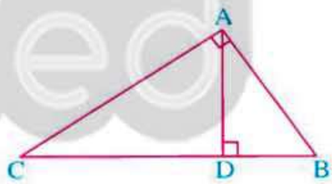
In the right-angled triangle , the area of the square on a side of the right angle is equal to the area of the rectangle whose dimensions are the length of the projection of this side on the hypotenuse and the length of the hypotenuse.

Euclides

- **In the opposite figure :**

If Δ ABC is right-angled at A

, D $\in \overline{BC}$ where $\overline{AD} \perp \overline{BC}$



, then :

$(AB)^2 = BD \times BC$
$(AC)^2 = CD \times CB$

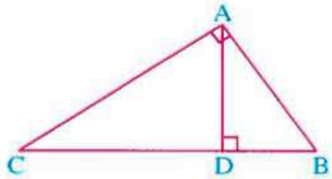
Notice that :

- BD is the length of the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$
- CD is the length of the projection of $\overline{AC}$ on $\overleftrightarrow{BC}$

Corollary

If Δ ABC is right-angled at A , D $\in \overline{BC}$ such that : $\overline{AD} \perp \overline{BC}$, then : $(AD)^2 = BD \times DC$

We can deduce the previous corollary as follows :



$\because$ Δ ABD is right-angled at D

$\therefore$ $(AB)^2 = (AD)^2 + (BD)^2$ (Pythagoras' theorem)

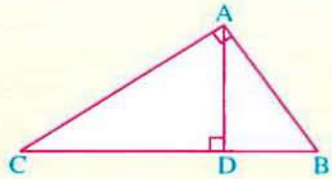
$\therefore$ $(AD)^2 = (AB)^2 - (BD)^2$ but $(AB)^2 = BD \times BC$ (Euclidean theorem)

$\therefore$ $(AD)^2 = BD \times BC - (BD)^2 = BD\ (BC - BD) = BD \times DC$ (Q.E.D.)

Remark

If Δ ABC is right-angled at A and $D \in \overline{BC}$

such that $\overline{AD} \perp \overline{BC}$, then : $AD = \frac{AB \times AC}{BC}$



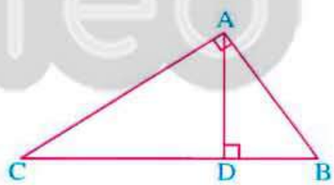
And we can prove that as follows :

The area of Δ ABC $= \frac{1}{2}\ BC \times AD = \frac{1}{2}\ AB \times AC$

$\therefore AD = \frac{AB \times AC}{BC}$

We can deduce the Euclidean theorem and its corollaries using similarity of two triangles as follows :

In the opposite figure :



ABC is a right-angled triangle at A

, $D \in \overline{BC}$ where $\overline{AD} \perp \overline{BC}$

In $\Delta\Delta$ ABC , DBA :

$\because$ m ($\angle$ BAC) = m ($\angle$ ADB) = 90° , $\angle$ B is common

$\therefore$ m ($\angle$ C) = m ($\angle$ BAD)

$\therefore$ Δ ABC ~ Δ DBA (1)

, by the same way in $\Delta\Delta$ ABC , DAC :

$\because$ m ($\angle$ BAC) = m ($\angle$ ADC) = 90° , $\angle$ C is common

$\therefore$ m ($\angle$ B) = m ($\angle$ DAC)

$\therefore$ Δ ABC ~ Δ DAC (2)

, from (1) , (2) : $\therefore \Delta ABC \sim \Delta DBA \sim \Delta DAC$

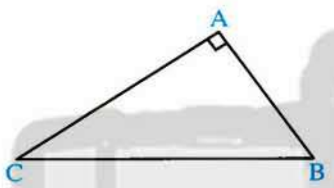
, $\because \Delta ABC \sim \Delta DBA$ $\therefore \frac{AB}{DB} = \frac{BC}{BA}$ $\therefore (AB)^2 = DB \times BC$

, $\because \Delta ABC \sim \Delta DAC$ $\therefore \frac{AC}{DC} = \frac{BC}{AC}$ $\therefore (AC)^2 = DC \times BC$

, $\because \Delta DBA \sim \Delta DAC$ $\therefore \frac{DB}{DA} = \frac{DA}{DC}$ $\therefore (DA)^2 = DB \times DC$

, $\because \Delta ABC \sim \Delta DBA$ $\therefore \frac{BC}{BA} = \frac{AC}{DA}$ $\therefore DA = \frac{BA \times AC}{BC}$

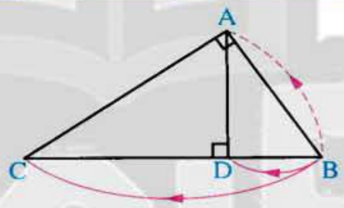
In the following , we write the summary of the relations of Pythagoras' theorem and Euclidean theorem :



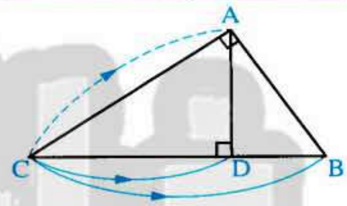
$(BC)^2 = (AB)^2 + (AC)^2$

$(AB)^2 = (BC)^2 - (AC)^2$

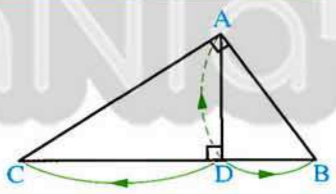
$(AC)^2 = (BC)^2 - (AB)^2$



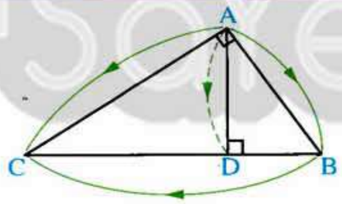
$(BA)^2 = BD \times BC$



$(CA)^2 = CD \times CB$



$(DA)^2 = DB \times DC$

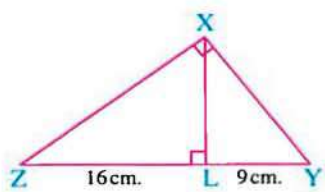


$AD = \frac{AB \times AC}{BC}$

Example 1 **In the opposite figure :**

XYZ is a right-angled triangle at X , $\overline{XL} \perp \overline{YZ}$

such that : $L \in \overline{YZ}$, YL = 9 cm. and LZ = 16 cm.



Find : 1 The length of $\overline{XY}$

2 The length of $\overline{XZ}$

3 The length of $\overline{XL}$

Solution

Given m (∠ YXZ) = m (∠ XLZ) = 90° , YL = 9 cm. , LZ = 16 cm.

R.T.F.
1. The length of $\overline{XY}$
2. The length of $\overline{XZ}$
3. The length of $\overline{XL}$

Proof ∵ Δ XYZ is right-angled at X , $\overline{XL} \perp \overline{YZ}$

∴ $(XY)^2 = YL \times YZ$ (Euclidean theorem)

∴ $(XY)^2 = 9 \times 25 = 225$

∴ XY = 15 cm. (First req.)

Similarly :

$(XZ)^2 = ZL \times ZY$ (Euclidean theorem)

∴ $(XZ)^2 = 16 \times 25 = 400$ ∴ XZ = 20 cm. (Second req.)

, ∵ $(XL)^2 = LY \times LZ$ (Corollary)

∴ $(XL)^2 = 9 \times 16 = 144$ ∴ XL = 12 cm. (Third req.)

Another solution to find the length of $\overline{XL}$

$$XL = \frac{XY \times XZ}{YZ} = \frac{15 \times 20}{25} = 12 \text{ cm.}$$

And we can find the length of $\overline{XL}$ using any of the two right-angled triangles XLZ or XLY using Pythagoras' theorem as follows :

In the right-angled triangle XLY

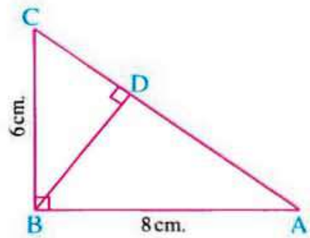
$(XL)^2 = (XY)^2 - (YL)^2 = (15)^2 - (9)^2 = 225 - 81 = 144$

∴ XL = 12 cm.

Example 2 **In the opposite figure :**

Δ ABC is right-angled at B and D ∈ $\overline{AC}$ such that $\overline{BD} \perp \overline{AC}$,

AB = 8 cm. and CB = 6 cm.



Find :
1. AC
2. DB
3. The length of the projection of $\overline{BC}$ on $\overleftrightarrow{AC}$
4. The length of the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$

Solution

Given Δ ABC is right-angled at B , $\overline{BD} \perp \overline{AC}$, AB = 8 cm. and CB = 6 cm.

R.T.F.

1 AC 2 DB

3 The length of the projection of $\overline{BC}$ on $\overleftrightarrow{AC}$

4 The length of the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$

Proof

$\because \Delta$ ABC is right-angled at B

$\therefore (AC)^2 = (AB)^2 + (BC)^2$ (Pythagoras' theorem)

$\therefore (AC)^2 = 64 + 36 = 100$ $\therefore$ AC = 10 cm. (First req.)

, $\because \overline{BD} \perp \overline{AC}$, m ($\angle$ ABC) = 90°

$\therefore DB = \frac{AB \times BC}{AC} = \frac{8 \times 6}{10} = 4.8$ cm. (Second req.)

, $\because$ The projection of $\overline{BC}$ on $\overleftrightarrow{AC}$ is $\overline{DC}$

, $(BC)^2 = CD \times CA$ (Euclidean theorem)

$\therefore 36 = CD \times 10$ $\therefore CD = \frac{36}{10} = 3.6$ cm. (Third req.)

, $\because$ The projection of $\overline{AB}$ on $\overleftrightarrow{AC}$ is $\overline{AD}$

, $(AB)^2 = AD \times AC$ (Euclidean theorem)

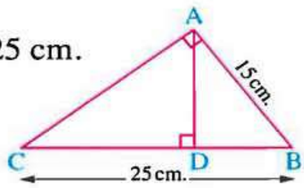
$\therefore 64 = AD \times 10$ $\therefore AD = \frac{64}{10} = 6.4$ cm. (Fourth req.)

TRY *by yourself*

In the opposite figure :

ABC is a triangle in which , m ($\angle$ BAC) = 90°

and D $\in \overline{BC}$ such that $\overline{AD} \perp \overline{BC}$, AB = 15 cm. and BC = 25 cm.



Complete the following :

1 $(AC)^2 = (BC)^2 - (\cdots\cdots)^2$ $\therefore$ AC = ... cm.

2 $(AB)^2 = BD \times$ .... $\therefore$ BD = ... cm.

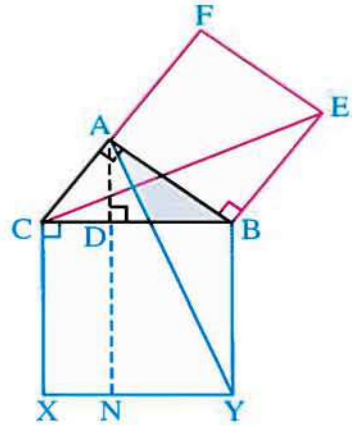
3 $(AC)^2 =$ .... $\times$ CB $\therefore$ CD = ... cm.

4 $(AD)^2 = BD \times$ .... $\therefore$ AD = ... cm.

Answers of try by yourself

1 AB , 20 2 BC , 9 3 CD , 16 4 CD , 12

Euclidean theorem proofs



- **In the opposite figure :**

ABC is a right-angled triangle at A
and the square ABEF is set up on $\overline{AB}$ (one of the sides of the right-angle)
and the square BCXY is set up on the hypotenuse $\overline{BC}$
If $\overrightarrow{AD}$ is drawn to be perpendicular to $\overline{BC}$ to cut it at D
and cut $\overline{XY}$ at N and draw $\overline{EC}$ and $\overline{AY}$

Then : m (∠ EBC) = 90° + m (∠ ABC), m (∠ ABY) = 90° + m (∠ ABC)

∴ m (∠ EBC) = m (∠ ABY)

∵ In ΔΔ EBC and ABY :

EB = AB (Two sides in the square ABEF)
BC = BY (Two sides in the square BCXY)
m (∠ EBC) = m (∠ ABY) (by proof)

∴ Δ EBC ≡ Δ ABY

∴ The area of Δ EBC = the area of Δ ABY

, ∵ The area of Δ EBC = $\frac{1}{2}$ the area of the square ABEF
and the area of Δ ABY = $\frac{1}{2}$ the area of the rectangle BDNY

∴ The area of the square ABEF = the area of the rectangle BDNY

, ∵ The area of the square ABEF = $(AB)^2$
And the area of the rectangle BDNY = BD × BY = BD × BC (**Notice that :** BY = BC)

∴ $(AB)^2 = BD \times BC$

i.e. The area of the square set up on $\overline{AB}$ (one of the sides of the right angle)
= the area of the rectangle whose dimensions are the length of $\overline{BD}$
(The projection of $\overline{AB}$ on the hypotenuse $\overline{BC}$) and the length of the hypotenuse $\overline{BC}$

Similarly, we can prove that : $(AC)^2 = CD \times BC$

i.e. The area of the square set up on $\overline{AC}$ (one of the sides of the right angle)
= the area of the rectangle whose dimensions are the length of $\overline{CD}$
(The projection of $\overline{AC}$ on the hypotenuse $\overline{BC}$) and the length of the hypotenuse $\overline{BC}$

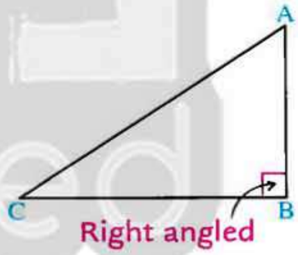
Lesson 5 Classifying triangles according to their angles

- You studied before that the type of the triangle according to its angles can be identified due to the type of the greatest angle in measure.
- If ABC is a triangle in which ∠ B is the greatest angle in measure , then :

1 If m (∠ B) = 90°

(**i.e.** ∠ B is a right angle) ,

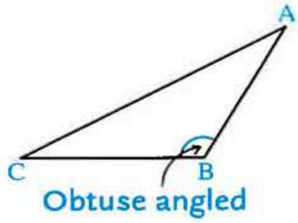
then ABC is a **right-angled** triangle.



2 If m (∠ B) > 90°

(**i.e.** ∠ B is an obtuse angle) ,

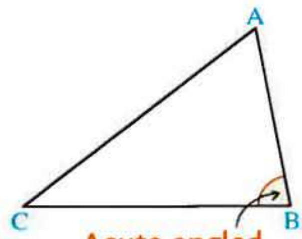
then ABC is an **obtuse-angled** triangle.



3 If m (∠ B) < 90°

(**i.e.** ∠ B is an acute angle) ,

then ABC is an **acute-angled** triangle.



Remark

In any triangle (right , acute or obtuse-angled triangle) , we find that :

The length of any side of the triangle is greater than the difference between the lengths of the other two sides and less than the sum of their lengths.

i.e. If ABC is a triangle , then :

- $BC - AC < AB < BC + AC$
- $AB - AC < BC < AB + AC$
- $AB - BC < AC < AB + BC$

Determining the type of the triangle according to its angles in case of knowing the lengths of its three sides

To determine the type of the triangle according to its angles in case of knowing the lengths of its three sides , we should compare between the square of the length of the longest side of the triangle and the sum of squares of the lengths of the other two sides , then this comparison will determine the type of the triangle as follows :

- Let ABC be a triangle in which $\overline{AC}$ is the longest side , then :

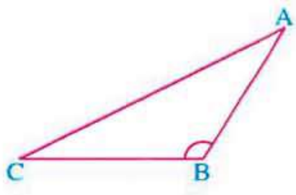
Remember that

From Pythagoras' theorem , if Δ ABC in which m ($\angle$ B) = 90° , then $(AC)^2 = (AB)^2 + (BC)^2$

1 If $(AC)^2 = (AB)^2 + (BC)^2$, then m ($\angle$ ABC) = 90° and ABC is a **right-angled** triangle.

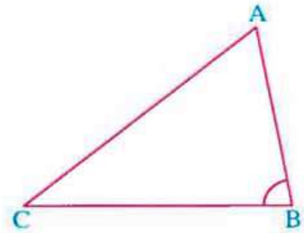
i.e. If the square length of the longest side equals the sum of the squares lengths of the other two sides , then the triangle is right-angled.

2 If $(AC)^2 > (AB)^2 + (BC)^2$, then m ($\angle$ ABC) > 90° and ABC is an **obtuse-angled** triangle.



i.e. If the square length of the longest side is greater than the sum of squares lengths of the other two sides , then the triangle is obtuse-angled.

3 If $(AC)^2 < (AB)^2 + (BC)^2$, then m (∠ ABC) < 90°
and ABC is an **acute-angled** triangle.



i.e. If the square length of the longest side is less than the sum of squares lengths of the other two sides , then the triangle is acute-angled.

Example 1 **In each of the following, determine the type of the triangle ABC according to its angles if :**

1. AB = 4 cm. , BC = 5 cm. and AC = 7 cm.
2. AB = 5 cm. , BC = 13 cm. and AC = 12 cm.
3. AB = 11 cm. , BC = 8 cm. and AC = 9 cm.

Solution

1. ∵ $\overline{AC}$ is the longest side
   , $(AC)^2 = (7)^2 = 49$
   , $(AB)^2 + (BC)^2 = (4)^2 + (5)^2 = 16 + 25 = 41$
   ∴ $(AC)^2 > (AB)^2 + (BC)^2$
   ∴ ABC is an obtuse-angled triangle.
2. ∵ $\overline{BC}$ is the longest side
   , $(BC)^2 = (13)^2 = 169$
   , $(AB)^2 + (AC)^2 = (5)^2 + (12)^2 = 25 + 144 = 169$
   ∴ $(BC)^2 = (AB)^2 + (AC)^2$
   ∴ ABC is a right-angled triangle.
3. ∵ $\overline{AB}$ is the longest side
   , $(AB)^2 = (11)^2 = 121$
   , $(BC)^2 + (AC)^2 = (8)^2 + (9)^2 = 64 + 81 = 145$
   ∴ $(AB)^2 < (BC)^2 + (AC)^2$
   ∴ ABC is an acute-angled triangle.

TRY 1 *by yourself*

In each of the following , determine the type of $\triangle$ XYZ according to its angles :

1. XY = 3 cm. , YZ = 5 cm. , ZX = 4 cm.
2. XY = 9 cm. , YZ = 8 cm. , ZX = 6 cm.
3. XY = 13 cm. , YZ = 7 cm. , ZX = 9 cm.

Remarks

1. To determine the type of an angle in a triangle , we compare between the square length of the side opposite to it and the sum of squares lengths of the other two sides.
2. The greatest angle in measure in the triangle is opposite to the longest side.
3. In any triangle , there are two acute angles at least.

Example 2 **In each of the following, determine the type of $\angle$ A in $\triangle$ ABC if :**

1. AB = 6 cm. , BC = 7 cm. and AC = 8 cm.
2. AB = 12 cm. , BC = 15 cm. and AC = 9 cm.
3. AB = 12 cm. , BC = 20 cm. and AC = 15 cm.

Solution

1. $\because (BC)^2 = (7)^2 = 49$,
, $(AB)^2 + (AC)^2 = (6)^2 + (8)^2 = 36 + 64 = 100$
$\therefore (BC)^2 < (AB)^2 + (AC)^2$
$\therefore m(\angle A) < 90°$
$\therefore \angle$ A is an acute angle.

Notice that : $\angle$ A is opposite $\overline{BC}$ in $\triangle$ ABC

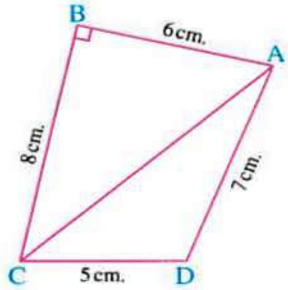
2. $\because (BC)^2 = (15)^2 = 225$, $(AB)^2 + (AC)^2 = (12)^2 + (9)^2 = 144 + 81 = 225$
$\therefore (BC)^2 = (AB)^2 + (AC)^2$ $\quad$ $\therefore m(\angle A) = 90°$
$\therefore \angle$ A is a right angle.

3. $\because (BC)^2 = (20)^2 = 400$
, $(AB)^2 + (AC)^2 = (12)^2 + (15)^2 = 144 + 225 = 369$
$\therefore (BC)^2 > (AB)^2 + (AC)^2$ $\quad$ $\therefore m(\angle A) > 90°$
$\therefore \angle$ A is an obtuse angle.

Unit 5

Example 3 **In the opposite figure :**

ABCD is a quadrilateral in which m (∠ B) = 90° ,
AB = 6 cm. , BC = 8 cm. , AD = 7 cm.
and DC = 5 cm.
Determine the type of the angle which has
the greatest measure in Δ ACD



Solution

Given m (∠ B) = 90° , AB = 6 cm. , BC = 8 cm. , AD = 7 cm. , DC = 5 cm.

R.T.F. The type of the greatest angle in measure in Δ ACD

Proof

∵ ABC is a triangle in which m (∠ B) = 90°

∴ $(AC)^2 = (BC)^2 + (AB)^2 = 64 + 36 = 100$ (1)

∴ AC = 10 cm.

, ∵ AD = 7 cm. and DC = 5 cm.

∴ $\overline{AC}$ is the longest side in Δ ACD

∴ ∠ D is the greatest angle in measure.

, ∵ $(AD)^2 + (DC)^2 = 49 + 25 = 74$ (2)

From (1) and (2) :

∴ $(AC)^2 > (AD)^2 + (DC)^2$

∴ ∠ D is an obtuse angle. (The req.)

TRY 2 by yourself

Determine the type of the greatest angle in measure in* Δ ABC *if :
AB = 4 cm. , BC = 7 cm. and AC = 5 cm.

Answers *of try by yourself*

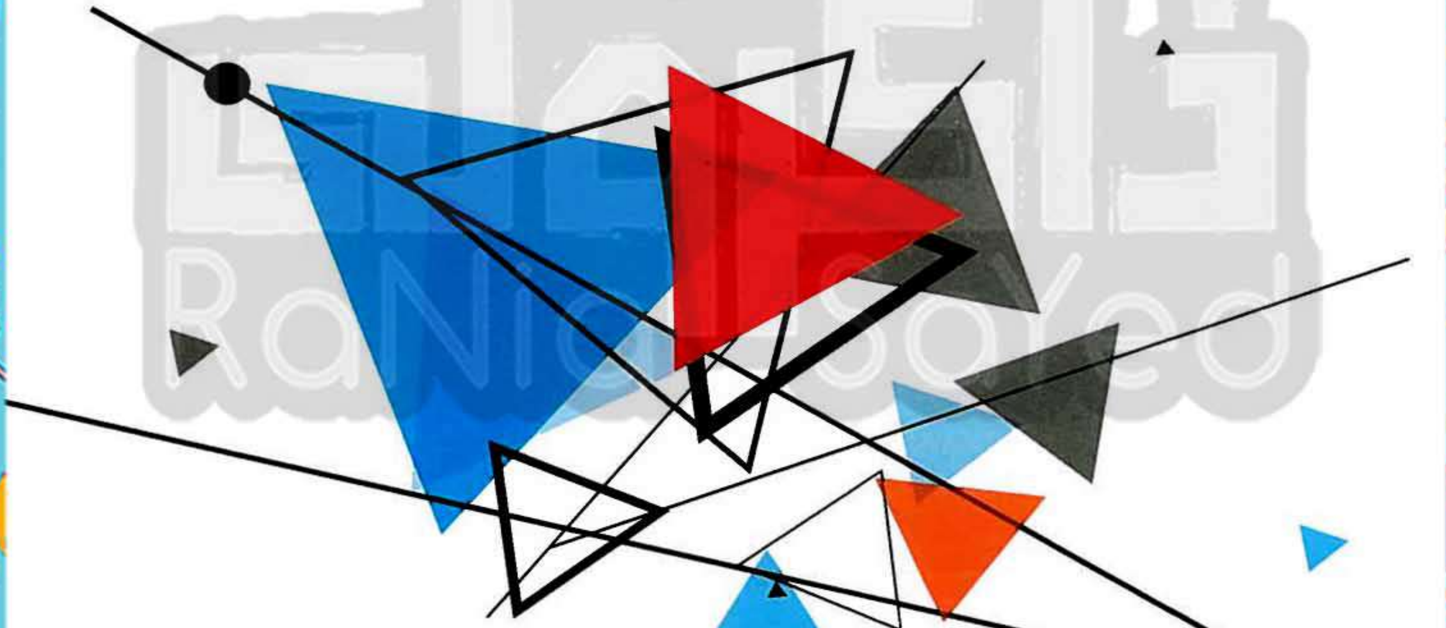
1 [1] Δ XYZ is right-angled triangle at X
[2] Δ XYZ is acute-angled triangle
[3] Δ XYZ is obtuse-angled triangle at Z

2 ∠ A is obtuse

EL-MOASSER

In Mathematics
Exercises

For 2nd Prep.
Second Term



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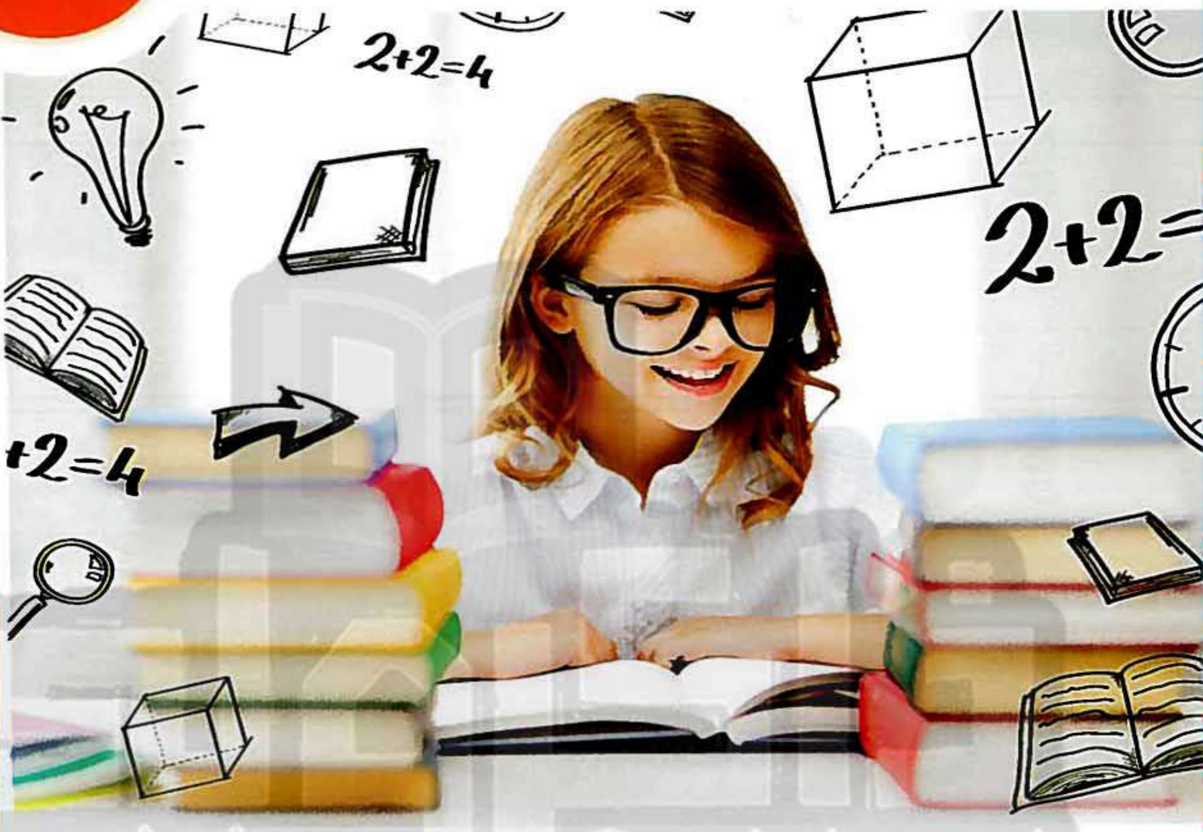
CONTENTS

First Algebra and Statistics



UNIT 1

Factorization

Exercises of the unit :

1. Factorizing quadratic trinomial in the form : $x^2 + b x + c$
2. Factorizing quadratic trinomial in the form : $a x^2 + b x + c$, where $a \neq \pm 1$
3. Factorizing the perfect square trinomials.
4. Factorizing the difference of two squares.

✪ Summary of the first part of unit one.

✪ Exams of the first part of unit one.

5. Factorizing the sum and difference of two cubes.
6. Factorizing by grouping.
7. Factorizing by completing the square.
8. Solving quadratic equations in one variable algebraically.
9. Applications on solving quadratic equations in one variable algebraically.

✪ Summary of the second part of unit one.

✪ Exams of the second part of unit one.

Exercise 1

Factorizing quadratic trinomial in the form : $x^2 + bx + c$

📖 From the school book

1 Find :

1. Two numbers such that their product = 30 and their sum = 11
2. 📖 Two numbers such that their product = 12 and their sum = – 8
3. Two numbers such that their product = – 18 and their sum = 3
4. 📖 Two numbers such that their product = – 15 and their sum = – 14

2 Factorize each of the following :

1. $x^2 + 8x + 15$
2. 📖 $x^2 + 11x + 10$
3. 📖 $x^2 - 7x + 12$
4. $x^2 - 17x + 30$
5. $x^2 + 5x - 14$
6. 📖 $x^2 + 4x - 12$
7. $x^2 - 6x - 16$
8. 📖 $x^2 - 3x - 10$

3 Factorize each of the following :

1. $x^2 + 5xy + 6y^2$
2. $a^2 + 11ab + 30b^2$
3. 📖 $b^2 + 3bc - 10c^2$
4. $a^2 + 22ab - 48b^2$
5. $x^2 - 15xy + 36y^2$
6. $a^2 - 13ab + 42b^2$
7. 📖 $x^2 - 5xy - 24y^2$
8. $x^2 - 7xy - 18y^2$

4 Factorize each of the following :

1. $15a + a^2 - 34$
2. $22a - 75 + a^2$
3. $-10 + x^2 + 3x$
4. $x^2 + 21 - 10x$

5 **Factorize each of the following :**

1. $x^4 + 9x^2 + 18$
2. $x^4 - 8x^2 + 15$
3. $\ell^6 - 6\ell^3 - 40$
4. $a^4 + a^2 b^2 - 56 b^4$

6 **Factorize each of the following :**

1. $5x^2 - 10x - 15$
2. $2a^2 + 28a + 96$
3. $y^3 + y^2 - 6y$
4. $x^3 - 3x^2 - 28x$
5. $3x^2 - 42 - 15x$
6. $18x - 15x^2 + 3x^3$
7. $-2x^2 - 2x + 40$
8. $-x^2 + 2x + 63$
9. $a^2 b^2 - 24ab^2 + 143b^2$
10. $2a^4 - 24a^2 b^2 - 26b^4$

7 **Factorize each of the following :**

1. $x(x + 7) + 10$
2. $x^2 - 4x - 3(x - 2)$
3. $(a - 4b)(a + 4b) + 6ab$
4. $x^2(x - 23) + 60x$
5. $(x - 4)(x - 9) - 2(x + 5)$

8 **Find the value of $c \in \mathbb{Z}$ such that the expression will be factorized then factorize it :**

1. $x^2 + cx - 15$
2. $x^2 - 7x + c$
3. $y^2 - cy + 29$
4. $a^2 + a - c$

9 **Complete :**

1. $x^2 - 11x + 18 = (x - \cdots\cdots)(x - \cdots\cdots)$
2. $x^2 + 5x + 6 = (\cdots\cdots)(x + 2)$
3. $x^2 + \cdots\cdots + 35 = (x + \cdots\cdots)(\cdots\cdots + 5)$
4. If $(x - 2)$ is a factor of the expression : $x^2 - 8x + 12$, then the other factor is ....
5. $(x - \cdots\cdots)$ is a factor of the expression : $x^2 - x - 6$
6. If $(x + 2y) = 4$ and $(x - y) = 1$, then the numerical value of the expression $x^2 + xy - 2y^2$ is ....

10 **Choose the correct answer from those given :**

1. If the expression : $x^2 + 7x + a$ can be factorized , then a may be equal to ....
   (a) 8 (b) 10 (c) 18 (d) 49
2. If the expression : $x^2 - 3x + c$ can be factorized , then c may be equal to ....
   (a) 1 (b) 2 (c) 4 (d) 6

3 For the expression : $x^2 - x - k$ can be factorized then $k \neq$ ....

(a) 12 (b) 30 (c) 6 (d) 8

4 If the expression : $x^2 + a x + 2$ can be factorized , then a equals ....

(a) 1 (b) 2 (c) 3 (d) 4

5 If the expression : $x^2 + b x - 10$ can be factorized , then b equals ....

(a) 3 (b) 2 (c) 1 (d) -1

6 If the expression : $x^2 - c x + 12$ can be factorized , then c equals ....

(a) -1 (b) 4 (c) 7 (d) 1

7 Which of the following numbers can be added to the expression : $x^2 - 8x + 5$ to be factorized ?

(a) 1 (b) 2 (c) 4 (d) 5

Geometric Application

11 The area of a rectangle is $(x^2 + 6x + 8)$ cm^2 and its length = $(x + 4)$ cm. Find each of its width and its perimeter in terms of x

For excellent pupils

12 **Factorize each of the following :**

1 $(x - 1)^2 - 2(x - 1) - 8$

2 $10 + 3(a + b) - (a + b)^2$

13 If $x^2 - 2xy - 3y^2 = 7$ and $x + y = 1$

Find the value of : $x - 3y$

Exercise **2**

Factorizing quadratic trinomial in the form : $a x^2 + b x + c$ where $a \neq \pm 1$

From the school book

1 Complete the missing terms :

1. $5x^2 - 2x - 7 = (5x - \cdots\cdots)(x + \cdots\cdots)$
2. $3x^2 + 10x + 8 = (\cdots\cdots + 4)(x + \cdots\cdots)$
3. $6x^2 - 11x - 10 = (2x - \cdots\cdots)(\cdots\cdots + 2)$
4. $3x^2 - 7x + 2 = (\cdots\cdots - 2)(\cdots\cdots - 1)$
5. $3x^2 + 7x - 6 = (3x - \cdots\cdots)(\cdots\cdots + \cdots\cdots)$
6. $2x^2 + x - 6 = (\cdots\cdots - \cdots\cdots)(x + \cdots\cdots)$
7. $2x^2 - \cdots\cdots - \cdots\cdots = (2x + 3y)(\cdots\cdots - 2y)$
8. $5x^2 - 3xy - \cdots\cdots = (x - y)(\cdots\cdots + \cdots\cdots)$

2 Factorize each of the following expressions :

1. $2x^2 + 3x + 1$
2. $3a^2 + 7a + 2$
3. $5z^2 - 7z + 2$
4. $3x^2 - 14x - 5$
5. $5x^2 + 4x - 12$
6. $3x^2 + 10x + 8$
7. $6x^2 - 11x + 3$
8. $5a^2 - 18a + 16$
9. $3y^2 + 7y - 6$
10. $8z^2 + 2z - 3$
11. $4y^2 + 5y - 21$
12. $12a^2 - a - 6$

Unit 1

3 Factorize each of the following expressions :

1. $2x^2 - 5xy + 2y^2$
2. $3x^2 - 20xy - 7y^2$
3. $6a^2 + 5ab + b^2$
4. $2y^2 + yx - x^2$
5. $10a^2 + 11ab - 18b^2$
6. $6x^2 - 47xy - 63y^2$
7. $7x^4 + 23x^2y - 30y^2$

4 Factorize each of the following expressions :

1. $6x^2 - 21x + 18$
2. $8x^2 - 28x - 60$
3. $25m - 10 + 15m^2$
4. $8x^3 - 27x^2 - 20x$
5. $6x^3 + 14x^2 + 8x$
6. $18x^5 + 33x^3 - 30x$
7. $21x^2y^2 + 6x^2y^3 - 15x^2y^4$
8. $12(c+d)x^2 + 68(c+d)x + 80(c+d)$

5 Factorize each of the following :

1. $2x(x+3) + 13x + 24$
2. $4x(3x+7y) - 5y^2$
3. $5y^2 - 4x(7y + 3x)$
4. $(5b-2)^2 - 4b - 5$

6
1. If $(x+1)$ is a factor of the expression : $5x^2 - 2x - 7$, then find the second factor.
2. If $(2x-7)$ is a factor of the expression : $4x^2 - 8x - 21$, then find the second factor.

7 Find the value of $c \in \mathbb{Z}$ such that the algebraic expression can be factorized then factorize it :

1. $cx^2 + x - 15$
2. $cx^2 - 13x + 6$

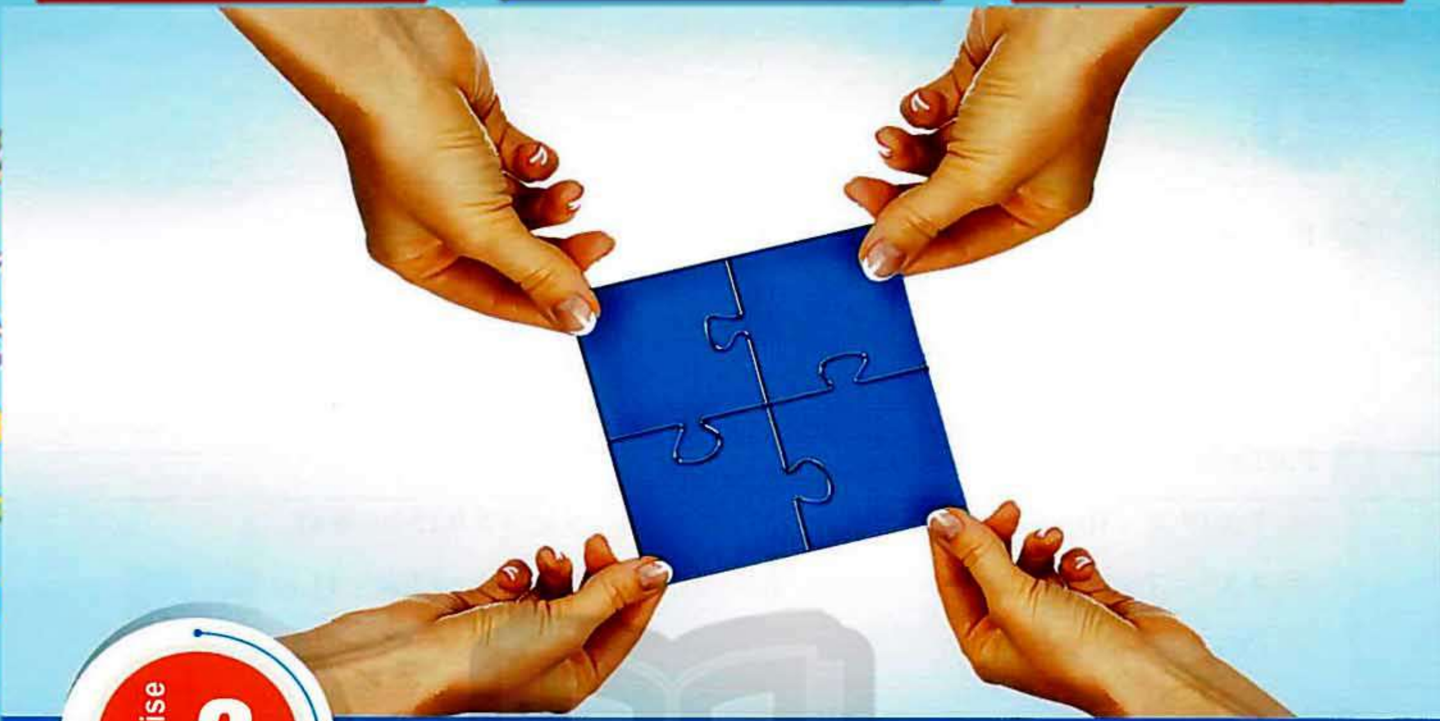
Geometric Application

8 The area of a rectangle is $(2x^2 + 19x + 35)$ cm². Find two possible dimensions of the rectangle in terms of x then find its perimeter as $x = 3$

For excellent pupils

9 Factorize each of the following :

1. $3 + 11(a+b) - 4(a+b)^2$
2. $3(2x+3y)^2 - (2x+3y)(x-y) - 2(x-y)^2$



Exercise 3 Factorizing the perfect square trinomials

From the school book

1 Show which of the following trinomials is a perfect square expression :

1. $a^2 + 9$
2. $a^2 - ab + b^2$
3. $x^2 - 12x + 36$
4. $25x^2 - 15x + 9$
5. $\ell^2 - 8\ell m + 16m^2$
6. $9x^2 + 15xy + 25y^2$
7. $4c^4 - 12c^2d - 9d^2$
8. $4 + 36a^3 + 81a^6$
9. $1 - 2a + a^2$
10. $4 - 6a + 8a^2$
11. $\frac{1}{4}y^2 - y + 4$
12. $0.01x^2 - 0.2x + 1$

2 Factorize each of the following :

1. $m^2 - 2m + 1$
2. $x^2 + 2xy + y^2$
3. $9x^2 + 12x + 4$
4. $25b^2 - 10b + 1$
5. $9a^2 + 6ab + b^2$
6. $4x^2 - 4xy + y^2$
7. $16a^2 - 40ab + 25b^2$
8. $1 + 14x + 49x^2$
9. $36 - 60k + 25k^2$
10. $1 - 10a^2 + 25a^4$

3 Factorize each of the following :

1. $18y^2 - 12y + 2$
2. $12x^2 + 36xy + 27y^2$
3. $24a^4 + 24a^2 + 6$
4. $6a^4 - 12a^2b^2 + 6b^4$
5. $20ay^2 - 60ay + 45a$
6. $24x + 24x^2 + 6x^3$
7. $3z + 42z^4 + 147z^7$
8. $4b^2c + bc^2 + 4b^3$
9. $60ab - 36a^2 - 25b^2$
10. $(c - d) + 2x(c - d) + x^2(c - d)$

Unit 1

4 **Factorize each of the following :**

1. $\frac{1}{4}y^2 - 2y + 4$
2. $\frac{1}{16}a^2 + \frac{1}{10}a + \frac{1}{25}$
3. $\frac{4}{25}x^2 - \frac{1}{10}x + \frac{1}{64}$
4. $0.01x^2 - 0.2x + 1$

5 **Factorize each of the following :**

1. $7x(7x - 10y) + 25y^2$
2. $9a^2 + 5b(5b - 6a)$
3. $4x^2 - 7y(4x - 7y)$
4. $m^2 - 11n(2m - 11n)$
5. $(x - y)^2 + 4xy$

6 **Complete the missing term in each of the following trinomial expressions to be a perfect square expression :**

1. $4x^2$ .... $+ 1$
2. $9a^2$ .... $+ 36$
3. $4a^2$ .... $+ 36b^2$
4. $\frac{1}{25}x^2$ .... $+ \frac{1}{4}y^2$
5. $36x^4$ .... $+ 9y^4$
6. z^4 .... $+ 49\ell^2$
7. $a^2 - 6a +$ ....
8. $4x^2 + 28x +$ ....
9. $a^2 - 6ab +$ ....
10. $25m^2 + 20mn +$ ....
11. .... $- 18y^2 + 81$
12. .... $- 24ab + 16b^2$

7 **Find the positive value of k which makes each of the following trinomial expressions a perfect square :**

1. $36x^2 + kx + 1$
2. $16y^2 + ky + 100$
3. $4x^2 + x + k$
4. $9\ell^2 + 12\ell + k$
5. $kx^2 - 6x + 1$
6. $ka^2b^2 - 12ab + 9$
7. $k^2x^2 - 4x + 16$
8. $1 + 14y + k^2y^2$

8 **Choose the correct answer from those given :**

1. If $x^2 + kx + 25$ is a perfect square , then k = ...
   (a) 5 (b) 10 (c) ± 10 (d) ± 5
2. If the expression : $x^2 + 14x + b$ is a perfect square , then b = ...
   (a) 2 (b) 7 (c) 14 (d) 49

3 The expression : $a x^2 - 40 x + 25$ is a perfect square when a = ...

(a) 2 (b) 4 (c) 9 (d) 16

4 If the expression : $c + 3 x + \frac{1}{4}$ is a perfect square , then c = ...

(a) 9 (b) $\frac{9}{4} x^2$ (c) $9 x^2$ (d) $4 x^2$

5 If $x = 6$, $y = 4$, then $x^2 - 2 x y + y^2$ = ...

(a) 2 (b) 4 (c) 10 (d) 100

6 If $a^2 + 2 ab + b^2 = 25$, then a + b = ...

(a) 5 (b) – 5 (c) ± 5 (d) 12.5

9 Use factorization to get the value of each of the following easily :

1 $(87)^2 + 2 \times 13 \times 87 + (13)^2$

2 $(99)^2 - 2 \times 99 \times 98 + (98)^2$

3 $(7.3)^2 + 2 \times 7.3 \times 2.7 + (2.7)^2$

4 $(20.7)^2 - 1.4 \times 20.7 + (0.7)^2$

5 $(997)^2 + 6 \times 997 + 9$

6 $(99)^2 + 2 \times 99 + 1$

7 $25 - 2 \times 45 + 81$

Geometric Application

10 The area of a square is $(9 x^2 + 30 x + m)$ cm^2. Find the value of m (given that the side length of the square is a rational number) , then find its perimeter when $x = 2$

For excellent pupils

11 Factorize each of the following :

1 $y^2 + 2 y (x + 1) + (x + 1)^2$

2 $(a + b)^2 - 4 c^2 (a + b) + 4 c^4$

3 $9 (x + 2)^2 + 12 (x + 2) (y - 3) + 4 (y - 3)^2$

Exercise 4 Factorizing the difference of two squares

📖 From the school book

1 **Factorize each of the following :**

1. 📖 $x^2 - 4$
2. $a^2 - 25$
3. $16x^2 - 9$
4. $49y^2 - 1$
5. $x^2 - 4y^2$
6. 📖 $225x^2 - y^2$
7. $49x^2 - 64y^2$
8. 📖 $625a^2 - 81b^2$
9. 📖 $9 - y^2$
10. 📖 $-9x^2 + 25$
11. $a^2b^2 - 1$
12. 📖 $a^2 - b^2c^4$
13. $x^4 - 100$
14. $16a^6 - b^6$
15. $x^2 - \frac{1}{16}$
16. $\frac{1}{9}y^2 - 2\frac{1}{4}$
17. $\frac{a^2}{25} - \frac{4b^2}{49}$
18. $0.04x^2 - 0.25y^2$

2 **Factorize each of the following perfectly :**

1. $x^4 - 1$
2. 📖 $x^4 - 16y^4$
3. 📖 $x^{100} - 1$

3 **Factorize each of the following :**

1. $2x^2 - 32$
2. $x^3 - 25x$
3. $x^4 - x^2$
4. 📖 $8x^2 - 50$
5. $x^3y - xy^5$
6. 📖 $27x^3 - 48xy^6$
7. $\frac{1}{3}x^2 - 3$
8. $3x^2 - \frac{3}{16}$
9. $\frac{1}{2}x^2 - \frac{1}{18}y^2$
10. $4b^2(2a - b) - 25a^2(2a - b)$

Exercise 4

4 **Factorize each of the following :**

1. $(a+b)^2-4$
2. $(2x-3)^2-1$
3. $1-(a-1)^2$
4. $9a^2-(2a+b)^2$
5. $a^2b^2-(ab-1)^2$
6. $(x+1)^2-(x-1)^2$
7. $9(m-1)^2-25(m+1)^2$
8. $(x+y+5)^2-(x-y-5)^2$
9. $(a-2b)(a+2b)-5b^2$

5 **Use factorization to get the value of each of the following easily :**

1. $(77)^2-(23)^2$
2. $(78)^2-(77)^2$
3. $(11.6)^2-(1.6)^2$
4. $(8.27)^2-(1.73)^2$
5. $(95)^2-25$
6. $(999)^2-1$
7. $2\times(25.87)^2-2\times(24.13)^2$

6 **Using the idea of factorizing the difference between two squares , find the value of each of the following :**

1. 31×29
2. 103×97

7 If $xy=8$, find the numerical value of the expression $(x+y)^2-(x-y)^2$

8 **Complete the following :**

1. $(2x+\cdots\cdots)(\cdots\cdots-3y)=4x^2-\cdots\cdots$
2. $(\cdots\cdots+3m)(\cdots\cdots-3m)=25x^2-\cdots\cdots$
3. $\cdots\cdots-64x^2=(4-\cdots\cdots)(4+\cdots\cdots)$
4. If $a-b=2$, $a+b=3$, then $a^2-b^2=\cdots\cdots$
5. If $x^2-y^2=20$, $x+y=10$, then $x-y=\cdots\cdots$
6. If $a^2-b^2=45$, $a-b=5$, then $\sqrt{a+b}=\cdots\cdots$
7. If $x^2-y^2=24$, $x+y=8$, then $3x-3y=\cdots\cdots$
8. If $x^2-y^2=x+y$, then $x-y=\cdots\cdots$
9. If $2(a-b)(a+b)=18$, then $a^2-b^2=\cdots\cdots$
10. If $a+b=7(a-b)=14$, then $a^2-b^2=\cdots\cdots$

9 **Choose the correct answer from those given :**

1. If $x^2-a=(x-3)(x+3)$, then $a=\cdots\cdots$

   (a) 3 (b) -3 (c) 9 (d) -9

2 If $x^2 + \ell - 4 = (x - 2)(x + 2)$, then $\ell = \cdots\cdots$

(a) zero (b) 2 (c) 4 (d) 8

3 If $x + 2y = 3$, $x^2 - 4y^2 = 21$, then $x - 2y = \cdots\cdots$

(a) 14 (b) 9 (c) 7 (d) 6

4 If $a - b = 7$, $a + b = 5$, then $2a^2 - 2b^2 = \cdots\cdots$

(a) 2 (b) 12 (c) 35 (d) 70

5 If $x^2 - y^2 = 16$, $y - x = 2$, then $x + y = \cdots\cdots$

(a) 4 (b) 8 (c) -8 (d) 2

6 If $a + b = 5$, $a - b = 4$, then $b^2 - a^2 = \cdots\cdots$

(a) -20 (b) -1 (c) 9 (d) 20

7 If $(25)^2 - (15)^2 = 10x$, then $x = \cdots\cdots$

(a) 40 (b) 30 (c) 20 (d) 10

8 $(x - y)(x + y)(x^4 - 2x^2y^2 + y^4) = \cdots\cdots$

(a) $x^6 - y^6$ (b) $(x - y)^3 (x + y)^3$

(c) $(x^3 - y^3)(x^3 + y^3)$ (d) $(x^2 + y^2)(x^2 - y^2)$

Geometric Application

10 A right-angled triangle whose hypotenuse = 41 cm. long and the length of one side of the right angle is 40 cm. Use factorization to get the length of the second side of the right angle.

For excellent pupils

11 **Factorize each of the following :**

1 $(a^2 - 2ab + b^2) - c^2$

2 $(2a + 3b)^3 - 8a^3 - 12a^2b$

12 If $(225)^2 = (25)^2 + 25 \times 20x$, find the value of : x

13 If $x > y$, $x^2 - 2xy + y^2 = 4$, $x + y = 8$,
find the numerical value of the expression : $x^2 - y^2$

Summary of the first part of Unit 1
"From lesson 1 to lesson 4"

- **Before factorizing the algebraic expression , you must do the following :**
  - Arrange the terms of the expression descendingly or ascendingly according to the indices (exponents) of one of the given algebraic symbols. It is better to be descending.
  - Take out the H.C.F. of the terms of the expression.
  - Perform operations included in arcs and simplify the algebraic expression.

- **Factorizing the trinomial which is in the form :** $x^2 + b\,x + c$ is to write it as the product of two factors such that :
  - The first term in each factor is x
  - The two other terms in the two factors are two numbers whose product is c which is the last term in the trinomial , and their sum is b which is the coefficient of x in the trinomial.

- **When you factorize the trinomial :** $x^2 + b\,x + c$ in the form $(x + \ell)\,(x + m)$ **, then :**
  1. If c is positive ***i.e.*** The product of the two numbers is positive , then ℓ and m have the same sign as b
  2. If c is negative ***i.e.*** The product of the two numbers is negative , then ℓ and m have different signs such that the great one (numerically) has the same sign as b

- **To factorize the trinomial :** $a\,x^2 + b\,x + c$ where $(a \neq \pm 1)$, do as follows :
  1. Factorize $a\,x^2$ into two factors "$\ell\,x$, $m\,x$" and write them inside two parentheses as shown in the opposite figure.
  2. Factorize the last term in the trinomial (c) into two factors "n and h" and write them as shown in the previous parentheses.
  3. Find : "The product of extremes (outer terms) + the product of means (inner terms)" If the sum equals the middle term in the trinomial , then the factorization is true. If not , then the factorization is false hence , you should try again to get the true factorization.

$(\ell\,x \quad + n)$

$(m\,x \quad + h)$

Factorizing the perfect square trinomial :

If the trinomial is a perfect square arranged descendingly or ascendingly according to the powers of one of its symbols , then we can factorize it to be in the form :

$$\left(\sqrt{\text{The first term}} \pm \sqrt{\text{The third term}}\right)^2$$

Notice that : The sign between the two terms inside the parentheses is the same sign of the middle term in the trinomial.

How to find a missing term of a perfect square trinomial :

1. Finding the middle term :

The middle term $= \pm\, 2 \times \sqrt{\text{the first term}} \times \sqrt{\text{the third term}}$

2. Finding the first term :

The first term $= \dfrac{(\text{the middle term})^2}{4 \times \text{the third term}}$

3. Finding the third term :

The third term $= \dfrac{(\text{the middle term})^2}{4 \times \text{the first term}}$

Factorizing the difference between two squares :

The difference of two squares of two quantities

= (the sum of the two quantities) × (the difference of the two quantities)

i.e. $a^2 - b^2 = (a + b)(a - b)$

Exams on the first part of Unit One
from lesson (1) to lesson (4)

Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1. If $x^2 + k x + 49$ is a perfect square , then k = ...
   (a) 7 (b) – 7 (c) ± 7 (d) ± 14
2. If $x + 2 y = 4$, $x^2 - 4 y^2 = 20$, then $x - 2 y$ = ...
   (a) 4 (b) 0 (c) 5 (d) 80
3. If the expression : $x^2 + 5 x + a$ can be factorized , then a may be equal to ....
   (a) 6 (b) 8 (c) 10 (d) 12
4. If the expression : $x^2 + 16 x + b$ is a perfect square , then b = ...
   (a) 8 (b) 25 (c) 16 (d) 64
5. If $2 x^2 + c x - 3 = (2 x + 3)(x - 1)$, then c = ...
   (a) 3 (b) 1 (c) – 2 (d) 4
6. If $x^2 - 3 x + a = (x - 5)(x + 2)$, then a = ...
   (a) – 3 (b) 5 (c) – 10 (d) 4

2 Complete the following :

1. $x^2 -$ .... $= (x + 9)(x - 9)$
2. $(3 a -$ ....$)^2 =$ .... $- 12 a +$ ....
3. $(x +$ ....$)(2 x - 3) =$ .... $+$ .... $- 15$
4. If $x^2 - k + 10 = (x - 3)(x + 3)$, then k = ...
5. If $(x - 3)$ is a factor of the expression : $x^2 + 2 x - 15$, then the other factor is ....

3 Factorize each of the following :

1. $x^2 - 7 x - 8$
2. $3 x^2 - 10 x - 8$
3. $8 x^3 - 2 y^2 x$
4. $4 x^2 + 12 x + 9$

4 Use factorization to get the value of each of the following :

1. $(99)^2 - 2 \times 99 \times 97 + (97)^2$
2. $(78)^2 - (22)^2$

5 The area of a rectangle is $(x^2 + 10 x + 24)$ cm^2 and its length is $(x + 6)$ cm.
Find each of its width and its perimeter in terms of x

Unit 1

Model 2

Answer the following questions :

1 Choose the correct answer from those given :

1. If $x^2 - m = (x - 7)(x + 7)$, then m = ...
   (a) 7 (b) – 7 (c) 49 (d) – 49
2. The expression : $a x^2 - 36 x + 36$ is a perfect square when a = ...
   (a) 3 (b) 6 (c) 9 (d) 4
3. If $x^2 - k x + 12 = (x - 3)(x - 4)$, then k = ...
   (a) 1 (b) 5 (c) 12 (d) 7
4. If $2 x^2 - 3 x - 35 = (2 x + a)(x - 5)$, then a = ...
   (a) 5 (b) 7 (c) 10 (d) 2
5. If $a + 5 b = 3$, $a^2 - 25 b^2 = 12$, then a – 5 b = ...
   (a) 9 (b) 16 (c) 3 (d) 4
6. If $x^2 + k x + 64$ is a perfect square , then k = ...
   (a) 8 (b) 16 (c) 32 (d) 64

2 Complete the following :

1. If $a + b = 5$, $b - a = 4$, then $b^2 - a^2$ = ...
2. If $x^2 + y^2 = 17$, $x y = -9$, then $(x - y)^2$ = ...
3. If $(2 x - 1)$ is one of the factors of the expression : $2 x^2 + 9 x - 5$, then the other factor is ....
4. $(3 x + ...)(.......... - 5 y) = 9 x^2 - ....$
5. $x^2 + x - 6 = (x + 3)(..........)$

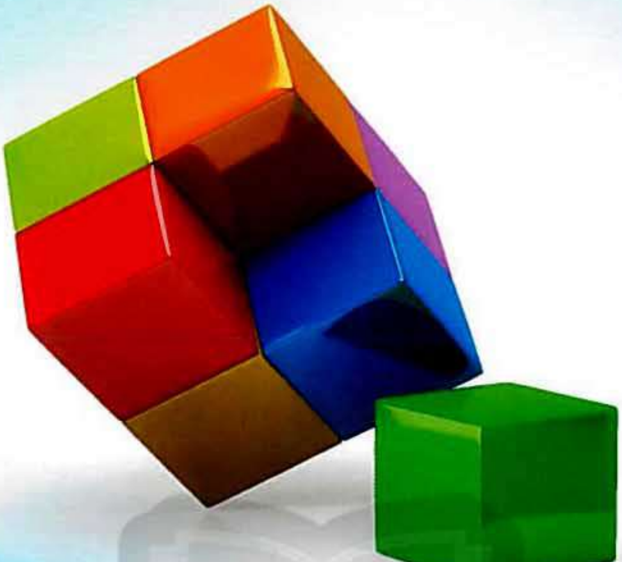
3 Factorize each of the following :

1. $x^2 + 21 - 10 x$
2. $25 a^4 - 10 a^2 + 1$
3. $8 a^2 - 50$
4. $5 z^2 - 7 z + 2$

4 A rectangle of area $(5 a^2 - 18 a + 16)$ cm².
Find two possible dimensions for it in terms of a , then its perimeter when a = 5

5 Factorize each of the following :

1. $4 x^2 y - 4 x y^2 + y^3$
2. $(a + b + 3)^2 - (a - b - 3)^2$



Exercise 5 — Factorizing the sum and difference of two cubes

📖 From the school book

1 Factorize each of the following :

1 📖 $x^3 + 8$	2 $x^3 - 1$	3 $64x^3 + 27$
4 📖 $8x^3 - 125$	5 $125 + a^3$	6 $343 - 27m^3$
7 📖 $m^3 + 64n^3$	8 📖 $512x^3 - y^3$	9 $x^3y^3 + 27$
10 $27x^3y^3 - 64$	11 📖 $\frac{1}{8}a^3 - 8b^3$	12 $\ell^3 - \frac{1}{125}$
13 $8a^3 + 0.001$	14 📖 $0.027m^3 - n^3$	15 $1 + 125b^6$
16 📖 $8x^3 - 343y^6$	17 $x^6 + y^6$	18 $x^6 - 64$

2 Factorize each of the following perfectly :

1 $2x^3 + 16$	2 $3x^3 - 81$	3 $\ell^4 + 64\ell$
4 📖 $\ell^3m - 27m^4$	5 $3x^4 + 3x$	6 $2x^5 - 54x^2$
7 $16x^3 + 250y^3$	8 📖 $16a^3b + 686b^4$	9 $54x^4y^2 - 16xy^5$
10 $500x^8y^2 - 256x^5y^5$	11 $\frac{1}{2}x^3 + 4$	12 $\frac{1}{3}x^3 - 9$

3 Choose the correct answer from those given :

1 📖 If $x + y = 3$, $x^2 - xy + y^2 = 5$, then $x^3 + y^3 =$ ....

(a) 15 (b) 25 (c) 8 (d) 7

2 If $x^3 - y^3 = 14$, $x^2 + xy + y^2 = 7$, then $x - y =$ ....

(a) 2 (b) 7 (c) 14 (d) -2

3 If $x^3 + y^3 = 28$, $x + y = 2$, then $x^2 - xy + y^2 =$ ....

(a) 28 (b) 14 (c) 2 (d) 7

Unit 1

4 If $y^3 - a = (y - 2)(y^2 + 2y + 4)$, then a = ...

(a) 2 (b) 4 (c) 8 (d) – 8

5 If $x^3 - 8 = (x + a)(x^2 + 2x + 4)$, then a = ...

(a) 4 (b) – 4 (c) 2 (d) – 2

6 If $x^3 + 27 = (x + 3)(x^2 + k + 9)$, then "k" equals ....

(a) $-6x$ (b) $-3x$ (c) $3x$ (d) $6x$

7 $x^3 - k^3 = (x - k)(x^2 + 4x + k^2)$, then k = ...

(a) 2 (b) 4 (c) 16 (d) 64

8 $(x - y)(x + y)(x^4 + x^2y^2 + y^4)$ = ...

(a) $x^3 - y^3$ (b) $x^3 + y^3$ (c) $x^6 - y^6$ (d) $x^6 + y^6$

4 Complete the following to get true statements :

1 $x^3 - 1 = (x - 1)$ (....................)

2 $8a^3 + 125$ = (.......... + ...) ($4a^2 - 10a$ + ...)

3 $x^{12} + y^{15}$ = (.......... + ...) (.......... – .... + ...)

4 $8a^3$ – .... = (.......... – ....) (.......... + ... + 9)

5 If $(x - 3)$ is a factor of the expression : $x^3 - 27$, then the second factor is ....

6 If $(4a^2 - 2a + 1)$ is a factor of the expression : $8a^3 + 1$, then the other factor is ....

5 If $x^2 - y^2 = 20$, $x - y = 2$, $x^2 - xy + y^2 = 28$ Find the value of : $x^3 + y^3$

6 Factorize each of the following :

1 $(a + b)^3 - b^3$

2 $(x + 5)^3 - 125$

3 $(m - 2n)^3 - 8n^3$

4 $8(m + n)^3 - n^3$

5 $2 - 2(x - 1)^3$

6 $(x + 5)^3 + (x - 5)^3$

7 $(x + y)^3 - (x - y)^3$

8 $(m - n) + (m - n)^4$

9 $(x^3 - 2)(x^3 + 2) - 4$

10 $(x - 3)(x^2 + 3x + 9) + 28$

7 Factorize each of the following :

1 $m^6 - 3m^3 + 2$

2 $y^6 + 26y^3 - 27$

3 $x^6 - 28x^3 + 27$

4 $x^6 - 7x^3 - 8$

For excellent pupils

8 Factorize perfectly : $(x + 5)^4 - x - 5$

9 If $xy = 2$, $x - y = 1$, then find the value of : $x^3 - y^3$

10 Two integers whose sum is 2 and the sum of their squares = 34 Find the sum of their cubes.

Exercise 6 — Factorizing by grouping

From the school book

1 Factorize each of the following perfectly :

1. $a x + b x + a y + b y$
2. $ab - bd + ah - dh$
3. $a x + y x + y + a$
4. $am - an + m - n$
5. $a x - c y - c x + a y$
6. $m x - m y - n x + n y$
7. $x y + 5 y + 7 x + 35$
8. $7 x - 28 + a x - 4 a$
9. $5 \ell - 10 m - a \ell + 2 am$
10. $3 a x - a - 6 b x + 2 b$

2 Factorize each of the following perfectly :

1. $c^2 + cd + dh + ch$
2. $6 m^2 - n + 2 m - 3 mn$
3. $8 mn - 2 m^2 + 12 n \ell - 3 m \ell$
4. $x^2 - 2 x z - 2 x y + 4 y z$
5. $a^2 + 2 ab + b^2 - c^2$
6. $25 x^2 - 10 x + 1 - y^2$
7. $1 - x^2 - 4 x y - 4 y^2$
8. $x^2 - y^2 + 4 x + 4 y$
9. $x^2 - 5 x - 4 y^2 + 10 y$
10. $9 x^2 - 4 a^2 + y^2 + 6 x y$
11. $2 x^2 y - x y^2 + 2 a x - a y$
12. $ab x^2 + b x - a x - 1$

3 Factorize each of the following perfectly :

1. $a^3 + a^2 + a + 1$
2. $x^3 - 3 x^2 + 6 x - 18$
3. $a^3 + b^3 - a - b$
4. $x^3 + 2 x^2 - x - 2$
5. $a^3 - 9 a + a^2 - 9$
6. $3 x^3 + 2 x^2 + 12 x + 8$
7. $y^3 + 6 y^2 + 12 y + 8$
8. $a^4 - 3 a^3 - 15 a + 5 a^2$
9. $a^5 - 2 a^2 + a^3 - 2$
10. $x^2 y^3 + 8 x^2 - y^3 - 8$

Unit 1

4 **Factorize each of the following perfectly :**

1) $x^5 - x^3 - x^2 + 1$

2) $4 m^4 - 9 m^2 + 6 m - 1$

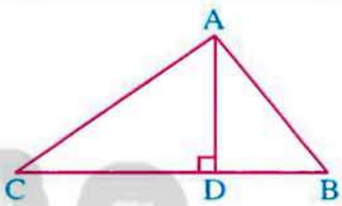
3) $121 x^4 - 100 x^2 - 20 x - 1$

Geometric Application

5 A rectangle whose area is $(x^3 + 5x^2 + 3x + 15)$ cm², and width $(x + 5)$ cm. , find its length in terms of x , then find its perimeter when $x = 2$ cm.

6 **In the opposite figure :**

ABC is a triangle of area $\frac{1}{2}(x^3 + 2x + 2x^2 + 4)$ cm². If its altitude AD is $(x + 2)$ cm. , what is the length of its base $\overline{BC}$?



For excellent pupils

7 **Factorize each of the following perfectly :**

1) $2x^3(x + 3) - 18x^2 - 54x$

2) $a(a + 4b) + 4b^2 - 9$

3) $a^3 - 4b^3 + ab(a - 4b)$

4) $a^2(b - 5) - 7a(b - 5) - 18b + 90$

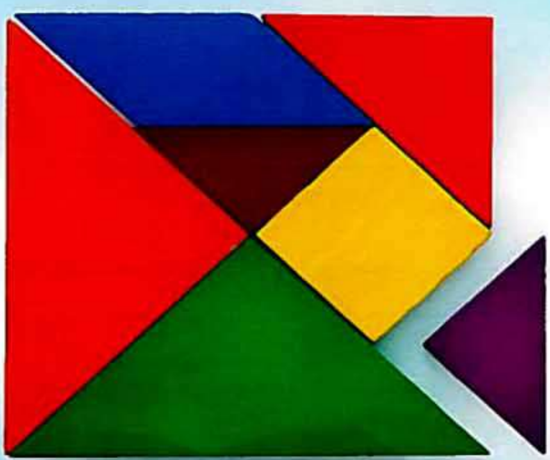
8 **Factorize each of the following perfectly :**

1) $x^2 - 4xy + x - 2y + 4y^2$

2) $3x^2 - 15x - 72 - xy + 8y$

3) $a^3 + a - 2$

4) $a^3 + a^2 + 4$



Exercise 7 Factorizing by completing the square

From the school book

1 **Factorize each of the following perfectly :**

1. $x^4 + 4$
2. $x^4 + 64$
3. $x^4 + 4y^4$
4. $x^4 + 64y^4$
5. $a^4 + 2500b^4$
6. $81x^4 + 4z^4$
7. $4x^4 + 625z^4$
8. $64x^4 + 81y^4$
9. $12x^4 + 3y^4$
10. $8x^4y^2 + 162z^4y^2$

2 **Factorize each of the following completely :**

1. $9x^4 + 2x^2 + 1$
2. $x^4 - 28x^2 + 16$
3. $x^4 + 9x^2 + 81$
4. $9x^4 - 25x^2 + 16$
5. $x^4 + 3x^2y^2 + 4y^4$
6. $m^4 - 11m^2n^2 + n^4$
7. $x^4 + x^2y^2 + 25y^4$
8. $a^4 + 4a^2b^2 + 16b^4$
9. $x^4 + y^4 - 7x^2y^2$
10. $16x^4 - 28x^2y^2 + 9y^4$
11. $4x^4 + 25y^4 - 29x^2y^2$
12. $3m^4 + 3n^4 - 54m^2n^2$
13. $50x^4 + 18y^4 - 68x^2y^2$
14. $18ab^4 - 114b^2c^2a + 128ac^4$

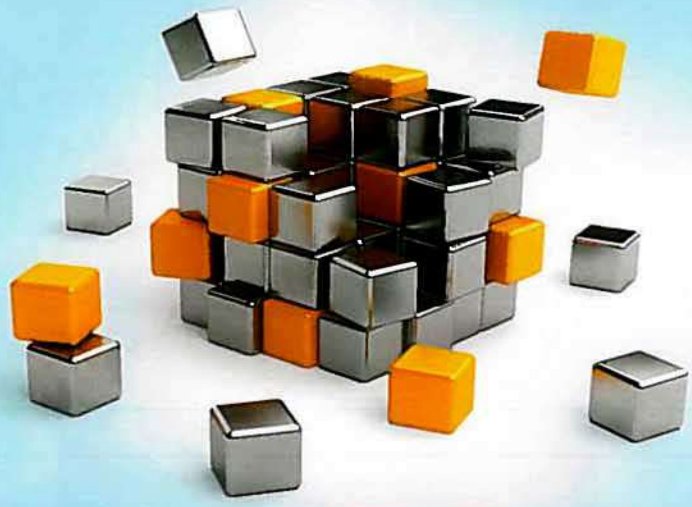
3 **Factorize each of the following completely :**

1. $x^2(9x^2 - 10y^2) + y^4$
2. $x^2(x^2 - 19y^2) + 25y^4$
3. $4x^2(4x^2 - 7y^2) + y^4$
4. $4a^2(a^2 - 6b^2) + 9b^4$

For excellent pupils

4 **Factorize each of the following completely :**

1. $x^8 - 16y^8$
2. $x^8 - 21x^4 - 100$
3. $x^8 - 5x^4y^4 - 36y^8$
4. $81x^8 - 17x^4y^4 - 64y^8$



GENERAL EXERCISE On factorizing the algebraic expressions

Factorize each of the following perfectly :

1. $25x^2 - 9y^2$
2. $2x^5 + 54x^2$
3. $2y^2 + 5y + 3$
4. $2x^4 - 18$
5. $2x^2 - 20x + 48$
6. $x^2 + 8x + 16$
7. $8x^3 + 27$
8. $y^2 - 50y - 51$
9. $25x^2 - 30x + 9$
10. $x^2 - 81$
11. $y^5 - y$
12. $3x^2 + 7x - 6$
13. $x^2 - 8x + 12$
14. $3x^3 + 2x^2 + 12x + 8$
15. $x^3 - 125$
16. $4x^2 - 12x + 9$
17. $a^3 + 3a^2 - 9a - 27$
18. $(x+2)^3 - 4x - 8$
19. $-2x^2 - 15x - 7$
20. $x^2 - 7x + 10$
21. $4x^4 + y^4$
22. $9x^4 - 16y^4$
23. $x^4 - 9x^2 + 20$
24. $1 - 4x^2$
25. $a^6 - 625b^6$
26. $(x+y)^3 - x^3$
27. $49x^2 + 70xy^2 + 25y^4$
28. $5x^2 - 3x - 2$
29. $x^4 - 11x^2y^2 + y^4$
30. $3x^4 - 15x^3 + 12x^2$
31. $3x^2 - 19x + 6$
32. $4x^2 + 28xy + 49y^2$
33. $x^6 - 64y^6$
34. $2y^4 - 4y^3 + 7y - 14$
35. $15a^4 - 21b^2 - 6a^2b$
36. $6x^2 + y(2y - 7x)$
37. $64x^4 + y^4$
38. $x^4 - 5x^2 - 24$
39. $20x^4 + 40x^2y^2 + 45y^4$
40. $9x^4 - 13x^2y^2 + 4y^4$

Exercise 8 Solving quadratic equations in one variable algebraically

From the school book

1 **Find in $\mathbb{R}$ the S.S. of each of the following equations :**

1. $x^2 - 6x = 0$
2. $x^2 - 16 = 0$
3. $4x^2 - 25 = 0$
4. $x^2 + 5x + 6 = 0$
5. $x^2 - 8x + 15 = 0$
6. $x^2 + 7x - 18 = 0$
7. $x^2 - x - 20 = 0$
8. $6x^2 - 7x - 3 = 0$
9. $2x^2 + 7x - 4 = 0$
10. $2x^2 - 5x - 3 = 0$
11. $x^2 + 4x + 4 = 0$
12. $9x^2 - 6x + 1 = 0$

2 **Find in $\mathbb{R}$ the S.S. of each of the following equations :**

1. $x^2 = x$
2. $3x^2 = 7x$
3. $4x^2 = 49$
4. $x^2 + x = 6$
5. $x^2 - 8x = -15$
6. $x^2 - 15 = 2x$
7. $2x^2 - 10x = -12$
8. $6x^2 - x = 22$
9. $5x^2 + 12x = 44$
10. $12x^2 = 47x - 45$
11. $5(x^2 + 3) = 60$
12. $x(x - 3) = 5x$

3 **Find in $\mathbb{R}$ the S.S. of each of the following equations :**

1. $x(x - 5) + 6 = 0$
2. $x(x + 3) = 10$
3. $(x - 3)(x + 1) = 5$
4. $(x + 8)(x - 3) = 3x$
5. $2x(x - 5) - 4(5 - x) = 0$
6. $(x + 3)^2 - 49 = 0$
7. $4(x + 5)^2 = 25$
8. $(x - 1)^2 + x = 3$

9) $2(x+3)^2 + 7(x+3) = 0$

10) $(2x+1)^2 = (3x-1)^2$

11) $(2x-1)^2 + (x-1)^2 = 10$

12) $(x+3)^2 + 3(x+3) - 10 = 0$

4 **Find in ℝ the S.S. of each of the following equations :**

1) $2x^3 - 8x = 0$

2) $4x^3 = 9x$

3) $x^4 - 5x^2 + 4 = 0$

4) $x^4 - 26x^2 = -25$

5) $x^4 - 16 = 0$

5 **Find in ℝ the S.S. of each of the following equations :**

1) $y^2 - \frac{7y}{3} = -\frac{4}{3}$

2) $x^2 - \frac{2x+3}{2} = \frac{9}{2}$

3) $x + \frac{2}{x} = 3$

4) $x - \frac{5}{x} = \frac{1}{2}$

5) $\frac{x-1}{5} = \frac{6}{x}$

6 **Choose the correct answer from those given :**

1) The S.S. of the equation : $x(x-2) = 0$ in ℝ is ....

(a) $\{0\}$ (b) $\{0, -2\}$ (c) $\{0, 2\}$ (d) $\{2\}$

2) The S.S. of the equation : $3(x-2)(x+5) = 0$ in ℝ is ....

(a) $\{0, 2, -5\}$ (b) $\{3, 2, -5\}$ (c) $\{2, -5\}$ (d) $\{-2, 5\}$

3) The S.S. of the equation : $x^2 - 4 = 0$ in ℝ is ....

(a) $\{4\}$ (b) $\{4, -4\}$ (c) $\{2\}$ (d) $\{2, -2\}$

4) The S.S. of the equation : $x^2 + 25 = 0$ in ℝ is ....

(a) $\{5\}$ (b) $\{5, -5\}$ (c) $\{-5\}$ (d) $\varnothing$

5) The S.S. of the equation : $(x-4)^2 = 0$ in ℝ is ....

(a) $\{4\}$ (b) $\{0, 4\}$ (c) $\{0, -4\}$ (d) $\{-4\}$

6) The S.S. of the equation : $x(x-3) = 5x$ in ℝ is ....

(a) $\{3\}$ (b) $\{0, 3, 5\}$ (c) $\{3, 5\}$ (d) $\{0, 8\}$

7) The S.S. of the equation : $\frac{4}{x} = \frac{x}{9}$ in ℝ is ....

(a) $\{4, 9\}$ (b) $\{6, -6\}$ (c) $\{6\}$ (d) $\{36\}$

8) The equation whose roots are 3 and 5 is ....

(a) $5x^2 + 8x + 3 = 0$ (b) $2x^2 + 8x - 15 = 0$

(c) $x^2 - 8x + 15 = 0$ (d) $3x^2 + 8x + 5 = 0$

7 **Complete the following :**

1. If -5 is a root of the equation : $x^2 + 2x - 15 = 0$
   , then the other root is ....
2. If $x = 2$ is a root of the equation : $x^2 - 6x + k = 0$, then k = ...
   and the other root is ....
3. If one of the roots of the equation : $2x^2 + 8x = 0$
   is a root of the equation : $x^2 + 5x + a = 0$, then a = ... or ....
4. The S.S. of the equation : $x - \frac{2}{x} = \frac{7}{2}$ is ....

8 If $x + \frac{1}{x} = 2$, then find the numerical value of the expression : $x^2 + \frac{1}{x^2}$

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9 If $x^2 + \frac{1}{x^2} = 34$, then find the numerical value of the expression : $x + \frac{1}{x}$

10 **Find in $\mathbb{R}$ the S.S. of the equation :** $\frac{x(x-2)}{6} - \frac{x(x+1)}{4} + \frac{7(x-3)}{3} - 2 = 0$

11 If 2 and 3 are two solutions of the equation : $x^2 - bx + c = 0$
, find the value of b and c

Exercise 9

Applications on solving quadratic equations in one variable algebraically

From the school book

1 **Choose the correct answer from those given :**

1 If the age of Bassim now is x years , then his age 3 years ago was .... years.
(a) $3x$ (b) $x+3$ (c) $x-3$ (d) x^3

2 If the age of Amgad now is x years , then his age after 7 years will be .... years.
(a) $7x$ (b) $x-7$ (c) $x+7$ (d) x^7

3 If the age of Ayman 5 years ago was x years , then his age now is .... years.
(a) $x-5$ (b) $x+5$ (c) $5x$ (d) $\frac{x}{5}$

4 If the age of Sally 2 years ago was x years , then her age after 3 years from now will be .... years.
(a) $x+2$ (b) $x+3$ (c) $x+5$ (d) $6x$

5 If the age of Magdy now is x years , then the square of his age after 2 years is ....
(a) x^2+2 (b) x^2+4 (c) $(x-2)^2$ (d) $(x+2)^2$

6 If the age of Samy now is x years , then twice his age 5 years ago is .... years.
(a) $x-5$ (b) $2x-5$ (c) $x-10$ (d) $2x-10$

7 Three times the square of the number x is ....
(a) $(3x)^2$ (b) x^2+3 (c) $3x^2$ (d) $\frac{x^2}{3}$

2 A positive integer whose square is more than five times the number by 36

Find the number. « 9 »

Exercise 9

3 An integer , if we add twice its square to the number 7 the result will be 135

Find the number. « 8 or – 8 »

4 Find the rational number whose four times its square equals 81 « $\frac{-9}{2}$ or $\frac{9}{2}$ »

5 A positive integer whose square equals six times the number. Find the number. « 6 »

6 What is the real number if it is added to its square , the result will be 12 ? « 3 or – 4 »

7 Find the positive rational number whose square is more than its twice by 48 « 8 »

8 Divide the number 20 into two numbers whose product is 75 « 15 , 5 »

9 Two real numbers , the difference between them is 5 and the sum of their squares is 73

Find the two numbers. « 3 , 8 or – 3 , – 8 »

10 Find two real numbers whose product is 45 and one of them is 4 more than the other.

« – 9 , – 5 or 5 , 9 »

11 The sum of the squares of two successive odd numbers is 130

Find the two numbers. « – 9 , – 7 or 7 , 9 »

12 The sum of three successive integers is equal to the square of their middle integer.

Find these numbers. « 2 , 3 and 4 or – 1 , 0 and 1 »

13 Two integers , the ratio between them is 7 : 8 and their product is more than nine times the greater number by 80 Find the two numbers. « 14 , 16 »

14 A positive integer , if we add twice its square to its additive inverse the result will be 91

Find this number. « 7 »

15 What is the real number which exceeds its multiplicative inverse by $\frac{5}{6}$? « $\frac{3}{2}$ or $\frac{-2}{3}$ »

16 A number is formed from two digits , its units digit is twice the tens digit and the product of the two digits exceeds their sum by 9

Find the number. « 36 »

Life Applications

17 The square of age of Said now is more than three times his age four years ago by 192
Find his age now. « 15 years »

18 Hatem is 4 years older than Hanan now , and the sum of squares of their ages now is 26
Find their ages now. « 5 years , one year »

19 If the age of Kamal now is more than the age of his brother Anees by 3 years and 4 years ago the product of their ages was 18
Find the age of each of them now. « 7 years , 10 year »

Geometric Applications

20 Find the dimensions of a rectangle whose length is 4 cm. more than its width and whose area is 21 cm^2. « 3 cm. , 7 cm. »

21 A rectangle whose area is 46 cm^2. and its length is 7.5 cm. more than its width.
Find its perimeter. « 31 cm. »

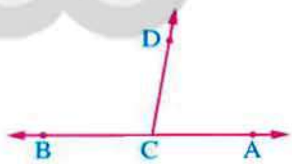
22 A rectangle whose length is more than its width by 5 cm. If its area is less than the area of a square whose side length is three times the width of the rectangle by 57 cm^2.
Find the two dimensions of the rectangle and the side length of the square.
« 3 cm. , 8 cm. , 9 cm. »

23 **In the opposite figure :**

$\overrightarrow{CD} \cap \overleftrightarrow{AB} = \{C\}$ If m (∠ BCD) = $(x^2)^\circ$,

m (∠ ACD) = 8 x°

Calculate the value of : x « 10° »



24 In the triangle ABC : m (∠ A) = $(x^2 + 61)^\circ$,

m (∠ B) = $(110 - 11x)^\circ$ and m (∠ C) = $(90 - 7x)^\circ$

Find the value of x and the measures of all angles. « 9° , 142° , 11° , 27° »

25 A right-angled triangle , the length of one side of the right angle is more than the length of the other side of the right angle by 2 cm. and its area = 24 cm^2.
Find the lengths of the sides of the right angle. « 8 cm. , 6 cm. »

26 A right-angled triangle whose two right angle sides lengths are $(5 X + 3)$ cm. and $(X + 5)$ cm. and its area is 24 cm^2. Calculate its perimeter. « 24 cm. »

27 A right-angled triangle whose sides lengths are $(2 X)$ cm. , $(2 X + 1)$ cm. and $(X - 11)$ cm. Find the value of X and calculate the perimeter and the area of the triangle.

« 20 , 90 cm. , 180 cm^2. »

28 A rectangle whose length is twice its width , if its length increases by 1 cm. and its width decreases by 1 cm. , then its area decreases by 7 cm^2.
Find the length and the width of the rectangle. « 6 cm. , 12 cm. »

For excellent pupils

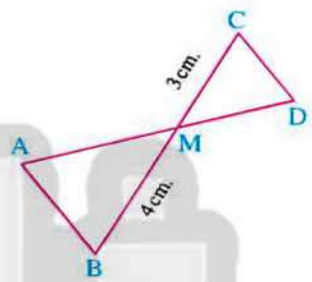
29 **In the opposite figure :**

Δ MCD ~ Δ MAB ,

MB = 4 cm. , MC = 3 cm. ,

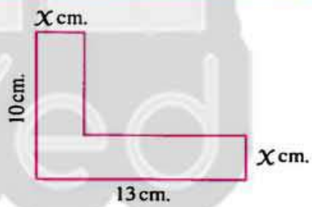
AD = 7 cm. , MA > MC

Find the length of $\overline{MA}$



« 4 cm. »

30 If the area of the opposite figure = 60 cm^2.
, find the value of X



« 3 cm. »

31 A room whose width is 9 metres and its length is 12 metres.
A decorator wanted to buy a carpet for the room in condition that he left around the carpet a rectangular tape of a fixed width uncovered. Calculate the width of the tape if the carpet covers half the area of the room. « 1.5 m. »

Summary of the second part of Unit 1
"From lesson 5 to lesson 9"

Factorizing the sum of two cubes.

The sum of the two cubes of two quantities
= (the first + the second) (the square of the first − the first × the second + the square of the second)

i.e. $a^3 \oplus b^3 = (a + b)(a^2 \ominus ab + b^2)$

Factorizing the difference between two cubes :

The difference between the two cubes of two quantities =
(the first − the second) (the square of the first + the first × the second + the square of the second)

i.e. $a^3 \ominus b^3 = (a - b)(a^2 \oplus ab + b^2)$

Factorizing by grouping :

The algebraic expression consisting of four terms can be factorized by one of the following two methods :

The first method

The algebraic expression consisting of four terms is divided into two expressions each of them consisting of two terms , such that you can find a common factor between the two terms.

The second method

The algebraic expression consisting of four terms is divided into a trinomial (must be perfect square) and a monomial (must be perfect square also) , such that the main expression should be factorized as a difference between two squares.

Factorizing by completing the square :

1. We add to the given expression twice the product of the two square roots of the two perfect square terms and subtract it again not to change the main expression.
2. Using the commutative and associative properties , we rewrite the expression after ordering its terms to get the form :
   a perfect square trinomial − a perfect square monomial
3. We factorize the resultant expression as a difference between two squares.
4. If it is possible , we should factorize the resultant expressions (resultant factors) in order that the factorization is perfect.

Solving the quadratic equation in one variable :

To solve the quadratic equation in one variable , do as follows :

1. Put the equation in the standard form : $a x^2 + b x + c = 0$
2. Factorize the expression in the left hand side into two factors.
3. Use the opposite fact to get the two roots of the equation.
4. Verify your answer by substituting by each value of x in the main equation.

Fact

If a and b are two real numbers and $a \times b = \text{zero}$, then $a = 0$ or $b = 0$

To solve word problems in algebra , translate the sentences into symbols and algebraic expressions and form an equation that can be solved by the methods of solving equations.

Exams on the second part of Unit One
from lesson (5) to lesson (9)

Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1. If $x + y = 4$, $x^2 - xy + y^2 = 5$, then $x^3 + y^3 =$ ....
   (a) $\frac{5}{4}$ (b) 20 (c) 9 (d) 10
2. The S.S. of the equation : $(x - 1)^2 = 0$ in $\mathbb{R}$ is ....
   (a) $\{0\}$ (b) $\{-1\}$ (c) $\{-1 , 1\}$ (d) $\{1\}$
3. If the age of Zyad now is x years , then his age 3 years ago is .... years.
   (a) $3x$ (b) $3 - x$ (c) $x - 3$ (d) $x + 3$
4. The S.S. of the equation : $5x(x - 3) = 0$ in $\mathbb{R}$ is ....
   (a) $\{5\}$ (b) $\{5 , 0 , 3\}$ (c) $\{5 , 3\}$ (d) $\{0 , 3\}$
5. If $y^3 - a = (y - 3)(y^2 + 3y + 9)$, then a = ...
   (a) 27 (b) 9 (c) 3 (d) 6
6. The S.S. of the equation : $x^2 + 4 = 0$ in $\mathbb{R}$ is ....
   (a) $\{-4\}$ (b) $\varnothing$ (c) $\{-2 , 2\}$ (d) $\{4 , -4\}$

2 Complete the following :

1. If $x + y = 3$, $a + b = 5$, then $ax + ay + bx + by =$ ....
2. If $x = 2$ is a root of the equation : $x^2 - 6x + k = 0$
   , then k = ... and the other root = ...
3. If $x^3 + 27 = (x + 3)(x^2 + k + 9)$, then k = ...
4. If $(x + 5)$ is a factor of the expression : $x^3 + 125$, then the other factor is ....
5. The S.S. of the equation : $\frac{x}{4} = \frac{25}{x}$ in $\mathbb{R}$ is ....

3 Factorize each of the following :

1. $y^3 - y^2 - 9y + 9$
2. $x^4 + 64y^4$
3. $4x^2 - 4xy - 16 + y^2$
4. $m^3 + 27n^3$

4 Find in $\mathbb{R}$ the S.S. of each of the following equations :

1. $x^2 - 8x + 15 = 0$
2. $4x^3 = 9x$
3. $(x - 3)(x + 1) = 5$
4. $5x^2 + 12x = 44$

Unit 1

5 [a] A real number if we added it to its square , the result will be 12 what is the number ?

[b] Find the dimensions of a rectangle whose length is 4 cm. more than its width and whose area is 21 cm^2.

Model 2

Answer the following questions :

1 Choose the correct answer from those given :

1 If $x^2 - 2xy + y^2 = 25$, then $x - y =$ ....

(a) 25 (b) – 5 (c) 5 (d) ± 5

2 If $x^3 - y^3 = 12$, $x^2 + xy + y^2 = 4$, then $x - y =$ ....

(a) 48 (b) 3 (c) 16 (d) 8

3 The S.S. of the equation : $5(x - 7)(x + 3) = 0$ in $\mathbb{R}$ is ....

(a) $\{-7 , 3\}$ (b) $\{7 , -3\}$ (c) $\{5 , 7 , -3\}$ (d) $\{-5 , 7 , -3\}$

4 If Sarah's age now is x years , then the square of her age after 2 years is ....

(a) $x^2 + 2$ (b) $x^2 + 4$ (c) $(x - 2)^2$ (d) $(x + 2)^2$

5 If $x^3 - 8 = (x + a)(x^2 + 2x + 4)$, then a = ...

(a) 4 (b) – 4 (c) 2 (d) – 2

6 The term should be added to the expression : $x^4 + 4y^4$ to be factorized as a perfect square is ....

(a) $2x^2y^2$ (b) $8x^2y^2$ (c) $4x^2y^2$ (d) $16xy$

2 Complete the following :

1 If (a + 5) is a factor of the expression : $ax - 3a + 5x - 15$, then the other factor is ....

2 The S.S. of the equation : $3x^2 - x =$ zero in $\mathbb{R}$ is ....

3 $(x + 5)(x^2 - 5x + 25) =$ ....

4 If $ax + ay - bx - by = 15$, $x + y = 5$, then a – b = ...

5 If 4 is a factor of the equation : $x^2 - x = 12$, then the other factor is ....

3 Factorize each of the following :

1 $81a^4 + 4b^4$

2 $5x - 10y - ax + 2ay$

3 $2x^3 - 54$

4 $x^4 + 4x^2y^2 + 16y^4$

4 Find in $\mathbb{R}$ the S.S. of each of the following equations :

1 $3a^2 + 7a = -2$

2 $x^4 - 10x^2 + 9 = 0$

3 $a^2 = a$

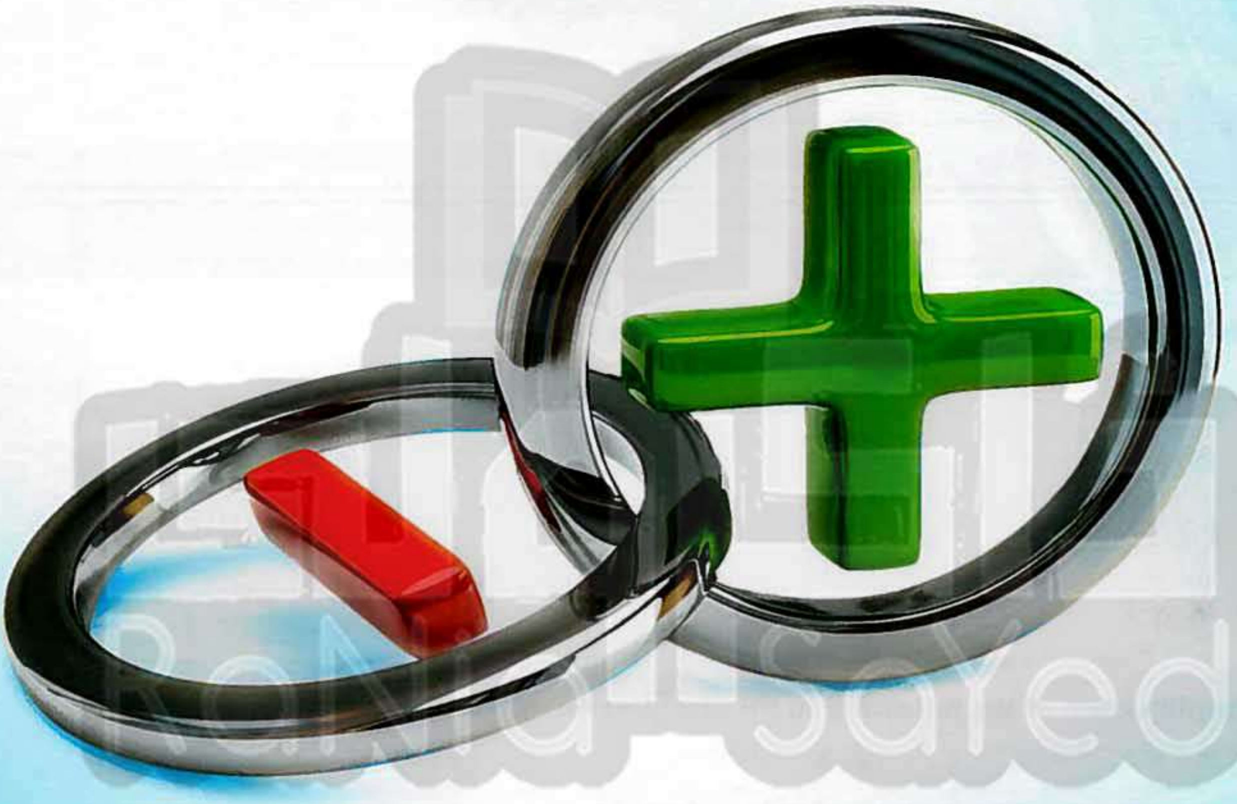
4 $x(x + 2) = 15$

5 [a] Find the number if its additive inverse add to its square the result will be 42

[b] A positive integer , its square is more than its 4 times by 5 , find the number.

UNIT 2

Non-negative and negative integer powers in ℝ



Exercises of the unit :

10. Non-negative and negative integer powers in ℝ
11. Solving the exponential equations in ℝ
12. Operations on integer powers.

- Summary of unit two.
- Unit exams.

Exercise 10 Non-negative and negative integer powers in $\mathbb{R}$

From the school book

1 **Find the value of each of the following in the simplest from :**

1. 3^{-2}
2. $\left(\frac{1}{4}\right)^{-1}$
3. $\left(\frac{2}{3}\right)^{-2}$
4. $\left(\sqrt{5}\right)^{4}$
5. $\left(\sqrt{3}\right)^{-2}$
6. $\left(-\sqrt{3}\right)^{-2}$
7. $\left(\sqrt[3]{5}\right)^{-3}$
8. $\frac{1}{\left(\sqrt{5}\right)^{-2}}$
9. $(0.01)^{-2}$
10. $(0.2)^{-2}$
11. $\left(\sqrt{2}\right)^{-3}$
12. $\left(\frac{\sqrt{3}}{3}\right)^{-5}$

2 **Simplify each of the following to the simplest form where $x \neq 0$:**

1. $x^{3} \times x^{-2} \times x^{-1}$
2. $x^{-4} \div x^{-3}$
3. $\left(x^{2}\right)^{-3} \times \left(x^{-3}\right)^{-2}$
4. $\frac{x^{2} \times x^{-3}}{x^{-4} \times x}$
5. $\frac{\left(x^{2}\right)^{-3} \times \left(x^{-1}\right)^{2}}{x^{-3} \times x^{-4}}$

3 **Simplify each of the following to the simplest form :**

1. $\left(\sqrt{2}\right)^{2} \times \left(\sqrt{2}\right)^{4}$ « 8 »
2. $\left(\sqrt{7}\right)^{5} \times \left(\sqrt{7}\right)^{-2} \times \left(\sqrt{3}\right)^{-1}$ « 7 »
3. $\left(\sqrt{2}\right)^{4} \times \left(-\sqrt{2}\right)^{2} \times \left(\sqrt{2}\right)^{-2}$ « 4 »
4. $\sqrt{3} \times \left(-\sqrt{3}\right)^{3} \times \left(-\sqrt{3}\right)^{4}$ « − 81 »
5. $\left(\sqrt{5}\right)^{-4} \div \left(\sqrt{5}\right)^{-6}$ « 5 »
6. $\left(-\sqrt{5}\right)^{9} \div \left(-\sqrt{5}\right)^{5}$ « 25 »
7. $\left(\frac{-1}{\sqrt{2}}\right)^{6}$ « $\frac{1}{8}$ »
8. $\left(\left(\sqrt{2}\right)^{3} \times \left(-\sqrt{2}\right)^{2}\right)^{2}$ « 32 »
9. $\left(\sqrt{3}\right)^{-4} \times \left(-\sqrt{2}\right)^{4}$ « $\frac{4}{9}$ »
10. $\left(\left(-5\right)^{3}\right)^{2} \times \left(-\sqrt{5}\right)^{-4}$ « 625 »

4 **Simplify each of the following to the simplest form :**

1) $\dfrac{(\sqrt{7})^{-4} \times (\sqrt{7})^{-3}}{(\sqrt{7})^{-9}}$ « 7 »

2) $\dfrac{(\sqrt{3})^{7} \times (\sqrt{3})^{8}}{(\sqrt{3})^{6}}$ « $81\sqrt{3}$ »

3) $\dfrac{(\sqrt{3})^{8} \times (-\sqrt{3})^{6}}{(\sqrt{3})^{12}}$ « 3 »

4) $\dfrac{(\sqrt{5})^{10} \times (-\sqrt{5})^{5}}{(\sqrt{5})^{11}}$ « – 25 »

5) $\dfrac{(2\sqrt{7})^{-2} \times (\sqrt{2})^{-4}}{(\sqrt{7})^{-2}}$ « $\frac{1}{16}$ »

6) $\dfrac{(3\sqrt{2})^{4} \times (\sqrt{2})^{2}}{(2\sqrt{3})^{2}}$ « 54 »

7) $\dfrac{(\sqrt{3})^{-4} \times (\sqrt{2})^{3} \times (3\sqrt{3})^{5}}{(3\sqrt{2})^{5} \times \sqrt{3}}$ « $\frac{1}{2}$ »

8) $\dfrac{(\sqrt{3})^{5} \times (\sqrt{3})^{4}}{(\sqrt{3})^{3} \times 27}$ « 1 »

9) $\dfrac{(\sqrt{2})^{5} \times (10)^{6}}{(\sqrt{2})^{3} \times 2^{3} \times 5^{5}}$ « 80 »

10) $\dfrac{(15)^{-2} \times (\sqrt{5})^{3} \times 3^{3}}{9 \times (\sqrt{5})^{-3}}$ « $\frac{5}{3}$ »

11) $\dfrac{(\sqrt{5})^{6} \times (\sqrt{2})^{8}}{(\sqrt{10})^{4}}$ « 20 »

12) $\dfrac{(10)^{2} \times (10)^{-7}}{(0.1)^{2} \times 0.001}$ « 1 »

5 **Simplify each of the following to the simplest form :**

1) $\left(\dfrac{2\sqrt{3}}{\sqrt{2}}\right)^{4}$ « 36 »

2) $\left(\dfrac{3\sqrt{2}}{2\sqrt{3}}\right)^{4}$ « $\frac{9}{4}$ »

3) $\left(\dfrac{-5\sqrt{2}}{2\sqrt{5}}\right)^{4}$ « $\frac{25}{4}$ »

4) $\left(\dfrac{\sqrt{3}}{\sqrt{2}}\right)^{2} \times \left(\dfrac{\sqrt{2}}{\sqrt{3}}\right)^{4}$ « $\frac{2}{3}$ »

6 **Simplify to the simplest form :**

1) $\dfrac{9^{x} \times 3^{x+2}}{(27)^{x}}$ « 9 »

2) $\dfrac{2^{2x} \times 3^{x-1}}{(12)^{x}}$ « $\frac{1}{3}$ »

3) $\dfrac{2^{x} \times 4^{x+1}}{8^{x}}$ « 4 »

4) $\dfrac{(36)^{n} \times 5^{2n}}{(30)^{2n}}$ « 1 »

5) $\dfrac{2^{x} \times (49)^{x-1}}{(98)^{x}}$ « $\frac{1}{49}$ »

6) $\dfrac{4^{x+2} \times 9^{3+x}}{6^{2x+3}}$ « 54 »

7) $\dfrac{4^{n} \times 6^{2n}}{2^{4n} \times 3^{2n}}$ « 1 »

8) $\dfrac{(81)^{x} \times 6^{2x}}{(27)^{2x-1} \times 4^{x}}$ « 27 »

9) $\dfrac{2^{n} \times 9^{n+1} \times (\sqrt{2})^{2}}{6 \times (18)^{n}}$ « 3 »

10) $\dfrac{3 \times (18)^{x+1} \times 2^{x}}{2 \times (36)^{x}}$ « 27 »

11) $\dfrac{6^{n} \times 4^{n+\frac{1}{2}}}{(24)^{n}}$ « 2 »

12) $\dfrac{8^{n-1} \times 32^{-n}}{32 \times 4^{-n}}$ « $\frac{1}{256}$ »

Unit 2

13 $\dfrac{(\sqrt{3})^{n+2} \times (15)^{n+1}}{(\sqrt{3})^{n} \times 3^{n} \times 5^{n+2}}$ « $\frac{9}{5}$ »

14 $\dfrac{4^{x+1} \times 9^{2-x}}{6^{2x}}$, then find the value of the result , when $x = 1$ « $2^2 \times 3^{4-4x}$, 4 »

15 $\dfrac{9^{x-1} \times (\sqrt{2})^{6x}}{8^{x} \times (\sqrt{3})^{2x}}$, then find the value of the result , when $x = 2$ « 1 »

16 $4^{x-1} \times 2^{3x+2} \times \left(\frac{1}{2}\right)^{3x}$, what is the value of the result if $2^{x} = 5$? « 25 »

7 Prove that :

1 $\dfrac{(\sqrt{2})^{2} \times 2^{1-x} \times 12^{2x-1}}{8^{x} \times 9^{x}} = \dfrac{1}{3}$

2 $\dfrac{(27)^{x-1} \times 8^{x}}{(2\sqrt{2})^{2x} \times (3\sqrt{3})^{2x}} = \dfrac{1}{27}$

8 If $a = \sqrt{3}$ and $b = \sqrt{2}$, find the value of :

1 $a^4 - b^4$

2 $\dfrac{a^4}{b^4}$

« 5 , $\frac{9}{4}$ »

9 If $x = 2\sqrt{2}$ and $y = 3$, find the value of : $(x^2 - y^2)^3$ « – 1 »

10 If $a = \dfrac{3}{\sqrt{2}}$ and $b = \dfrac{\sqrt{2}}{\sqrt{3}}$, find the value of : $2\,(ab)^2 \times \left(\frac{b}{a}\right)^{-2}$ « $\frac{81}{2}$ »

11 If $x = \dfrac{\sqrt{3}}{2}$, $y = \dfrac{1}{\sqrt{3}}$ and $z = \dfrac{\sqrt{2}}{2}$, find the value of : $x^2 + (xz)^2 \times y^2$ « $\frac{7}{8}$ »

12 If $a = \dfrac{3\sqrt{2}}{2}$ and $b = \dfrac{\sqrt{3}}{\sqrt{2}}$, prove that : $\left(\frac{a}{b}\right)^2 - 3\left(\frac{b}{a}\right)^2 = 2$

13 If $x = 2$ and $y = \sqrt{3}$, find the value of the following in the simplest form :

1 $3\,(x + y)^4\,(x - y)^4$

2 $\left(\dfrac{x + y}{x - y}\right)^{-2}$

« 3 , $97 - 56\sqrt{3}$ »

14 If $a = \dfrac{1}{\sqrt{2}}$, $b = -1$, then calculate the value of : $7\,a^6 + (1 - b)^{-3}$ « 1 »

15 If $x = 3$, $y = \sqrt{2}$, find in the simplest form the value of each of the following :

1 $x^{-2}\,y^{-4}$ « $\frac{1}{36}$ »

2 $(x^{-2} \times y^{4})^{-2}$ « $\frac{81}{16}$ »

3 $\left(\dfrac{x}{y}\right)^{-3}$ « $\frac{2\sqrt{2}}{27}$ »

16 **Choose the correct answer from those given :**

1. $5^2 + 5^2 =$ ....
(a) 10^2 (b) 10^4 (c) 5^4 (d) 50

2. $3^5 \times 2^5 =$ ....
(a) 5^{10} (b) 6^{10} (c) 6^5 (d) 6^{25}

3. $(5\ a)^{zero} =$ .... , $a \neq 0$
(a) 5 (b) a (c) 5 a (d) 1

4. $3\ x^{zero} =$ .... , $x \neq 0$
(a) zero (b) 1 (c) 3 (d) $3\ x$

5. $3^{(2^3)} =$ ....
(a) 3^6 (b) 3^5 (c) 3^8 (d) 3^{32}

6. $(5^2)^3 =$ ....
(a) 5^6 (b) 5^5 (c) 5^{32} (d) 5

7. $4^3 + 4^3 + 4^3 + 4^3 =$ ....
(a) 4^3 (b) 4^4 (c) 4^{12} (d) 4^{81}

8. $2^5 + 2^5 + 2^5 + 2^5 =$ ....
(a) 2^4 (b) 2^6 (c) 2^7 (d) 2^{20}

9. The quarter of the number : $4^{20} =$ ....
(a) 1^{20} (b) 4^{19} (c) 4^{16} (d) 4^5

10. 4 times the number : $2^8 =$ ....
(a) 2^{32} (b) 8^8 (c) 2^{10} (d) 4^8

11. Sixth the number : $2^{12} \times 3^{12}$ is ....
(a) 6^2 (b) 6^4 (c) 6^{11} (d) 6^{23}

12. Fifth the number : $(\sqrt[3]{5})^6$ is ....
(a) 5 (b) 5^5 (c) 5^6 (d) 5^{12}

13. $(\sqrt{3})^6 \times 3^4 =$ ....
(a) $(\sqrt{3})^{24}$ (b) 3^{10} (c) 3^7 (d) $(\sqrt{3})^{10}$

14. The value of : $2^5 + (\sqrt{2})^{10} =$ ....
(a) 2^6 (b) 2^{10} (c) $(\sqrt{2})^{15}$ (d) $(\sqrt{2})^{20}$

15. The value of : $2^{20} + 2^{21} =$ ....
(a) 2×2^{40} (b) 2×2^{41} (c) 3×2^{20} (d) 3×2^{21}

16 What of the following is closest to $11^2 + 9^2$?

(a) 22 + 18 (b) 211 + 29 (c) 120 + 20 (d) 120 + 80

17 If $3^{x} = 4$, then $3^{-x} =$ ....

(a) – 4 (b) $\frac{1}{4}$ (c) 4 (d) 12

18 If $2^{x} = 5$, then $8^{x} =$ ....

(a) 5 (b) 15 (c) 25 (d) 125

19 If $6^{x} = 11$, then $6^{x+1} =$ ....

(a) 12 (b) 22 (c) 66 (d) 72

20 If $5^{x} = 4$, then $5^{x-1} =$ ....

(a) 1.25 (b) 0.8 (c) 0.125 (d) 0.08

21 $0.002 \times 0.05 =$ ....

(a) 10^{-5} (b) 10^{-4} (c) 10^{4} (d) 10^{5}

22 If $x = \frac{\sqrt{9}}{\sqrt{3}}$, then $x^{-1} =$ ....

(a) $\frac{\sqrt{3}}{3}$ (b) $\frac{\sqrt{3}}{\sqrt{2}}$ (c) $\sqrt{3}$ (d) 2

23 $x^{m-1} \times$ .... $= 1$, $x \neq 0$

(a) x^{m-1} (b) x^{-m-1} (c) x^{m+1} (d) x^{-m+1}

24 $(\sqrt{3}+\sqrt{2})^9(\sqrt{3}-\sqrt{2})^9 =$ ....

(a) 1 (b) $\sqrt{5}$ (c) $\sqrt{6}$ (d) 5

25 The numerical value of the expression : $\frac{2^{2n+1} \times 5^{2n+1}}{10^{2n}}$ is ....

(a) $\frac{1}{10}$ (b) 7 (c) 10 (d) 100

17 Complete the following :

1 $(\sqrt{3})^4 \times (\sqrt{3})^2 = 3^{\cdots}$

2 $\frac{((\sqrt{7})^3)^2}{((\sqrt{7})^2)^3} =$ ....

3 $((\sqrt{5})^9)^{11} - ((\sqrt{5})^{11})^9 =$ ....

4 $a^{\cdots} \times a^6 = a^6$

5 $(a^2 b^{\cdots})^4 = a^8 b^{12}$

6 The simplest form of the expression : $2^{zero} \times 2^{-1} \times \left(\frac{-1}{\sqrt{2}}\right)^2 =$ ....

7 The greater number from the two numbers $(-\sqrt{2})^{25}$ and $(-\sqrt{2})^{24}$ is ....

8 If 4 times a number is 4^2 , then $\frac{3}{4}$ this number is ....

9 If $(x-5)^{zero} = 1$, then $x \in$ ....

10 If $a = 7^{x}$ and $b = 7^{-x}$, then $a \times b =$ ....

11 If $x = \left(\sqrt{2}+3\right)^{5}$ and $y = \left(\sqrt{2}+3\right)^{-5}$, then $x y =$ ....

12 $\left(\frac{5}{6}\right)^{-4} = \left(-\frac{\dots\dots}{\dots\dots}\right)^{2}$

13 If $\left(\frac{1}{2}\right)^{x} = 5$, then $(8)^{-x} =$ ....

14 If $2^{x} = 7$, $2^{y} = 5$, then $2^{x+y} =$ ....

15 If $5^{x} = 3$, $5^{-y} = 7$, then $5^{x+y} =$ ....

For excellent pupils

18 Complete the following :

1 If $x^{3} y^{-3} = 8$, then $y^{2} x^{-2} =$ ....

2 If $x = \sqrt{2}$, $y = \left(\sqrt{2}\right)^{-1}$, then $x^{101} y^{100} =$ ....

3 If $3^{x+2} = 18$, then $(81)^{x} =$ ....

4 If $2^{x} = 3$, $2^{y} = 5$, then $4^{2x+y} =$ ....

19 Choose the correct answer from those given :

1 $5 \times 5 \times 5 \times 2 \times 2 \times 2 \times 2 \times 2 = 4 \times$ ....

(a) 5^{3} (b) 2^{3} (c) 10^{3} (d) $5^{3} + 2^{3}$

2 $2^{2011} = 2^{2010} +$ ....

(a) 2 (b) 2010 (c) 2^{2010} (d) 2^{2011}

3 The expression : $2^{1000} + 256^{125} =$ ....

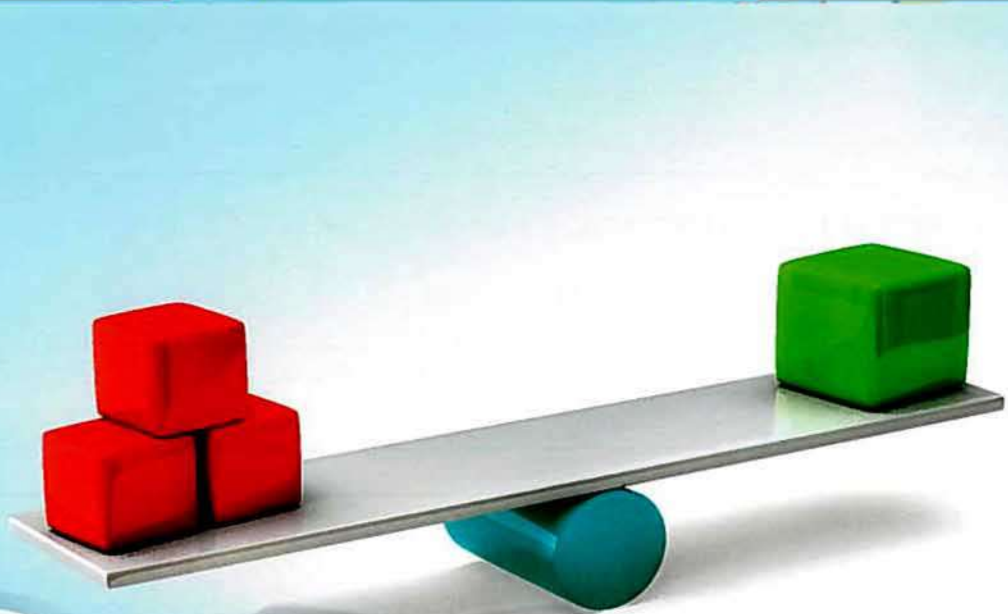
(a) 258^{125} (b) 258^{1125} (c) 2^{1001} (d) 4^{1000}

4 If $x \neq 0$ and $x + \frac{1}{x} = \sqrt{5}$, then $x^{2} + \frac{1}{x^{2}} =$ ....

(a) 1 (b) 3 (c) 5 (d) 7

5 The digit in the units place of : $3^{12} \times 2^{14}$ is ....

(a) 2 (b) 3 (c) 4 (d) 6



Exercise 11 Solving the exponential equations in $\mathbb{R}$

📖 From the school book

1 Find the value of n in each of the following when $n \in \mathbb{Z}$:

1. $5^{n} = 25$ « 2 »
2. $2^{-n} = 32$ « – 5 »
3. $2^{n-1} = 32$ « 6 »
4. 📖 $3^{n-2} = 81$ « 6 »
5. $3^{n-2} = 1$ « 2 »
6. $2^{n-3} = \frac{1}{4}$ « 1 »
7. 📖 $3^{n-2} = \frac{1}{9}$ « 0 »
8. 📖 $(\sqrt{3})^{n-1} = 9$ « 5 »
9. 📖 $\left(\frac{2}{5}\right)^{2n-1} = \frac{8}{125}$ « 2 »
10. 📖 $\left(\frac{3}{5}\right)^{n+2} = \frac{125}{27}$ « – 5 »
11. 📖 $\left(\frac{2}{3}\right)^{n-4} = 2\frac{1}{4}$ « 2 »
12. 📖 $\left(\frac{2}{3}\right)^{n+5} = \left(3\frac{3}{8}\right)^{-2}$ « 1 »
13. $5^{2n-4} = 7^{2n-4}$ « 2 »
14. $3^{n-4} = n^{n-4}$ « 3 or 4 »
15. $9 \times 3^{n-4} = 1$ « 2 »
16. $2 \times 4^{n+3} = \frac{1}{32}$ « – 6 »
17. $8 \times (\sqrt{2})^{n-3} = 1$ « – 3 »
18. $\frac{1}{27}(\sqrt{3})^{n+2} = 1$ « 4 »

2 Find the S.S. of each of the following equations in $\mathbb{R}$:

1. $6^{x^2-4} = 7^{x^2-4}$ « $\{-2, 2\}$ »
2. $2^{x^2-9} = 1$ « $\{-3, 3\}$ »
3. $2^{x^2-x} = 4$ « $\{-1, 2\}$ »
4. $5^{|x|} = 125$ « $\{-3, 3\}$ »
5. 📖 $(32)^{x-3} = 8^{2x+1}$ « $\{-18\}$ »
6. 📖 $3^{x-3} = (\sqrt{3})^{x+5}$ « $\{11\}$ »
7. $3^{n-1} \times 2^{1-n} = \frac{27}{8}$ « $\{4\}$ »
8. 📖 $25 \times 3^{x-1} = 9 \times 5^{x-1}$ « $\{3\}$ »

3 **Find the value of n in each of the following where $n \in \mathbb{Z}$:**

1. $\frac{2^n \times 9^{n+1}}{(18)^n} = 3^n$ « 2 »
2. $\frac{8^n \times 9^n}{(18)^n} = 64$ « 3 »
3. $\frac{6^{2n-3}}{2^{n-1} \times 3^{n-1}} = 6$ « 3 »
4. $\frac{(12)^{n-1}}{2^{n-1} \times 3^{n-1}} = 1$ « 1 »
5. $\frac{3^n \times 8^n}{(12)^{n+1}} = \frac{1}{3}$ « 2 »
6. $\frac{4^{2n} \times \left(\sqrt{3}\right)^{4n}}{9^n \times 4^n} = \frac{1}{16}$ « −2 »
7. $\frac{4^{n-1} \times 2^{n+3}}{8^n} = 2n^2$ « ±1 »
8. $\frac{(14)^{2n} \times 4^{n+1}}{4 \times 7^n \times 16^n} = 49$ « 2 »

4 **Find the S.S. of each of the following equations in $\mathbb{R}$:**

1. $(x-4)^5 = 32$ « {6} »
2. $\frac{1}{(x+9)^4} = 0.0001$ « {1 , −19} »
3. $(x^2 - x)^5 = 32$ « {−1 , 2} »
4. $\left(\sqrt{3}\right)^{x^2 - x} = 1$ « {0 , 1} »
5. $5^{x^2 - 5x} = 0.0016$ « {1 , 4} »
6. $5^{x^2} = 25^{x+4}$ « {4 , −2} »
7. $9^{x^2 - 1} = \frac{1}{(27)^x}$ « $\{-2 , \frac{1}{2}\}$ »

5 If $\frac{6^{2n} \times 2^{2n}}{4^{2n} \times 3^{2n+4}} = 9^{-x}$, then find the value of : x « 2 »

6 If $\frac{(81)^x \times 4^x}{6^{2x} \times 3^{2x}} = 3^{y-1}$, find the value of : y « 1 »

7 If $\frac{(27)^{x-1} \times 8^x}{\left(2\sqrt{2}\right)^{2x} \times \left(3\sqrt{3}\right)^{2x}} = 3^x$, find the value of : x « −3 »

8 If $\frac{7^x \times 6^x}{(14)^x} = 3^{2-y}$, find the value of : $x + y$ « 2 »

9 If $\left(\sqrt{\frac{3}{2}}\right)^x = \frac{4}{9}$, calculate the value of : $\left(\frac{3}{2}\right)^{x+1}$ « $\frac{8}{27}$ »

10 If $\frac{49^n \times 25^{2n} \times 3^{4n}}{7^{-n} \times 15^{4n}} = 343$, then calculate the value of : 6^{2n} « 36 »

11 If $3^x = 27$, $4^{x+y} = 1$, calculate the value of each of : x and y « 3 , −3 »

12 **Choose the correct answer from those given :**

1 If $3^{x+1} = 5^{x+1}$, then $x =$ ....

(a) 4 (b) 3 (c) -1 (d) 1

2 If $3^{2+x} = 5^{x+2}$, then $7^{x+2} =$ ....

(a) 7 (b) -7 (c) -14 (d) 1

3 If $\left(\frac{2}{3}\right)^9 = \left(\frac{3}{2}\right)^x$, then $x =$ ....

(a) -9 (b) 9 (c) 32 (d) 23

4 If $2^x = \frac{1}{8}$, then $x^2 =$ ....

(a) $\frac{1}{4}$ (b) 9 (c) -9 (d) $-\frac{1}{9}$

5 If $5^{|x-3|} = 25$, then $x =$ ....

(a) 5 (b) 2 (c) 1 (d) 5 or 1

6 If $3^{x-1} = \sqrt[3]{\frac{1}{27}}$, then $x =$ ....

(a) 1 (b) zero (c) -1 (d) -2

7 If $\left(\sqrt{3}\right)^{x+1} = 3\sqrt{3}$, then $x =$ ....

(a) 1 (b) 2 (c) zero (d) 3

8 If $2^{x-2} = 2^{1-2x}$, then $x =$ ....

(a) 2 (b) $\frac{1}{2}$ (c) 1 (d) zero

9 If $3^x = 9$, then $2^x - 1 =$ ....

(a) 7 (b) 3 (c) 8 (d) 5

10 If $(x+1)^3 = \left(\frac{1}{8}\right)^{-3}$, then $x =$ ....

(a) -7 (b) 7 (c) zero (d) 9

11 If $2^{2x} = 4$, then $2^{5x} =$ ....

(a) 32 (b) 16 (c) 10 (d) 8

12 If $0.05 \times 0.002 = 10^x$, then $x =$ ....

(a) -4 (b) zero (c) 2 (d) 4

13 If $2^{x-1} \times 3^{1-x} = \frac{9}{4}$, then $x =$ ....

(a) -3 (b) -1 (c) 1 (d) 3

14 If $2^{x} = \left(2\sqrt{5} + 3\sqrt{2}\right)\left(2\sqrt{5} - 3\sqrt{2}\right)$, then $x =$ ....

(a) 1 (b) – 1 (c) 2 (d) – 2

15 If $3^{x} = 7$, $7^{y} = 9$, then $x\,y =$ ....

(a) 5 (b) 2 (c) 7 (d) 9

13 Complete the following :

1 If $5^{x(x-1)} = 1$, then the value of $x =$ ....

2 If $3^{n} \times 3^{5} = 1$, then n = ...

3 If $2^{y} \times 5^{y} = 100$, then y = ...

4 If $\left(\frac{3}{5}\right)^{x-7} = 1$, then $x =$ ....

5 If $3^{x} \times 2^{-x} = 1.5$, then $x =$ ....

6 If $2^{x} \times 5^{-x} = 2.5$, then $x =$ ....

7 If $4^{x-6} = 64$, then $\sqrt{x} =$ ....

8 If $4^{x-10} = \frac{1}{16}$, then $\sqrt[3]{x} =$ ....

9 If $\left(\sqrt{5}\right)^{2x} = \frac{1}{5}$, then $x =$ ....

10 If $\frac{2^{x} \times 3^{x}}{12^{x}} = \frac{1}{2}$, then $x =$ ....

11 If $3^{x} + 3^{x} + 3^{x} = 1$, then $x =$ ....

12 If $\{3 , a^{x-2}\} = \{1 , 3\}$, then the value of $x =$ ....

13 If $(2^{x} , 125) = (16 , y^{3})$, then $x =$ .... and y = ...

Life Application

14 If the area of Sameh's chess board is 2^{n} cm^{2} , given that the side length of each small square which the board consists of is 4 cm.
, find the value of n « 10 »

For excellent pupils

15 Find the value of x in each of the following where $x \in \mathbb{R}$:

1 $x^{x+2} = 4^{x+2}$ « ± 4 or – 2 »

2 $a^{x+3} - 1 = (a-1)(a+1)(a^{2}+1)(a^{4}+1)$ « 5 »

Exercise 12

Operations on integer powers

📖 From the school book

1 Complete the following :

1. 📖 The simplest form of the expression : $2^{-3} \times 2^{-2} \div 4^{-3} = \dots\dots$
2. The simplest form of the expression : $2^{-3} \times 3^{-2} \div 6^{-4} = \dots\dots$
3. 📖 The simplest form of the expression : $(3^{-2})^3 \div 9^{-3} \times (-2)^{-1} = \dots\dots$
4. 📖 The simplest form of the expression : $4^3 \times 3^{-2} \times (\sqrt[3]{-8})^{-5} = \dots\dots$

2 Find the result of each of the following in its simplest form :

1. 📖 $(\sqrt{5})^5 \div 5\sqrt{5} + 2\sqrt{3} \times \sqrt{3}$ « 11 »
2. $(2\sqrt{3})^3 \times \sqrt{3} - (\sqrt{2})^7 \div 4\sqrt{2}$ « 70 »
3. $(\sqrt{3})^{-3} \times 3\sqrt{3} + (\sqrt{3})^{-4} \div (\sqrt{3})^{-10}$ « 28 »
4. $(2\sqrt{5})^4 - (\sqrt{5})^{-3} \times (5\sqrt{5})^2 \div 5\sqrt{5}$ « 399 »

3 Find the result of each of the following in its simplest form :

1. $\dfrac{(\sqrt{3})^7 \times (\sqrt{3})^{-5} - (\sqrt{3})^2}{(\sqrt{3})^7 \times (\sqrt{3})^{-5} + (\sqrt{3})^2}$ « zero »
2. $\dfrac{2(\sqrt{3})^5 \div 3\sqrt{3}}{2\sqrt{3} + (\sqrt{3}-1)^2}$ « $\frac{3}{2}$ »
3. $\dfrac{(2\sqrt{2})^3 \times 3\sqrt{2}}{(\sqrt{6}+\sqrt{2})^2 - 2\sqrt{12}}$ « 12 »

4 If $a = \sqrt{2}$, $b = \sqrt{3}$, find the numerical value of :

1. $\frac{b^4 - a^4}{b^2 + a^2}$ « 1 »
2. $\frac{a^3 + b^3}{a + b}$ « $5 - \sqrt{6}$ »

5 **Choose the correct answer from those given :**

1. The expression : $\frac{3^x \times 3^x \times 3^x}{3^x + 3^x + 3^x}$ equals ....

(a) 3^{2x-1} (b) 3^{1-2x} (c) $3^{x^3 - 3x}$ (d) $3^{3x - x^3}$

2. $(5^{x+2} - 5^{x+1}) \div 5^x =$ ....

(a) 5 (b) 10 (c) 15 (d) 20

3. The value of the expression : $3^5 + (\sqrt{3})^{10} - 2(3)^5 =$ ....

(a) zero (b) 3^5 (c) $(\sqrt{3})^5$ (d) $2(3)^5$

4. The simplest form of the expression : $\sqrt{4 \times \sqrt{16} \div \sqrt[3]{8} - 2^2} =$ ....

(a) 2 (b) 4 (c) 8 (d) 16

5. If $x = \sqrt{3}$, $y = \sqrt{5}$, then $\frac{x^8 - y^8}{x^4 + y^4} =$ ....

(a) 4 (b) – 4 (c) 16 (d) – 16

Geometric Application

6 If the total area of a cube = 3.375×10^2 cm^2.

Find :

1. The edge length of the cube.
2. The volume of the cube.

« 7.5 cm. , 421.875 cm^3. »

7 If the volume of the sphere = $\frac{4}{3} \pi r^3$

Find the radius length of the sphere whose volume is 3.8808×10^4 cm^3.

$(\pi = \frac{22}{7})$

« 21 cm. »

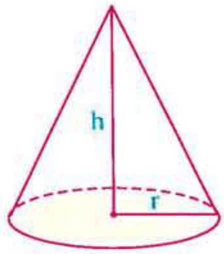
Unit 2

8 If the volume of the right circular cone is given

by the relation : $v = \frac{1}{3} \pi r^2 h$

Find the height of the cone h if the volume is 7.7×10^2 cm^3.

and its diameter length is 14 cm. $\left(\pi = \frac{22}{7}\right)$



« 15 cm. »

Life Applications

9 **Connecting with commercial business :**

If $c = m(1 + r)^n$ where (c) is the total sum (m) in pounds , (r) is the yearly profit per pound and (n) is the number of years , then calculate (c) to the nearest pound if :

$m = 2.5 \times 10^4$, $r = 9.8 \times 10^{-2}$, n = 12 « 76766 pounds »

10 **Population :**

If the number of population (y) in millions in a country is identified by the relation :

$y = 11.7(1.02)^x$ where x is the number of years starting from year 2005

Calculate the number of population expected for this country to the nearest million :

1. year 2011
2. year 2000

« 13 millions , 11 millions »

For excellent pupils

11 If $x = 2 + \sqrt{3}$, $y = 2 - \sqrt{3}$

, then find the value of the expression : $\frac{x^7 y^8 - y}{(x + y)^9}$ in the simplest form. « zero »

12 If $\ell = \frac{1}{2}(3^x + 3^{-x})$, $m = \frac{1}{2}(3^x - 3^{-x})$

Prove that : $\ell^2 - m^2 = 1$

Summary Unit 2

- If $a \in \mathbb{R}$, $n \in \mathbb{Z}^+$

  , then $a^n = a \times a \times a \times \cdots\cdots \times a$ where a is repeated as a factor n times.

- If $a \in \mathbb{R}^*$, then $a^0 = 1$
- $(-a)^n = a^n$ if n is an even number
- $(-a)^n = -a^n$ if n is an odd number
- If a is a real number , $a \neq 0$ and n is a positive integer

  , then $a^{-n} = \frac{1}{a^n}$, $a^n = \frac{1}{a^{-n}}$

Laws of integer powers in $\mathbb{R}$

1. $a^m \times a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $(a\,b)^n = a^n\,b^n$
4. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
5. $(a^m)^n = a^{mn}$

- If a is a real number , m and n are two integers and $a^m = a^n$, then $m = n$

  where : $a \neq 0$, $a \neq \pm 1$

- If a and b are two real numbers , m is an integer and $a^m = b^m$ then
  - $a = b$ if m is an odd number
  - $a = \pm b$ if m is an even number
  - $m = 0$ if $a \neq \pm b$
- If $a^n = 1$, then $n = 0$ where : $a \neq 0$ and $a \neq \pm 1$
- The order of operating the mathematical operations as follows :

1. Do the operations inside the parenthesis (the interior, then the exterior). →
2. Calculate the powers of the numbers (indices). →
3. Do the multiplication and the division in order from left to right. →
4. Do the addition and the subtraction in order from left to right.

Exams on Unit Two

Model 1

Answer the following questions :

1 **Choose the correct answer from those given :**

1. $3^{-2} = \cdots\cdots$
   (a) -9 (b) $\frac{-1}{9}$ (c) $\frac{1}{9}$ (d) 9
2. If $6^{x} = 7$, then $6^{x+1} = \cdots\cdots$
   (a) 8 (b) 13 (c) 36 (d) 42
3. If $3^{x} = 5$, then $27^{x} = \cdots\cdots$
   (a) 9 (b) 25 (c) 125 (d) 729
4. $4^3 + 4^3 + 4^3 + 4^3 = \cdots\cdots$
   (a) 4^3 (b) 4^4 (c) 4^{12} (d) 4^{81}
5. If $3^{x+1} = 5^{x+1}$, then $x = \cdots\cdots$
   (a) 4 (b) 3 (c) -1 (d) 1
6. One sixth of the number : $2^{12} \times 3^{12}$ is $\cdots\cdots$
   (a) 6^2 (b) 6^4 (c) 6^{11} (d) 6^{23}

2 **Complete the following :**

1. If $3^{x-2} = 1$, then $x = \cdots\cdots$
2. $(\sqrt{5})^4 \times (\sqrt{5})^2 = 5^{\cdots\cdots}$
3. $2^{-n} = 32$, then $n = \cdots\cdots$
4. The simplest form of the expression : $\left((\sqrt{7})^2\right)^3 - \left((\sqrt{7})^3\right)^2 = \cdots\cdots$
5. If five times a number is 5^3 , then $\frac{4}{5}$ of this number is $\cdots\cdots$

3 [a] If $\frac{x^{-5} \times x^{-4}}{x^{-12}} = 8$, find the value of x

[b] If $\left(\frac{3}{2}\right)^{x+2} = 3\frac{3}{8}$, find the value of x

4 [a] **Simplify :** $\frac{9^{x} \times (25)^{x}}{(15)^{2x}}$

[b] If $3^{x} = 27$, $5^{x+y} = 1$, find the values of x and y

5 [a] **Simplify :** $\frac{(\sqrt{2})^9 \times 3^{-2}}{3 \times (\sqrt{2})^5}$

[b] If $a = \frac{1}{\sqrt{2}}$, $b = -1$, find the numerical value of : $7a^6 + (1-b)^{-3}$

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1. $2^5 + (\sqrt{2})^{10} = \dots\dots\dots$
   (a) $(\sqrt{2})^{20}$ (b) 2^{10} (c) 2^6 (d) $(\sqrt{2})^{15}$
2. If $x^3 y^{-3} = 8$, then $\frac{y}{x} = \dots\dots\dots$
   (a) $\frac{1}{512}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) 2
3. Twice the number $2^8 = \dots\dots\dots$
   (a) 2^9 (b) 4^{16} (c) 2^{16} (d) 2^7
4. If $5^{x-1} = 125$, then $x = \dots\dots\dots$
   (a) 1 (b) 2 (c) 3 (d) 4
5. If $\left(\frac{3}{5}\right)^x = \frac{25}{9}$, then $x = \dots\dots\dots$
   (a) 2 (b) -2 (c) 1 (d) zero
6. If $(x-7)^{zero} = 1$, then $x \in \dots\dots\dots$
   (a) $\{7\}$ (b) $\mathbb{R}$ (c) $\mathbb{R} - \{-7\}$ (d) $\mathbb{R} - \{7\}$

2 **Complete the following :**

1. $(\sqrt{2})^5 \times (\sqrt{2})^3 = 2^{\dots\dots\dots}$
2. If $x^{-3} = 125$, then $x = \dots\dots\dots$
3. If $5^x \times 5^3 = 1$, then $x = \dots\dots\dots$
4. If $2^x = 7$, $2^y = 3$, then $2^{x+y} = \dots\dots\dots$
5. If $3^x = 4$, then $3^{x+2} = \dots\dots\dots$

3 **[a] If $x = 3$, $y = \sqrt{2}$, find in the simplest form the value of :**

1. $x^{-2} y^{-4}$
2. $\left(\frac{x}{y}\right)^{-3}$

[b] If $4^{x-2} = 64$, find x^2

4 **[a] Simplify :** $\frac{3^{x-2} \times 9^{x+1}}{27^x}$

[b] Simplify : $((-5)^3)^2 \times (-\sqrt{5})^{-8}$

5 **[a]** If $3^x = 27$, find the value of : 4^x

[b] Simplify : $\frac{4^{x+1} \times 9^{2-x}}{6^{2x}}$, then find the numerical value at $x = 1$

UNIT 3 Probability

Exercise of the unit :

13. Probability.

- Summary of unit three.
- Unit exam.

Exercise 13 Probability

From the school book

1 **Complete the following :**

1. The probability of the impossible event equal ....
   and the probability of the certain event equal ....
2. For every event A , we find that P (A) $\in$ ....
3. If a fair coin is tossed once , then the probability of appearance of a head equal ....
4. 10 cards are numbered from 1 to 10 , A card is drawn randomly , then the probability that the card carries a prime number equal ....
5. A bag has cards numbered from 0 to 10 , if a card is drawn randomly , then the probability that the card carries an even number is ....
6. A box contains 5 white balls, 7 red balls and 3 blue balls. If a ball is drawn from the box randomly , then the probability that the drawn ball is blue = ...
7. A bag contains 12 balls , 4 of them are red , 6 are green and the rest are blue. If one ball is chosen randomly , then the probability of getting a blue ball equal ....
8. In the experiment of throwing a fair die and observing the number on the upper face , then the probability of getting a number greater than 4 is ....
9. In the experiment of throwing a fair die and observing the number on the upper face , then the probability of getting a number less than 1 equals ....
10. A box contains 48 oranges , 4 of them are bad. If we draw an orange at random , then the probability that the drawn orange is bad equal ....
    and the probability that it is not bad equal ....
11. If the probability of the occurrence of an event is $\frac{5}{8}$, then the probability of the non-occurrence of this event is ....

Unit 3

12 A room has 3 doors numbered from 1 to 3 One student goes out from one door.
The probability that he goes out from the second door is ....

13 A city has 200000 people. The probability that a person gets infected by a disease in this city is 0.003 The expected number of infection is .... people.

14 A factory produces 400 lamps daily, if the probability that the lamp is defective = 0.02, then the expected number of good lamps produced daily is ....

2 A numbered card is selected randomly from a set of similar cards numbered from 1 to 24 Find the probability of getting a card that carries :

1 a multiple of 4
2 a multiple of 6
3 a multiple of 4 and 6 together
4 a multiple of 4 or 6
5 a number divisible by 25
6 a positive integer less than 25

3 Selecting randomly a card out of 40 similar cards in a box numbered from 1 to 40 Find the probability of getting a card that carries :

1 an even number
2 a number divisible by 3
3 a number is not divisible by 10
4 an even number is divisible by 3
5 a prime number is less than 20

4 A card is drawn randomly from 25 cards numbered from 1 to 25 Find the probability of drawing a card that carries a number :

1 divisible by 4
2 divisible by 5
3 divisible by 4 or 5
4 divisible by 4 and 5
5 greater than or equals 20
6 not divisible by 5 or 4

5 If a fair dice is thrown once , what is the probability of each of the following events ?

1 Getting an even number less than or equal to 4
2 Getting a number between 0 and 10
3 Getting a number divisible by 7
4 Getting a number that is not divisible by 2

6 Drawing randomly a coloured marble out of a box containing 12 red marbles, 18 white marbles and 20 blue marbles. Find the probability of selecting :

1 a white marble.
2 a red marble.
3 a yellow marble.
4 a non-red marble.
5 a red or blue marble.

7 A class contains 40 pupils , 20 pupils of them play football , 10 play volleyball and 6 play basketball , if one pupil is chosen randomly from the class , find the probability that the chosen pupil doesn't play any of the previous sports.

8 Wael has a bag containing 22 marbles. 12 marbles are black and the remained are red. Two marbles are drawn without returning them to the bag , they were red then a third marble is drawn without looking at it. What is the probability that it is black ?

9 A class has 50 students , the number of girls is less than the number of boys by 10
If a student is chosen randomly , find the probability that the student is a boy.

10 A box contains 80 similar balls. Some of them are red and the remained are blue.
If the probability of drawing a red ball is $\frac{1}{4}$, find the number of blue balls.

11 Drawing randomly a coloured marble out of a bag containing 32 similar marbles coloured red , white , green and yellow , the probability of getting a red marble is $\frac{3}{8}$
Estimate how many red marbles are in the bag.

12 Two players in a football team. During the training , one of them kicked 21 penalty kicks, he scored 18 goals , the other kicked 32 penalty kicks , he scored 25 goals. Which of them do you select to kick a penalty kick during the match ? Why ?

13 In football league , the probability of a team to win is 0.6 and the probability of a draw is 0.3. If the number of matches supposed to be played by that team is 30 matches.
How many matches do you predict the team wins ?
How many matches do you predict the team loses ?

Unit 3

14 53 of 100 school students prefer reading the books of the Family Library on a survey has been conducted.
How many students don't read such books out of 400 students ?

15 An insurance company for cars pays L.E. 2000 as compensations to the cars that have accidents.
If the probability for a car to get damaged is 0.004 and the numbers of subscribers in this document is 7000 subscribers. What is your prediction of what the company pays as compensation ?

16 A garment factory in the Tenth of Ramadan City produces 6000 units daily. As a sample of 1000 units was examined , 20 defective units were found.
Calculate the number of defective units.

17 In a fruit packing plant, 30% of fruits is not suitable for exporting because the size is too small.
How many tons can be exported in 10 days if 20 tons of fruits are delivered back daily to the factory ?

18 A calculator manufacturing company examined randomly electronic circuits in a sample of 200 units. The defective production was 6%

1. How many units are out of order in this sample ?
2. If the total production in one month was 1500 units, how many units are functional units of marketing ?

19 **A life insurance company has found in a sample of 10000 men , between 40 and 50 years old , 67 are dead in one year.**

1. What is the probability of a man to die between 40 and 50 years old in one year ?
2. Why are these results important for life insurance companies ?
3. If the company signed life-insurance contracts with 50000 men between 40 and 50 years old , then how many death-benefits should be paid in one year ?

20 **The following data shows the result of a survey about means of transport pupils use to go to school :**

Means of transport	Bicycle	Bus	Private car	On foot
The number	12	16	8	12

If a pupil is selected randomly, what is the probability that the pupil :

1 goes to school by bus. 2 goes to school on foot. 3 doesn't ride bicycles.

21 **The following table shows the evaluation of 50 students in one month :**

A student is randomly selected. What is the probability of getting a score of :

1 Excellent. 2 Good. 3 Failed. 4 Less than good.

Estimate	Excellent	Very good	Good	Pass	Fail
Number	6	9	11	16	8

22 **The following table shows the registered calls of 150 persons in one of the communication centres :**

Call duration	Less than 2 minutes	From 2 to 4 minutes	From 4 to 6 minutes	More than 6 minutes
No. of persons	120	15	10	5

Find the probability that a person makes a call of duration :

1 Less than 2 minutes
2 From 2 to 4 minutes
3 More than 6 minutes
4 2 minutes and more

23 **A survey has been conducted on 100 students about their favourite games which they practise. The result was as follows :**

Favourite game	Football	Handball	Athletics	Tennis	Hockey
Number of students	44	27	12	4	13

1 Find the probability if a student prefers :
(a) Practising football. (b) Practising handball. (c) Practising athletics.
(d) Practising tennis. (e) Practising hockey.

2 If the number of students is 600, how many students are predicted to practise hockey ?

Unit 3

24 **A garment factory produces two types of shirts. The factory made a survey to adjust the production quantity according to the market requirements. Samples of 100 shirts are chosen from 5 shopping centres of the factory. The following table lists the results :**

Number of shopping centres	1	2	3	4	5
Sold amount of first type	39	82	34	22	53
Sold amount of second type	61	18	66	78	47

1. Which type is more demanded ? What is the advice you give to the company ?
2. If the total production of this factory was 4000 shirts, what is your estimated number of shirts of the first type ?

25 **In producing 300 electric lamps , 18 units were found defective.**

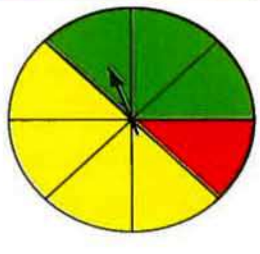
1. What is the probability of a unit to be a defective unit ?
2. What is the probability of a functional unit ?
3. Is it possible for a unit to be a functional unit and out of order unit at the same time ?
4. Find the sum of the probability of a defective unit and the probability of a functional unit. What do you observe ?
5. If a daily production of this factory was 1600 electric lamps, find the number of the functional units in that day.

26 **In a survey of favourite weight of a package of washing powder, the manufacturing company asked a group of 300 ladies using this product. The following table lists the results :**

Weight (in gm)	125	250	375	500	Sum
Number of ladies	120	45	96	39	300

1. Selecting randomly a lady, what is the probability to choose :
   (a) 125 gm.   (b) 250 gm.   (c) 375 gm.   (d) 500 gm.
2. What is your advice to the manager of this company according to the results of this survey ?

27 **The opposite figure represents a spinner game :**



1. Find the probability that the pointer stops at :
   (a) the red colour.   (b) the green colour.
   (c) the yellow colour.
2. The probability that the pointer does not stop at the red colour.

28 **Choose the correct answer from those given :**

1. Which of the following may be the probability of an event ?
   (a) 1.2 (b) – 0.4 (c) 315% (d) 75%

2. In an experiment of throwing a fair die , then the probability of appearing a number not equal to 2 in the upper face is ....
   (a) $\frac{1}{6}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $\frac{5}{6}$

3. If a coin is thrown 400 times , then the most expected number of appearing tail is ....
   (a) 204 (b) 199 (c) 240 (d) 195

4. Ahmed is a pupil in 2nd preparatory. In his class , there are 36 pupils.16 of them are girls. If a pupil is selected randomly , what is the probability that the pupil is a boy ?
   (a) $\frac{4}{9}$ (b) $\frac{1}{2}$ (c) $\frac{5}{9}$ (d) $\frac{1}{36}$

5. There are 25 boys and 20 girls in a classroom. One pupil is chosen randomly. The probability that the chosen pupil is a girl equals ....
   (a) $\frac{1}{20}$ (b) $\frac{4}{9}$ (c) $\frac{1}{25}$ (d) $\frac{5}{9}$

6. If the probability that a pupil succeeds is 70% , then the probability of his failure is ....
   (a) 0.7 (b) 0.07 (c) 0.3 (d) 0.03

7. A bag contains a number of similar balls , half of them are red , $\frac{1}{3}$ of them are black and the rest are white. One ball is chosen. The probability that the chosen ball is white equals ....
   (a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) zero

8. If the probability that a worker go to his work on foot is twice the probability of using any other mean of transport , then the probability that the worker use a mean of transport = ...
   (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 2

9. A box contains balls coloured with red , green , blue and yellow. If the box contains 20 yellow balls and the probability of selecting a yellow ball randomly is $\frac{1}{4}$, what is the number of balls in the box ?
   (a) 5 (b) 25 (c) 60 (d) 80

10. The number of pupils in a class of 2nd year preparatory is 36 pupils , the probability of selecting a pupil whose age is less than or equal to 13 years is $\frac{1}{6}$
    What is the number of pupils in the class whose ages are more than 13 years ?
    (a) 23 (b) 24 (c) 30 (d) 32

11 In a mixed school, the ratio between the number of boys to the number of girls is 7 : 9
A pupil is selected randomly from this school.
The probability that the selected pupil is a boy equals ....

(a) zero (b) $\frac{7}{16}$ (c) $\frac{9}{16}$ (d) 7

12 In a mixed school , there are 1500 pupils. A random sample formed from 200 pupils is selected. It is found that the number of girls equals 90
What is the expected number of girls in the school ?

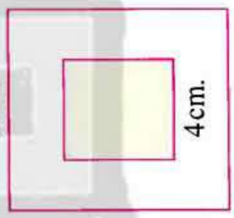
(a) 600 girls (b) 625 girls (c) 650 girls (d) 675 girls

13 The following table shows the numbers of 160 pupils in a school who like to practise a certain game. If a pupil is selected randomly from this sample , what is the probability that he is practising handball ?

Game	Swimming	Handball	Athletics	Football	Gymnastics	Boxing
Number	20	40	30	50	10	10

(a) $\frac{1}{16}$ (b) 25% (c) $\frac{1}{4}$% (d) $\frac{5}{16}$

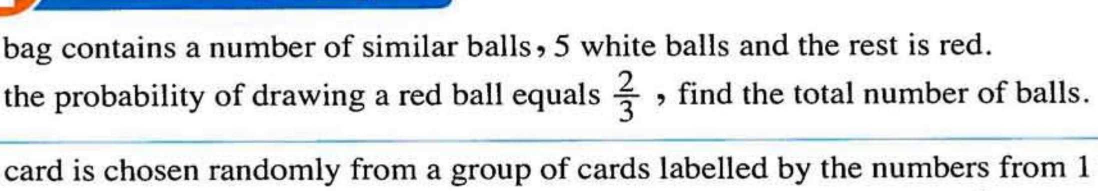
14 In the opposite board two squares are drawn , if a person points at it as a target , then the probability of hitting the shaded region is ....



(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

15 A spinner game is divided into two parts x and y , then the point is rolled 400 rounds , it stopped 98 times in the region x , then which of the following figures the pointer points to the region x ?



(a) (b) (c) (d)

For excellent pupils

29 A bag contains a number of similar balls, 5 white balls and the rest is red.
If the probability of drawing a red ball equals $\frac{2}{3}$, find the total number of balls.

30 A card is chosen randomly from a group of cards labelled by the numbers from 1 to n
If the probability that the drawn card carries a number greater than 8 is $\frac{1}{3}$,
find the value of n

Summary of Unit 3

- The probability of occurrence of a certain event = $\frac{\text{the number of times of repeating this outcome}}{\text{the number of all possible outcomes}}$
- The expected number for occurrence of a certain event

  = the probability of its occurrence × the total number of the given individuals
- The random experiment is an experiment, where all its possible outcomes are known before doing it but we can't determine the actual outcome.
- The sample space is the set of all possible outcomes of a random experiment and it is denoted by S
- The event is a subset of the sample space.
- The probability of occurrence of an event $A \subset S$ is denoted by P (A)

  It is found by using the relation :

  $$P(A) = \frac{\text{the number of elements of A}}{\text{the number of elements of the sample space}} = \frac{n(A)}{n(S)}$$

Exam on Unit Three

Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1. If the probability of the success of a student is 80 % , then the probability of his failure = ...

   (a) 0 (b) 0.8 (c) 20 (d) 0.2

2. If a fair die is rolled once , then the probability of appearing the number 7 is ....

   (a) zero (b) $\frac{1}{7}$ (c) $\frac{1}{6}$ (d) 1

3. Which of the following may be a probability of an event ?

   (a) – 0.5 (b) 1.2 (c) $\frac{4}{3}$ (d) 37 %

4. If a coin is thrown 200 times , then the expected number of appearing a tail is ....

   (a) 103 (b) 99 (c) 200 (d) 95

5. A basket has cards labelled by the numbers from 1 to 15 if a card is selected randomly , what is the probability that the number on the card is divisible by 5 ?

   (a) $\frac{1}{5}$ (b) $\frac{1}{3}$ (c) 3 (d) $\frac{2}{3}$

6. As throwing a fair die once , then the probability of appearing a prime number = ...

   (a) 45 % (b) 0.35 (c) 25 % (d) $\frac{1}{2}$

2 Complete the following :

1. The probability of the certain event = ...
   and probability of the impossible event = ...

2. If the probability that a student solves a question is 0.7 , then the expected number of questions to be solved out of 20 questions = ...

3. A garden has 5 doors numbered from 1 to 5 , a visitor goes out the probability that he goes out of the fourth door is ....

4. For any event A , P (A) $\in$ [.......... , ...]

5. In the experiment of throwing a fair die once and observing the upper face , then the probability of appearing a number less than 4 is ....

3 [a] A group of cards numbered from 1 to 20, a card is drawn randomly.

Calculate the probability of the drawing card carries :

1. A number divisible by 3
2. A prime number.
3. A number greater than 8

[b] A box contains 3 red balls, 4 yellow balls and 5 green balls.

A ball is selected randomly.

Find the probability of selecting :

1. A red ball.
2. A white ball.
3. Not a yellow ball.

4 If a fair dice is thrown once.

Find the probability of the following events:

1. Getting a number greater than 3
2. Getting a number less than 9
3. Getting an odd prime number.
4. Getting a number less than 1

5 [a] A box contains 60 similar balls some are red and the rest are blue, if the probability of drawing a red ball is $\frac{1}{4}$

Find the number of blue balls.

[b] A class has 48 pupils 18 of them are girls, if a pupil is chosen randomly.

Find the probability that the chosen pupil is a boy.

TIMSS Problems

Accumulative basic skills

1 Choose the correct answer from the given ones :

1. $\frac{5}{2} \div \frac{5}{2} = \dots\dots\dots$

   (a) 1 (b) $\frac{4}{25}$ (c) $\frac{25}{4}$ (d) $\frac{10}{4}$

2. If two thirds of a number equals 6 , then this number equals ....

   (a) 4 (b) 7 (c) 9 (d) 24

3. Which two fractions of the following are not equal ?

   (a) $\frac{1}{2}$, $\frac{2}{4}$ (b) $\frac{4}{3}$, $\frac{8}{6}$ (c) $\frac{5}{25}$, $\frac{2}{10}$ (d) $\frac{6}{9}$, $\frac{9}{27}$

4. Which of the following numbers is divisible by 4 ?

   (a) 1258 (b) 2421 (c) 1536 (d) 4010

5. Which of the following number is the greatest ?

   (a) $(-9)^{15}$ (b) $\left(\frac{1}{8}\right)^{12}$ (c) 501^{zero} (d) $(0)^{100}$

6. $2^{10} \times 2^{-10} = 3^{\dots}$

   (a) zero (b) 1 (c) 2 (d) 3

7. If $5x = 35$, then $2x + 1 = \dots\dots\dots$

   (a) 7 (b) 8 (c) 15 (d) 71

8. If $a^{x} = 2$, $a^{-y} = 3$, then $a^{x+y} = \dots\dots\dots$

   (a) 1 (b) -1 (c) $\frac{2}{3}$ (d) 6

9. Which of the following numbers lies between 2.2 and 2.3 ?

   (a) 1.3 (b) 2.4 (c) 2.25 (d) 2.1

10 $\sqrt{100-64} = 10 - \dots\dots$

(a) 6 (b) 4 (c) – 6 (d) – 4

11 $\frac{3}{4} + 50\ \% = \dots\dots$

(a) 75 % (b) $50\frac{3}{4}$ (c) 125 % (d) $\frac{3}{2}$

12 If $\sqrt{x+5} = 3$, then $\sqrt{x} = \dots\dots$

(a) zero (b) 2 (c) 4 (d) 9

13 If (3 , k) satisfies the relation : $2x + y = 7$, then k = ...

(a) 1 (b) 2 (c) 3 (d) 4

14 If $\frac{5(x+2)-7}{6} = \frac{13-(4-x)}{9}$, then $x = \dots\dots$

(a) $\frac{9}{17}$ (b) $\frac{9}{13}$ (c) $\frac{33}{17}$ (d) $\frac{33}{13}$

15 $2^{30} + 2^{30} + 2^{30} + 2^{30} = \dots\dots$

(a) 8^{120} (b) 8^{30} (c) 2^{120} (d) 2^{32}

2 Complete the following :

1 $2 \times 6 - 12 \div 3 = \dots\dots$

2 $\frac{(17)^2 - 2 \times 17 + 17}{17} = \dots\dots$

3 $3\frac{1}{4} \times \dots\dots = 1$

4 $-7 + |-7| = \dots\dots$

5 $298 + 502 = 300 + \dots\dots$

6 $(3x^{-1})^2 = \frac{9}{\dots\dots}$ where $x \neq 0$

7 If $x + y = \frac{3}{5}$, then $(5x + 5y)^3 = \dots\dots$

8 If $\frac{a}{b} = \frac{2}{5}$, then $\frac{5a}{2b} = \dots\dots$

9 If $x : 49 = 2 : 7$, then $x = \dots\dots$

10 If the sum of five numbers equals 20 , then the arithematic mean of these numbers = ...

11 1 , 2 , 4 , 7 , 11 , ... (in the same pattern)

12 1 , 4 , 9 , 16 , ... (in the same pattern)

13 If $\frac{2x}{5} = -2$, then $x^2 = \dots\dots$

14 If $0.000\,37 = 3.7 \times 10^{n}$, then n = ...

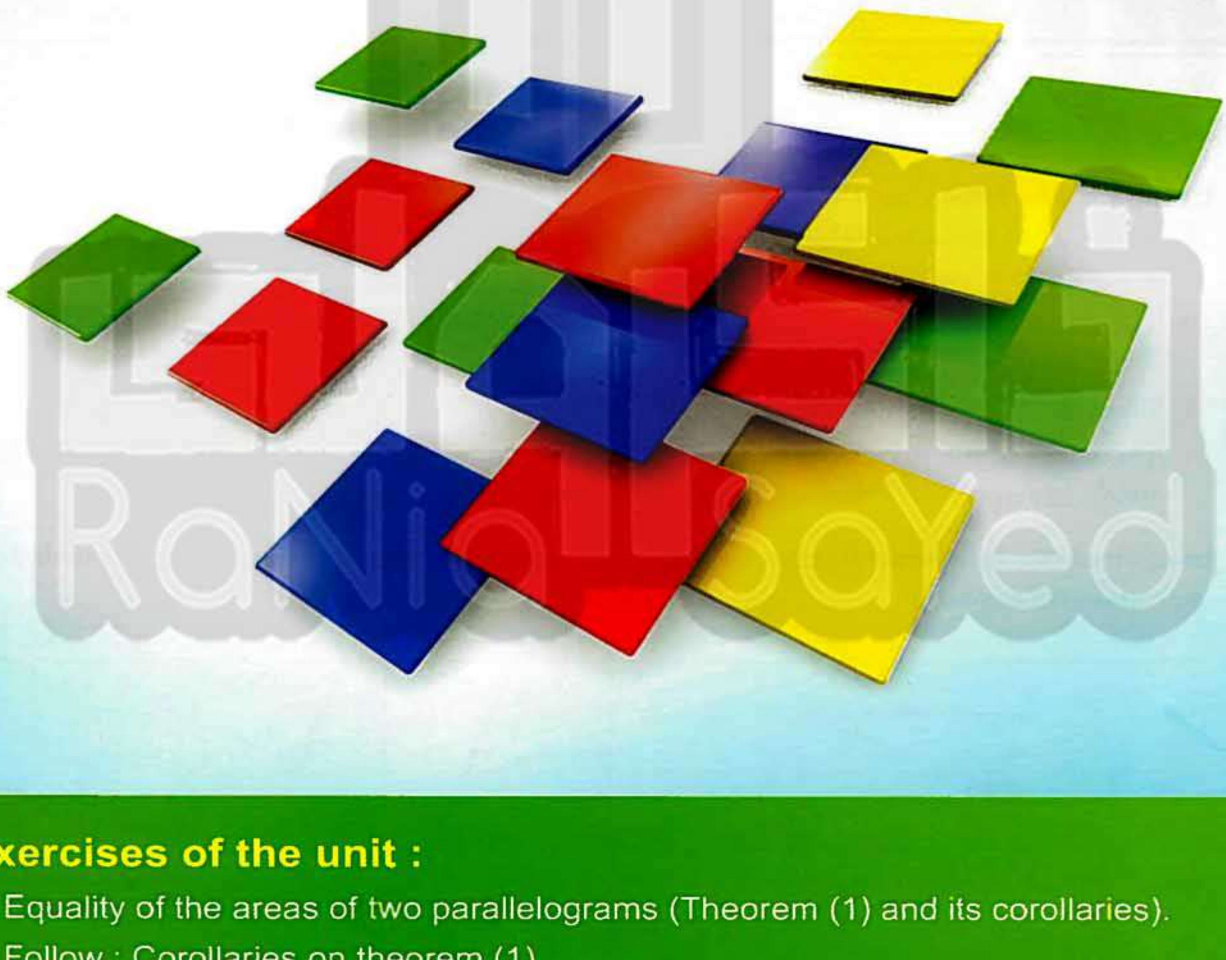
15 If M (1 , 3) , N (0 , 1) , then the slope of $\overleftrightarrow{MN} = \dots\dots$

Second

Geometry

UNIT 4

Areas



Exercises of the unit :

1. Equality of the areas of two parallelograms (Theorem (1) and its corollaries).
2. Follow : Corollaries on theorem (1)
3. Equality of the areas of two triangles (Theorem (2) and its corollaries).
4. Follow : Equality of the areas of two triangles (Theorem (3))
5. Areas of some geometric figures.

- Summary of unit four.
- Unit exams.

Exercise 1

Equality of the areas of two parallelograms

From the school book

1 Complete the following :

1. Surfaces of two parallelograms with common base and between two parallel straight lines , one is carrying this base , are ....
2. The parallelogram and .... with common base and between two parallel straight lines are equal in area.
3. The area of the parallelogram = ... × ....
4. The areas of the parallelograms with bases equal in length and lying on a straight line , while the opposite sides to these bases are on another straight line , are ....

2 In each of the following figures , determine a base and its corresponding altitude for each parallelogram :

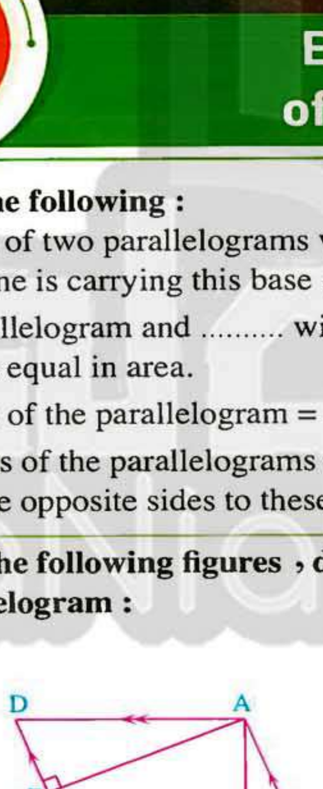


Fig. (1)

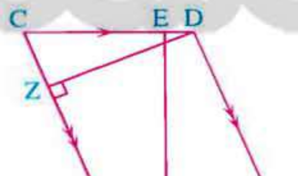


Fig. (2)

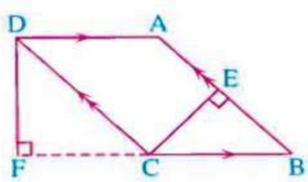


Fig. (3)

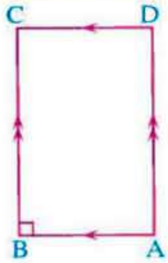
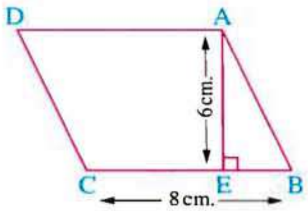


Fig. (4)

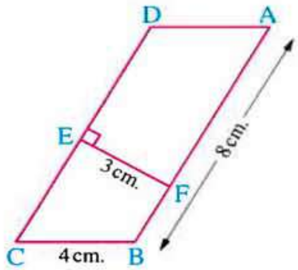
3 **In each of the following , if ABCD is a parallelogram , complete the statements below each figure :**

1



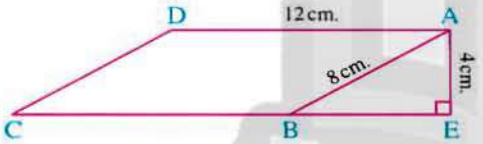
The area of ▱ ABCD = ... cm^2.

2



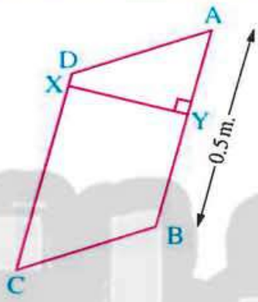
The area of ▱ ABCD = ... cm^2.

3



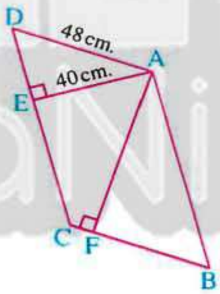
The area of ▱ ABCD = ... cm^2.

4



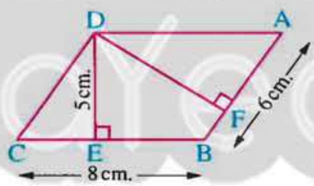
If the area of ▱ ABCD is 1.7 m^2 , then XY= .... m.

5



If the area of ▱ ABCD is 2400 cm^2 , then DC = ... cm.
, AF = ... cm.

6



DF = ... cm.

4 **Choose the correct answer from those given :**

1 If the base length of a parallelogram is 7 cm. and the corresponding height is 4 cm. , then its area equals ....

(a) 11 cm^2. (b) 14 cm^2. (c) 22 cm^2. (d) 28 cm^2.

2 If the area of a parallelogram is 35 cm^2 and its height is 5 cm. , then the length of the corresponding base is ....

(a) 5 cm. (b) 7 cm. (c) 9 cm. (d) 30 cm.

Unit 4

3 If the area of a parallelogram is 50 cm² and its base length = 10 cm. , then the corresponding height of this base = ...

(a) 500 cm. (b) 5 cm. (c) 250 cm. (d) 100 cm.

4 If the lengths of two adjacent sides of a parallelogram are 8 cm. and 10 cm. and its greater height is 5 cm. , then its area equals ....

(a) 80 cm² (b) 50 cm² (c) 40 cm² (d) 18 cm²

5 If ABCD is a parallelogram in which , AB = 5 cm. , BC = 10 cm. and its smaller height is 4 cm. , then its greater height equals ....

(a) 2 cm. (b) 4 cm. (c) 8 cm. (d) 10 cm.

6 A parallelogram whose area = 50 cm² and the length of its base equals twice the corresponding height , then this height equals ....

(a) 50 cm. (b) 25 cm. (c) 10 cm. (d) 5 cm.

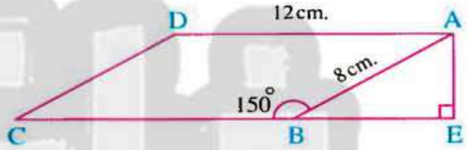
5 **In the opposite figure :**

ABCD is a parallelogram in which m (∠ ABC) = 150° ,

AD = 12 cm. , AB = 8 cm.

, $E \in \overrightarrow{CB}$ and $\overline{AE} \perp \overrightarrow{CB}$

Find : The area of ▱ ABCD



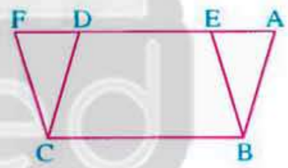
« 48 cm² »

6 **In the opposite figure :**

ABCD and EBCF are two parallelograms ,

$E \in \overleftrightarrow{AD}$ and $F \in \overleftrightarrow{AD}$

Prove that : The area of Δ ABE = the area of Δ DCF

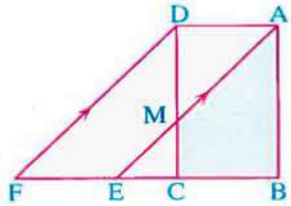


7 **In the opposite figure :**

ABCD is a rectangle , $\overline{AE}$ // $\overline{DF}$, $E \in \overrightarrow{BC}$ and $F \in \overrightarrow{BC}$

Prove that :

The area of the figure ABCM = the area of the figure DMEF



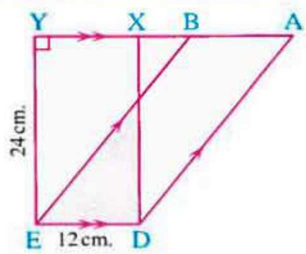
8 **In the opposite figure :**

$\overleftrightarrow{AB}$ // $\overleftrightarrow{DE}$, $X \in \overrightarrow{AB}$, $Y \in \overrightarrow{AB}$

, XDEY is a rectangle and $\overline{AD}$ // $\overline{BE}$

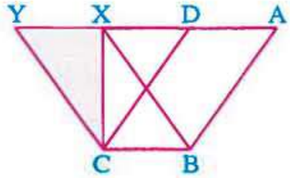
1 Find the area of the figure ABED

2 If AD = 30 cm. , find the length of the perpendicular from B to $\overline{AD}$



« 288 cm² , 9.6 cm. »

9 **In the opposite figure :**



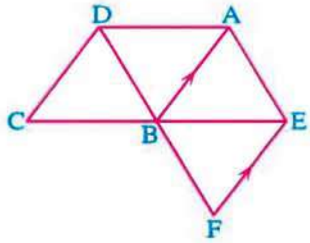
ABCD and XBCY are two parallelograms $X \in \overrightarrow{AD}$

and the area of Δ XCY = 15 cm².

Find : The area of ▱ ABCD

« 30 cm². »

10 **In the opposite figure :**



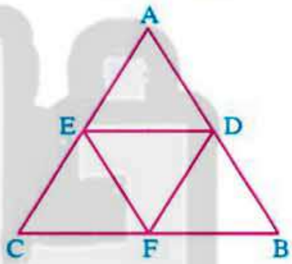
ABCD and AEBD are two parallelograms and $F \in \overrightarrow{DB}$

such that $\overline{EF} \parallel \overline{AB}$

Prove that :

1. AEFB is a parallelogram.
2. The area of ▱ ABCD = the area of ▱ AEFB

11 **In the opposite figure :**



DBFE and DFCE are two parallelograms

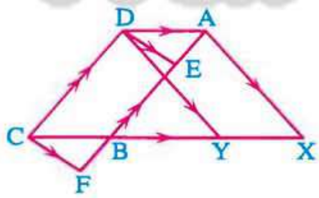
and $F \in \overline{BC}$

Prove that :

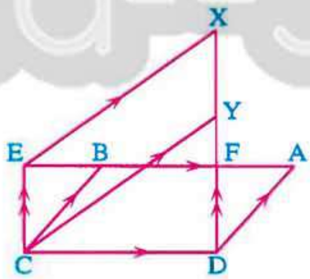
The area of the figure ABFE = the area of the figure ADFC

12 **In the following figures , show that all the three parallelograms have equal areas :**

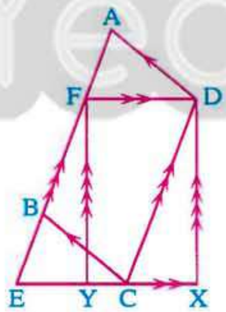
1



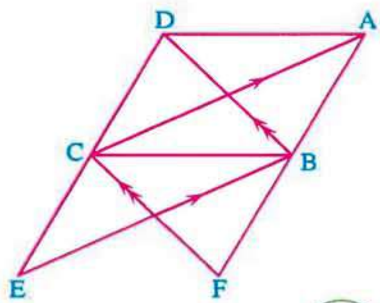
2



3



13 **In the opposite figure :**



ABCD is a parallelogram , $\overline{BE} \parallel \overline{AC}$,

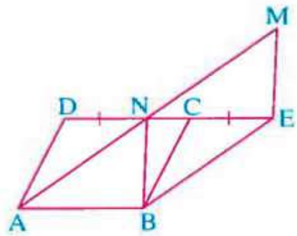
where $\overline{BE} \cap \overrightarrow{DC} = \{E\}$

and $\overline{CF} \parallel \overline{DB}$ where $\overline{CF} \cap \overrightarrow{AB} = \{F\}$

Prove that : The area of ▱ BFCD = the area of ▱ ABEC

Unit 4

14 **In the opposite figure :**



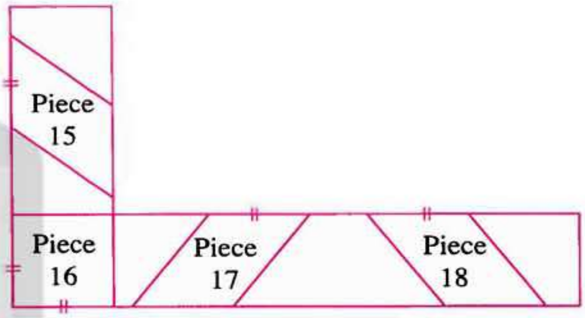
ABCD and BEMN are two parallelograms and EC = DN

where $E \in \overrightarrow{DC}$ and $M \in \overrightarrow{AN}$

Prove that : The area of ▱ ABCD = the area of ▱ BEMN

Life Application

15 In the national project "Build up your home" , a piece of land is divided as illustrated in the opposite figure :



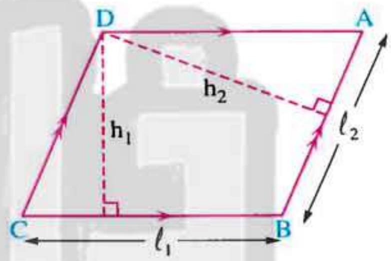
Is the area of piece number 15 = the area of the piece number 16 ? State the number of pieces of equal areas. Explain your answer.

For excellent pupils

16 ABCD is a parallelogram whose area is 240 cm^2.

$\ell_1 : h_1 = 5 : 3$, $\ell_1 : \ell_2 = 4 : 3$

Find : h_2



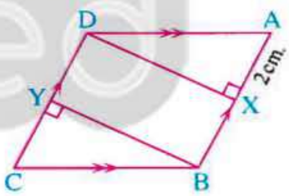
« 16 cm. »

17 **In the opposite figure :**

ABCD is a parallelogram whose area is 24 cm^2.

and XBYD is a rectangle whose area is 12 cm^2, AX = 2 cm.

Find the perimeter of the rectangle XBYD



« 16 cm. »

Exercise 2

Follow : Corollaries on theorem (1)

From the school book

1 Choose the correct answer from those given :

1 The area of the triangle is .... the area of the parallelogram which has a common base with it and its vertex lies on the straight line parallel to this base.

(a) equal to (b) half (c) twice (d) quarter

2 The area of the triangle = ... the base length × the corresponding height.

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

3 The ratio between the area of the parallelogram and the area of the triangle whose base is common and are included between two parallel straight lines = ...

(a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 2 : 3

4 If the base length of a triangle is 4 cm. and the corresponding height = 3 cm. , then its area = ...

(a) 6 cm^2 (b) 12 cm^2 (c) 24 cm. (d) 34 cm^2

5 The triangle whose base length is 12 cm. and its area is 48 cm^2 , the corresponding height = ...

(a) 3 cm. (b) 4 cm. (c) 6 cm. (d) 8 cm.

6 If the area of the triangle is 42 cm^2 and its height = 7 cm. , then the length of the corresponding base = ...

(a) 15 cm. (b) 12 cm. (c) 8 cm. (d) 4 cm.

7 The area of a right-angled triangle in which the lengths of the sides of the right angle are 6 cm. and 9 cm. equals ....

(a) 54 cm^2 (b) 60 cm^2 (c) 27 cm^2 (d) 15 cm^2

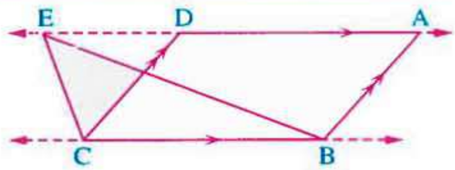
8 The area of the rectangle whose dimensions are 6 cm. and 4 cm. .... the area of the triangle whose base length is 12 cm. and the corresponding height is 4 cm.

(a) < (b) > (c) = (d) ≠

9 If ABCD is a parallelogram with area 100 cm^2 and E $\in \overline{AD}$, then the area of Δ EBC = ...

(a) 25 cm^2 (b) 50 cm^2 (c) 100 cm^2 (d) 200 cm^2

2 **In the opposite figure :**



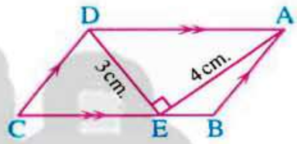
ABCD is a parallelogram and E $\in \overleftrightarrow{AD}$

Complete the following :

1 The area of Δ EBC = ... the area of ▱ ABCD

2 If the area of Δ EBC = 20 cm^2 , then the area of ▱ ABCD = ... cm^2

3 **In the opposite figure :**

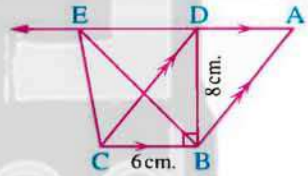


ABCD is a parallelogram , AE = 4 cm. , ED = 3 cm. , m (∠ AED) = 90° and E $\in \overline{BC}$

Complete :

1 The area of Δ AED = ... cm^2 2 The area of ▱ ABCD = ... cm^2

4 **In the opposite figure :**

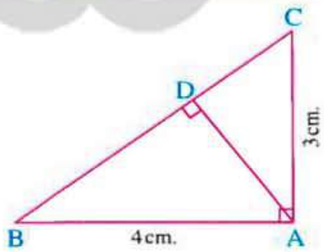


ABCD is a parallelogram in which , BC = 6 cm. , $\overline{DB} \perp \overline{BC}$, such that , DB = 8 cm. and E $\in \overleftrightarrow{AD}$

Complete :

1 The area of ▱ ABCD = ... cm^2 2 The area of Δ EBC = ... cm^2

5 **In the opposite figure :**



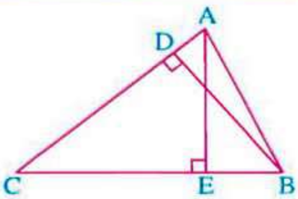
ABC is a right-angled triangle at A ,

$\overline{AD} \perp \overline{BC}$, AB = 4 cm. and AC = 3 cm.

Find : 1 The area of Δ ABC 2 The length of $\overline{AD}$

« 6 cm. , 2.4 cm. »

6 **In the opposite figure :**



ABC is a triangle in which BC = 6.5 cm.

, AC = 6 cm. , $\overline{AE} \perp \overline{BC}$, $\overline{BD} \perp \overline{AC}$ and BD = 5 cm.

Find : 1 The area of Δ ABC 2 The length of $\overline{AE}$

« 15 cm.^{2} , $4\frac{8}{13}$ cm. »

7 **In the opposite figure :**

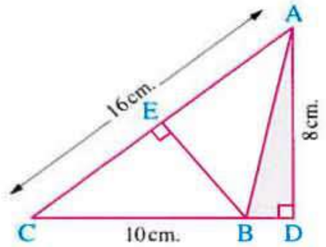
$\overline{AD} \perp \overrightarrow{CB}$, $\overline{BE} \perp \overline{AC}$, AC = 16 cm. ,

BC = 10 cm. and AD = 8 cm.

Find :

1 The area of Δ ABC

2 The length of $\overline{BE}$

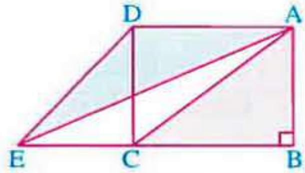


« 40 cm.^{2} , 5 cm. »

8 **In the opposite figure :**

ABCD is a rectangle and E ∈ $\overrightarrow{BC}$

Prove that : The area of Δ DAE = the area of Δ ABC



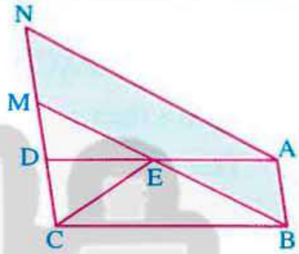
9 **In the opposite figure :**

ABCD and ABMN are two parallelograms

and M ∈ $\overrightarrow{CD}$

Prove that :

The area of Δ EBC = $\frac{1}{2}$ the area of ▱ ABMN



10 **In the opposite figure :**

ABCD is a rectangle , ABEF is a parallelogram

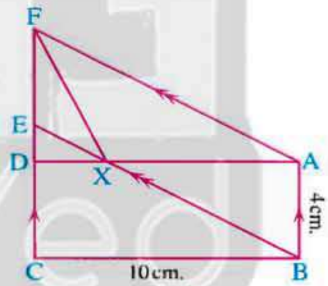
, D ∈ $\overline{CF}$, X ∈ $\overline{BE}$, E ∈ $\overline{CF}$

, AB = 4 cm. and BC = 10 cm.

Find by proof :

1 The area of ▱ ABEF

2 The area of Δ XAF



« 40 cm.^{2} , 20 cm.^{2} »

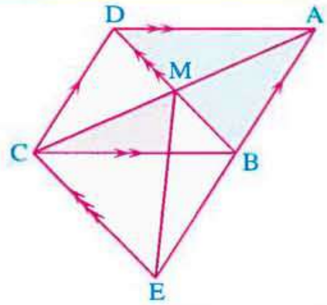
11 **In the opposite figure :**

ABCD and BECD are two parallelograms , where

$\overline{AC} \cap \overline{BD} = \{M\}$

Prove that :

The area of Δ ABD = the area of Δ MEC



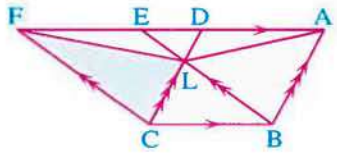
12 **In the opposite figure :**

ABCD and EBCF are two parallelograms , $\overline{BE} \cap \overline{CD} = \{L\}$

, D ∈ $\overline{AF}$ and E ∈ $\overline{AF}$

Prove that : 1 The area of Δ ABL = the area of Δ FCL

2 The area of the figure ABCL = the area of the figure FCBL



Unit 4

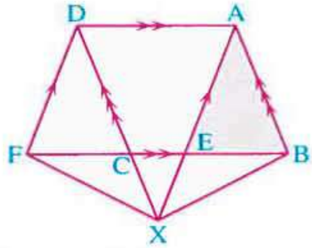
13 **In the opposite figure :**

ABCD and AEFD are two parallelograms

and $\overrightarrow{AE} \cap \overrightarrow{DC} = \{X\}$

Prove that :

The area of Δ ABX = the area of Δ DFX



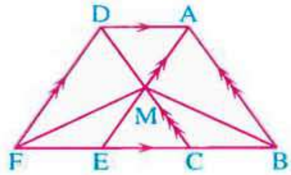
14 **In the opposite figure :**

ABCD and AEFD are two parallelograms and $\overline{AE} \cap \overline{CD} = \{M\}$

where $E \in \overline{BF}$ and $C \in \overline{BF}$

Prove that :

The area of Δ ABM = the area of Δ DMF

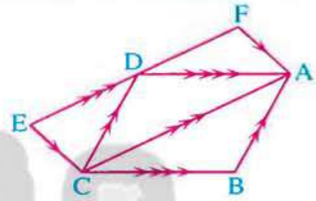


15 **In the opposite figure :**

ABCD and ACEF are two parallelograms and $D \in \overline{FE}$

Prove that :

The area of ▱ ABCD = the area of ▱ ACEF

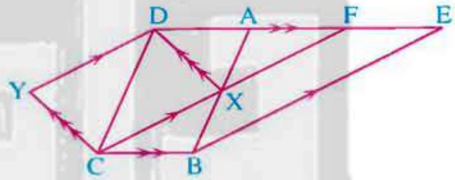


16 **In the opposite figure :**

$\overleftrightarrow{ED} // \overleftrightarrow{BC}$, $\overleftrightarrow{XD} // \overleftrightarrow{CY}$,

$\overleftrightarrow{EB} // \overleftrightarrow{FC} // \overleftrightarrow{DY}$, $X \in \overline{FC}$

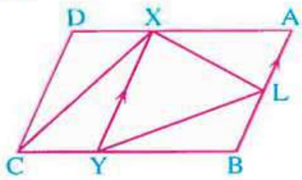
, $F \in \overline{ED}$ and $A \in \overline{ED}$



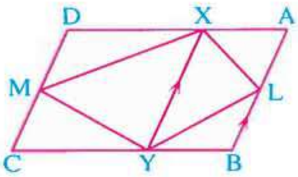
Prove that : The area of ▱ EBCF = the area of ▱ ABCD = the area of ▱ DXCY

17 **In each of the following figures , $\overline{XY} // \overleftrightarrow{AB}$, show that the area of coloured part is equal to half of the area of parallelogram ABCD :**

1



2



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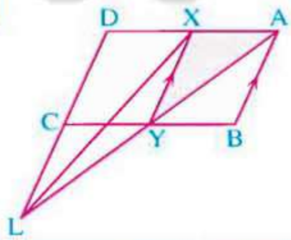

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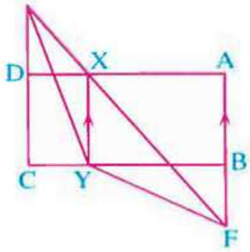
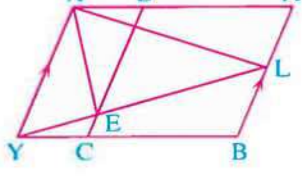
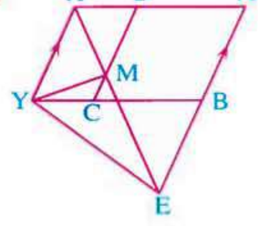


5

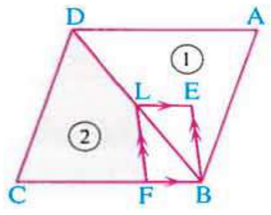


6

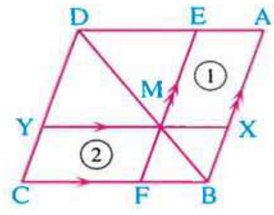


18 **In each of the opposite figures :**

ABCD is a parallelogram.
Why is the area of figure (1) equal to
the area of figure (2) ?



(a)



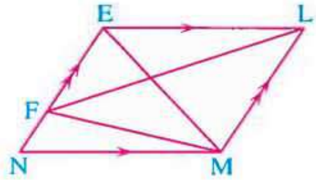
(b)

19 **In the opposite figure :**

LMNE is a parallelogram.

Prove that :

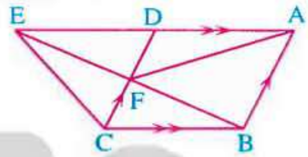
The area of Δ LEF + the area of Δ MFN = the area of Δ LEM



20 **In the opposite figure :**

ABCD is a parallelogram , $E \in \overrightarrow{AD}$ and $\overline{BE} \cap \overline{CD} = \{F\}$

Prove that : The area of Δ AFD = the area of Δ EFC



21 **In the opposite figure :**

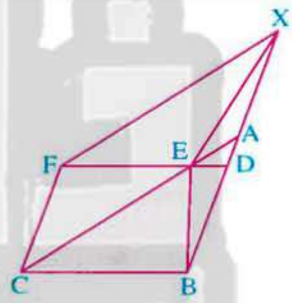
DBCF is a parallelogram with area equal to 80 cm^2 and $E \in \overline{DF}$

ACFX is a parallelogram and $A \in \overline{BX}$

Prove that :

1 The area of Δ EBC = the area of Δ EFX

2 The area of Δ AXE + the area of Δ ECF = 40 cm^2

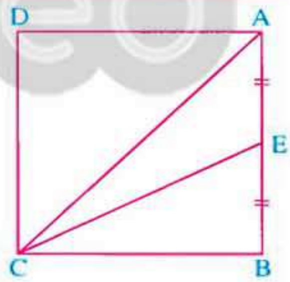


22 **In the opposite figure :**

ABCD is a square E is the midpoint of $\overline{AB}$

The perimeter of the square ABCD = 48 cm.

Find : The area of Δ AEC



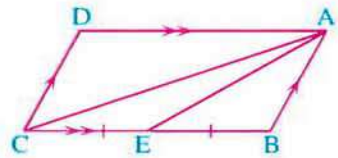
« 36 cm^2 »

23 **In the opposite figure :**

ABCD is a parallelogram whose perimeter is 48 cm. ,
BC = 2 AB and the area of Δ ABC = 56 cm^2 ,
E is the midpoint of $\overline{BC}$

Find : 1 The two heights of ▱ ABCD

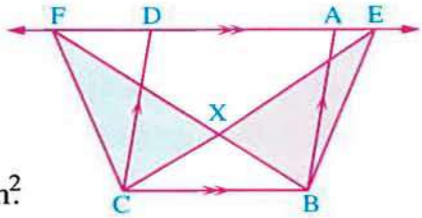
2 The area of Δ AEC



« 14 cm. , 7 cm. , 28 cm^2 »

Unit 4

24 **In the opposite figure :**



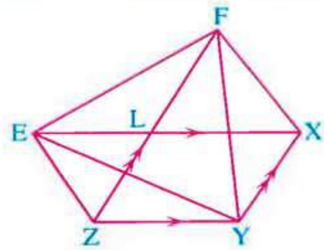
ABCD is a parallelogram , $E \in \overleftrightarrow{AD}$ and $F \in \overleftrightarrow{AD}$

Prove that : The area of Δ EBX = the area of Δ FCX

and if the area of Δ XBC = $\frac{1}{6}$ the area of ▱ ABCD = 12 cm².

Calculate : The area of Δ FCX « 24 cm². »

25 **In the opposite figure :**



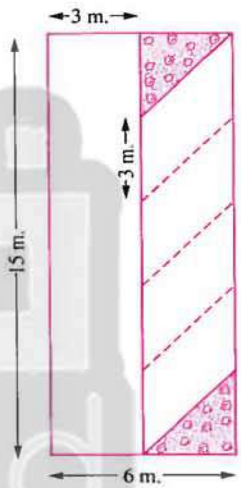
XYZL is a parallelogram , $E \in \overrightarrow{XL}$ and $F \in \overrightarrow{ZL}$

Prove that :

The area of the figure FXYE = the area of the figure FYZE

Life Application

26 The opposite figure shows a design of a rectangular piece of land of dimensions 15 m. and 6 m. , divided into four congruent places for parking cars , each of them is in the shape of a parallelogram , and two congruent places for planting flowers , each of them is in the shape of a triangle , besides a path for cars in the shape of a rectangle of width 3 m.

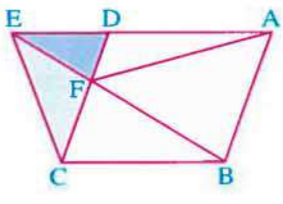


Find : 1 The area used for parking cars. « 36 m². »

2 The area used for planting flowers. « 9 m². »

For excellent pupils

27 **In the opposite figure :**



ABCD is a parallelogram , $F \in \overline{CD}$ and $\overrightarrow{BF} \cap \overrightarrow{AD} = \{E\}$

Prove that :

The area of Δ AFE = the area of Δ DCE

28 Δ ABC is a right-angled at B , in which m ($\angle$ C) = 30° , $\overline{BD} \perp \overline{AC}$ intersecting it at D

Prove that : $BD = \frac{2\,AD \times BC}{AC}$

Exercise 3 Equality of the areas of two triangles

From the school book

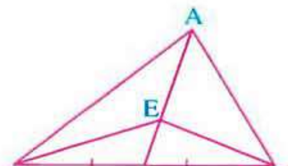
1 Complete the following :

1. The two triangles drawn on a common base and their vertices located on a straight line parallel to the base are ....
2. Triangles with congruent bases and drawn between two parallel lines are ....
3. The median in the triangle divides its area into ....
4. If ABC is a triangle , D is the midpoint of $\overline{BC}$, then :
   The area of Δ ABD = the area of Δ ....
5. If $\overline{XL}$ is a median in Δ XYZ , then the area of Δ XYZ = ... the area of Δ XYL
6. The triangle XYZ in which L $\in \overline{YZ}$ such that YL = $\frac{1}{2}$ LZ , then :
   The area of Δ XYL = ... the area of Δ XYZ

2 In the opposite figure :

ABC is a triangle with a median $\overline{AD}$,

E $\in \overline{AD}$, draw $\overline{BE}$ and $\overline{CE}$



Prove that : The area of Δ ABE = the area of Δ ACE

Complete :

$\because$ $\overline{AD}$ is a median in the triangle ....

$\therefore$ The area of Δ ABD = the area of .... (1)

$\because$ .... is a median in Δ EBC

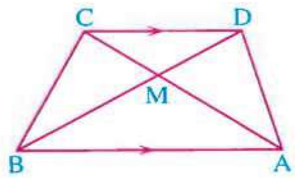
$\therefore$ The area of Δ EBD = the area of .... (2)

Subtracting sides of (2) from sides of (1) , then the area of Δ ABE = ...

Unit 4

3 **In the opposite figure :**

$\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$ and $\overline{AC} \cap \overline{BD} = \{M\}$

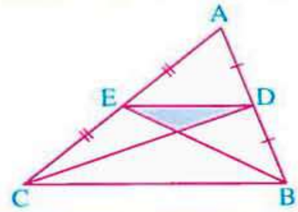


Complete and justify each step of your answer :

1. The area of Δ ADB = the area of ………. because ……….
2. The area of Δ DAC = the area of ………. because ……….
3. The area of Δ DAM = the area of ………. because ……….

4 **In the opposite figure :**

D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$



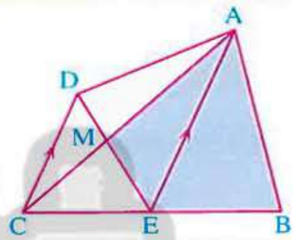
Prove that :

The area of Δ BDE equals the area of Δ CDE

5 **In the opposite figure :**

ABCD is a quadrilateral. $E \in \overline{BC}$ such that $\overline{AE} \parallel \overline{DC}$,

$\overline{AC} \cap \overline{DE} = \{M\}$

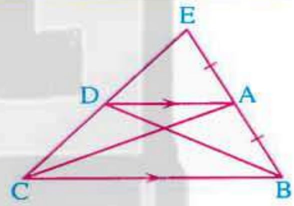


Prove that : The area of Δ ABC = the area of figure ABED

6 **In the opposite figure :**

ABCD is a quadrilateral in which $\overline{AD} \parallel \overline{BC}$ and $\overrightarrow{BA} \cap \overrightarrow{CD} = \{E\}$

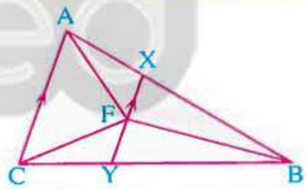
such that BA = AE



Prove that : The area of Δ ADC = the area of Δ ADE

7 **In the opposite figure :**

$\overline{AC} \parallel \overline{XY}$ and F is the midpoint of $\overline{XY}$

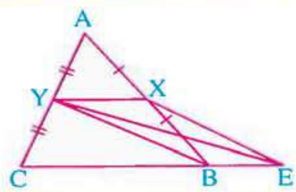


Prove that : The area of Δ ABF = the area of Δ CBF

8 **In the opposite figure :**

ABC is a triangle. X is the midpoint of $\overline{AB}$,

Y is the midpoint of $\overline{AC}$ and $E \in \overrightarrow{CB}$

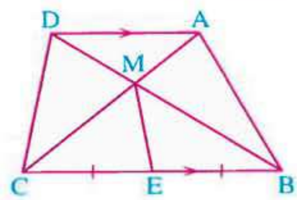


Prove that : The area of Δ XYE = the area of Δ AXY

9 **In the opposite figure :**

$\overline{AD} \parallel \overline{BC}$, $\overline{AC} \cap \overline{BD} = \{M\}$,

E is the midpoint of $\overline{BC}$



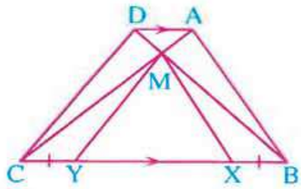
Prove that : The area of the figure ABEM = the area of the figure DMEC

10 **In the opposite figure :**

$\overline{AD} // \overline{BC}$, $\overline{AC} \cap \overline{BD} = \{M\}$ and BX = CY

Prove that :

The area of the figure ABXM = the area of the figure DCYM

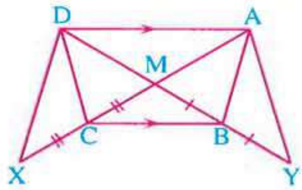


11 **In the opposite figure :**

$\overline{AD} // \overline{BC}$ and B is the midpoint of $\overline{YM}$,

C is the midpoint of $\overline{MX}$

Prove that : The area of Δ AYB = the area of Δ DCX



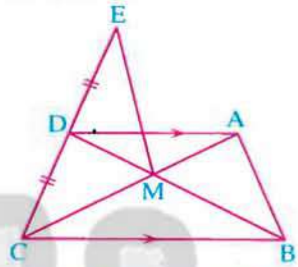
12 **In the opposite figure :**

$\overline{AD} // \overline{BC}$ and $\overline{AC} \cap \overline{BD} = \{M\}$,

D is the midpoint of $\overline{EC}$

Prove that :

The area of Δ MDE = the area of Δ AMB



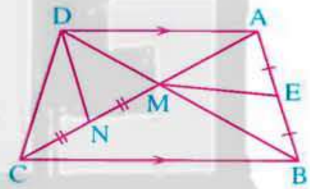
13 **In the opposite figure :**

ABCD is a quadrilateral whose diagonals intersect at M,

$\overline{AD} // \overline{BC}$ and E is the midpoint of $\overline{AB}$,

N is the midpoint of $\overline{MC}$

Prove that : The area of Δ AEM = the area of Δ DNC

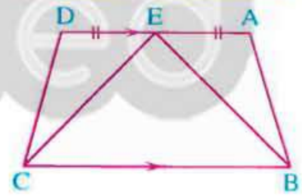


14 **In the opposite figure :**

$\overline{AD} // \overline{BC}$ and E is the midpoint of $\overline{AD}$

Prove that :

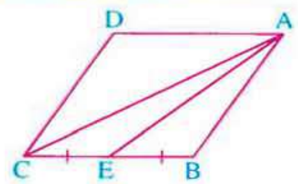
The area of the figure ABCE = the area of the figure DEBC



15 **In the opposite figure :**

ABCD is a parallelogram. E is the midpoint of $\overline{BC}$

Prove that : The area of Δ ABE = $\frac{1}{4}$ the area of ▱ ABCD

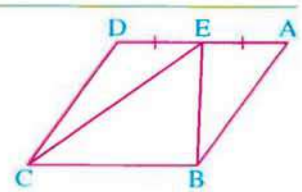


16 **In the opposite figure :**

ABCD is a parallelogram. E is the midpoint of $\overline{AD}$

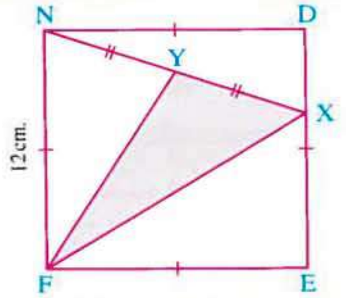
The area of ▱ ABCD = 48 cm^2.

Find : The area of Δ ABE



« 12 cm^2. »

17 **In the opposite figure :**



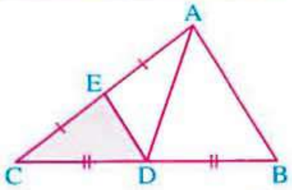
DEFN is a square whose side length is 12 cm.,

$X \in \overline{DE}$ and Y is the midpoint $\overline{XN}$

Find : The area of Δ XYF

« 36 cm^2 »

18 **In the opposite figure :**



D is the midpoint of $\overline{BC}$,

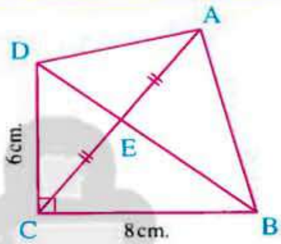
E is the midpoint of $\overline{AC}$,

the area of Δ DEC = 5 cm^2

Calculate : The area of Δ ABC

« 20 cm^2 »

19 **In the opposite figure :**

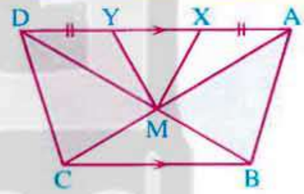


ABCD is a quadrilateral in which m ($\angle$ C) = 90° , BC = 8 cm.

, DC = 6 cm. , E is the midpoint of $\overline{AC}$

Prove that : The area of the figure ABCD = 48 cm^2

20 **In the opposite figure :**



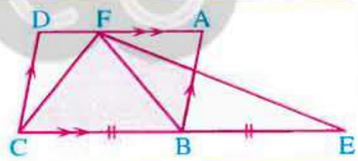
ABCD is a quadrilateral whose diagonals intersect at M ,

$\overline{AD}$ // $\overline{BC}$, $X \in \overline{AD}$ and $Y \in \overline{AD}$

such that : AX = DY

Prove that : The area of the figure ABMX = the area of the figure DCMY

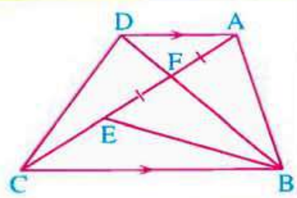
21 **In the opposite figure :**



ABCD is a parallelogram. $E \in \overrightarrow{CB}$, where BC = BE

Prove that : The area of Δ FEC = the area of ▱ ABCD

22 **In the opposite figure :**

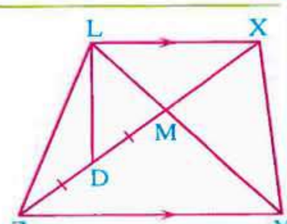


$\overline{AD}$ // $\overline{BC}$, $F \in \overline{AC}$ and $E \in \overline{AC}$

such that : AF = FE

Prove that : The area of Δ BFE = the area of Δ DFC

23 **In the opposite figure :**

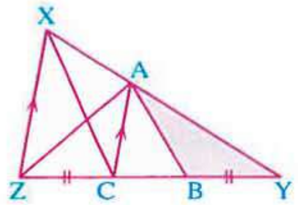


$\overline{XL}$ // $\overline{YZ}$ and $\overline{XZ} \cap \overline{YL} = \{M\}$,

D is the midpoint of $\overline{MZ}$

Prove that : The area of Δ LMD = $\frac{1}{2}$ the area of Δ XMY

24 **In the opposite figure :**

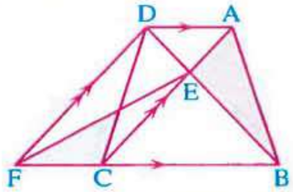


XYZ is a triangle and YB = CZ

and $\overline{XZ} \parallel \overline{AC}$

Prove that : The area of Δ AYB = the area of Δ AXC

25 **In the opposite figure :**

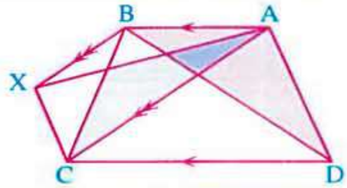


$\overline{AD} \parallel \overline{BC}$, $\overline{DF} \parallel \overline{EC}$ and $\overline{AC} \cap \overline{BD} = \{E\}$,

$\overrightarrow{DF} \cap \overrightarrow{BC} = \{F\}$

Prove that : The area of Δ ABE = the area of Δ ECF

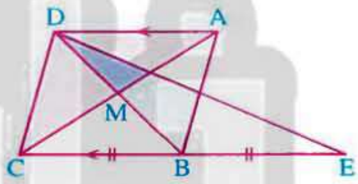
26 **In the opposite figure :**



ABCD is a trapezium in which $\overline{AB} \parallel \overline{DC}$ and $\overline{BX} \parallel \overline{AC}$

Prove that : The area of Δ ABD = the area of Δ ACX

27 **In the opposite figure :**

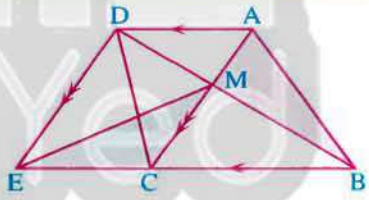


ABCD is a parallelogram. Its diagonals intersect at M

in which $\overline{AD} \parallel \overline{BC}$ and B is the midpoint of $\overline{EC}$

Prove that : The area of Δ EBD = the area of Δ ACD

28 **In the opposite figure :**



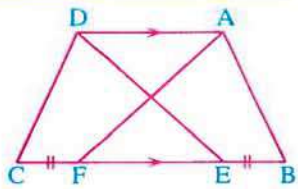
$\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$, E ∈ $\overrightarrow{BC}$ and $\overleftrightarrow{AC} \parallel \overleftrightarrow{DE}$,

$\overline{AC} \cap \overline{BD} = \{M\}$

Prove that :

1. The area of Δ ABM = the area of Δ DCM = the area of Δ EMC
2. The area of Δ DBC = the area of Δ EBM

29 **In the opposite figure :**

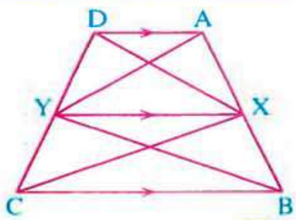


$\overline{AD} \parallel \overline{BC}$ and BE = CF

Prove that :

The area of the figure ABED = the area of the figure AFCD

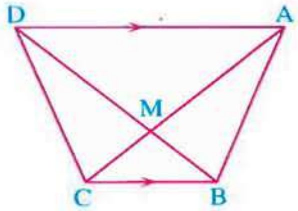
30 **In the opposite figure :**



$\overline{AD} \parallel \overline{XY} \parallel \overline{BC}$

Prove that : The area of Δ YAB = the area of Δ XDC

31 **In the opposite figure :**



$\overline{AD}$ // $\overline{BC}$ **Prove that :** The area of Δ ABM = the area of Δ DMC

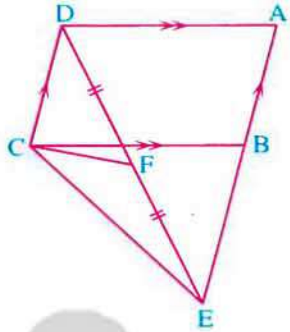
and if the area of Δ MBC = 20 cm^2 ,

the area of Δ ABM = 3 times the area of Δ MBC

Calculate : The area of the rectangle drawn on $\overline{BC}$ such that its other base is on $\overleftrightarrow{AD}$

« 160 cm.^{2} »

32 **In the opposite figure :**



ABCD is a parallelogram.

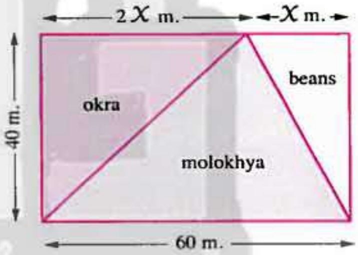
E ∈ $\overleftrightarrow{AB}$, $\overline{CF}$ is a median of Δ CDE

Find : The ratio between the area of Δ CFD and the area of ▱ ABCD

« 1 : 4 »

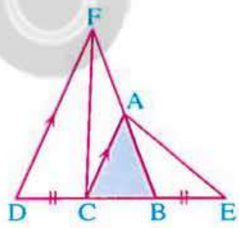
Life Application

33 A rectangular piece of land of dimensions 60 m. and 40 m. wanted to be planted using three kinds of vegetables as in the opposite figure. Find the planted area with each kind.



For excellent pupils

34 **In the opposite figure :**

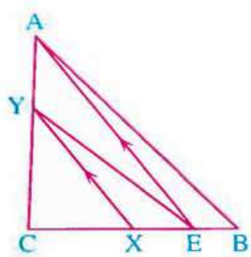


ABC is a triangle. D ∈ $\overleftrightarrow{BC}$ and E ∈ $\overleftrightarrow{BC}$ such that BE = CD ,

$\overrightarrow{DF}$ // $\overline{CA}$ and intersects $\overrightarrow{BA}$ at F

Prove that : The area of Δ FBC = the area of Δ ACE

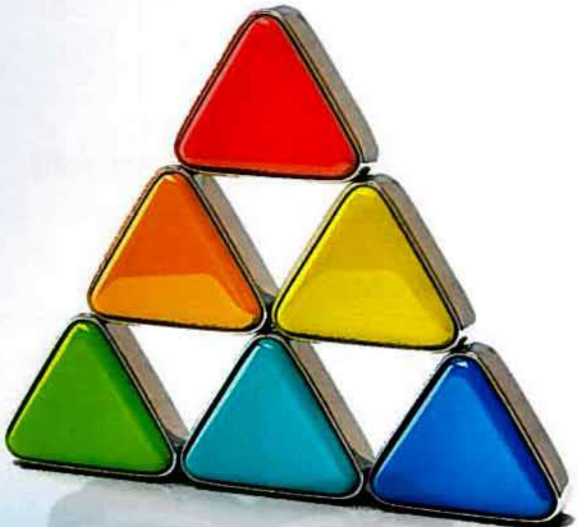
35 **In the opposite figure :**



ABC is a triangle in which X is the midpoint of $\overline{BC}$ and E ∈ $\overline{BX}$

Draw $\overrightarrow{XY}$ // $\overline{EA}$ to cut $\overline{AC}$ at Y

Prove that : The area of Δ EYC = $\frac{1}{2}$ the area of Δ ABC



Exercise 4

Follow : Equality of the areas of two triangles

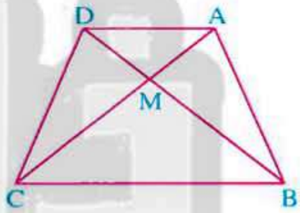
From the school book

1 **In the opposite figure :**

ABCD is a quadrilateral , its diagonals intersect at M

and the area of Δ ABM = the area of Δ DCM

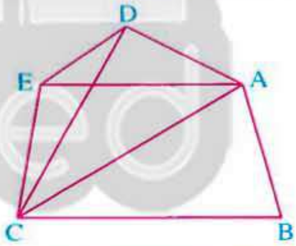
Prove that : $\overline{AD} \parallel \overline{BC}$



2 **In the opposite figure :**

The area of the figure ABCD = the area of the figure ABCE

Prove that : $\overline{DE} \parallel \overline{AC}$

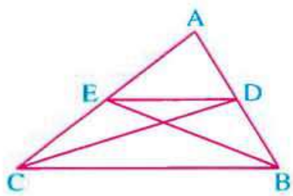


3 **In the opposite figure :**

ABC is a triangle in which D $\in \overline{AB}$ and E $\in \overline{AC}$

such that the area of Δ ABE = the area of Δ ACD

Prove that : $\overline{DE} \parallel \overline{BC}$

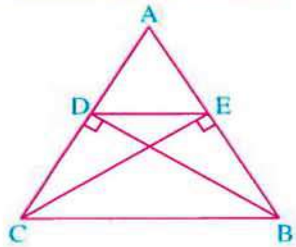


4 **In the opposite figure :**

AB = AC , $\overline{BD} \perp \overline{AC}$ and $\overline{CE} \perp \overline{AB}$

Prove that :

1. $\overline{ED} \parallel \overline{BC}$
2. The area of Δ ADB = the area of Δ AEC



Unit 4

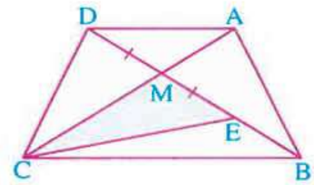
5 **In the opposite figure :**

ABCD is a quadrilateral whose diagonals are intersecting at M

and $E \in \overline{BM}$, where ME = MD

The area of Δ AMB = the area of Δ CME

Prove that : $\overline{AD} \parallel \overline{BC}$



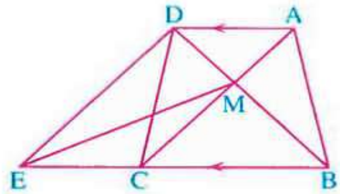
6 **In the opposite figure :**

ABCD is a quadrilateral in which $\overline{AD} \parallel \overline{BC}$

, $E \in \overrightarrow{BC}$ and $\overline{AC} \cap \overline{BD} = \{M\}$

The area of Δ ABM = the area of Δ ECM

Prove that : $\overline{DE} \parallel \overleftrightarrow{AC}$

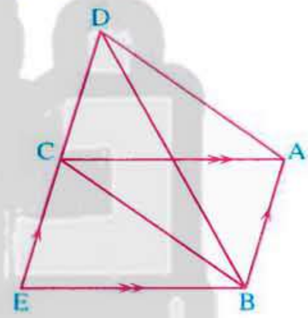


7 **In the opposite figure :**

ABEC is a parallelogram.

$D \in \overrightarrow{EC}$ such that the area of Δ DBC = the area of Δ EBC

Prove that : $\overline{AD} \parallel \overline{BC}$

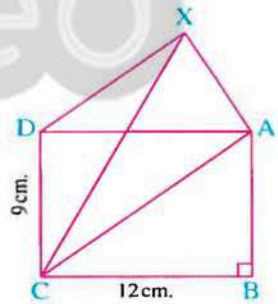


8 **In the opposite figure :**

ABCD is a rectangle. BC = 12 cm., CD = 9 cm.

and the area of Δ XAC = 54 cm^2.

Prove that : $\overline{XD} \parallel \overline{AC}$

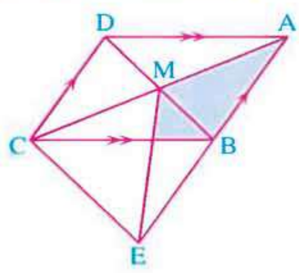


9 **In the opposite figure :**

ABCD is a parallelogram , $\overline{AC} \cap \overline{BD} = \{M\}$

and $E \in \overrightarrow{AB}$, where the area of Δ AME = the area of Δ ABC

Prove that : The figure BECD is a parallelogram.



10 **In the opposite figure :**

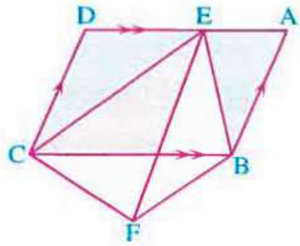
ABCD is a parallelogram. $E \in \overline{AD}$

and F is a point outside the parallelogram.

Draw $\overline{FC}$, $\overline{FE}$ and $\overline{FB}$ such that :

The area of Δ FCE = the area of Δ EAB + the area of Δ ECD

Prove that : $\overline{BF} \parallel \overline{EC}$

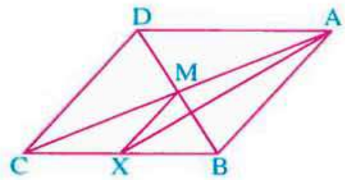


11 **In the opposite figure :**

ABCD is a parallelogram.

The area of Δ ABX = the area of Δ DMC

Prove that : $\overline{MX} \parallel \overline{AB}$



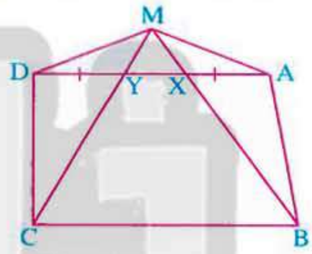
12 **In the opposite figure :**

ABCD is a quadrilateral.

$X \in \overline{AD}$ and $Y \in \overline{AD}$ such that AX = YD

The area of Δ ABM = the area of Δ DCM

Prove that : $\overline{AD} \parallel \overline{BC}$

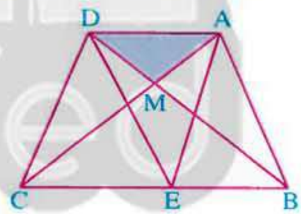


13 **In the opposite figure :**

ABCD is a quadrilateral.

The area of Δ ABD = the area of Δ ACD

Prove that : The area of Δ AED = the area of Δ ACD



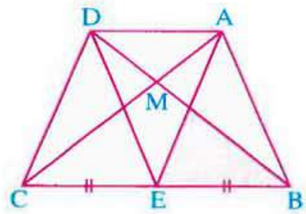
14 **In the opposite figure :**

E is the midpoint of $\overline{BC}$, $\overline{AC} \cap \overline{BD} = \{M\}$

The area of Δ ABE = the area of Δ DEC

Prove that :

The area of Δ AMB = the area of Δ DMC

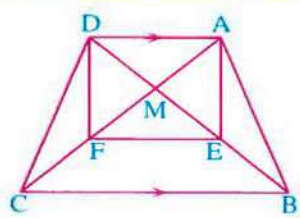


15 **In the opposite figure :**

If $\overline{AD} \parallel \overline{BC}$

and the area of Δ ABE = the area of Δ DFC

Prove that : $\overline{EF} \parallel \overline{BC}$



16 In the opposite figure :

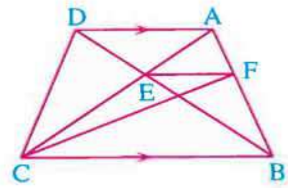
ABCD is a quadrilateral in which $\overline{AD} \parallel \overline{BC}$ and its diagonals intersect at E

Prove that : The area of Δ AEB = the area of Δ DEC

and if $F \in \overline{AB}$ such that :

The area of Δ AFC = the area of Δ DEC

Prove that : $\overline{FE} \parallel \overline{AD}$



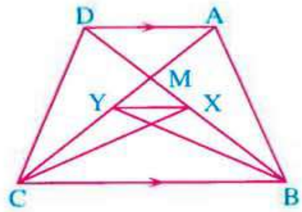
17 In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, $\overline{AC} \cap \overline{BD} = \{M\}$,

$\overline{CX}$ is a median in Δ CBD

and $\overline{BY}$ is a median in Δ BAC

Prove that : $\overline{XY} \parallel \overline{BC}$



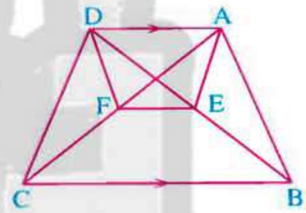
18 In the opposite figure :

ABCD is a quadrilateral in which

$\overline{AD} \parallel \overline{BC}$, E is the midpoint of $\overline{BD}$

and F is the midpoint of $\overline{AC}$

Prove that : $\overline{EF} \parallel \overline{BC}$



19 In the opposite figure :

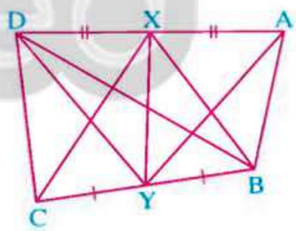
ABCD is a quadrilateral.

X is the midpoint of $\overline{AD}$,

Y is the midpoint of $\overline{BC}$

The area of Δ DBC = the area of Δ YAD

Prove that : $\overline{XB} \parallel \overline{DY}$



20 In the opposite figure :

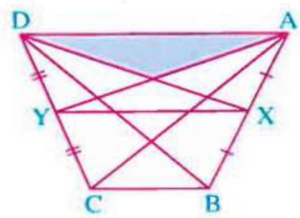
ABCD is a quadrilateral.

$\overline{DX}$ is a median in Δ DAB ,

$\overline{AY}$ is a median in Δ ACD

The area of Δ XAD = the area of Δ YAD

Prove that : $\overline{AD} \parallel \overline{BC} \parallel \overline{XY}$



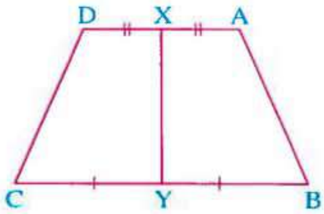
21 **In the opposite figure :**

ABCD is a quadrilateral. X is the midpoint of $\overline{AD}$

and Y is the midpoint of $\overline{BC}$ such that :

The area of the figure ABYX = the area of the figure DCYX

Prove that : $\overline{AD} \parallel \overline{BC}$



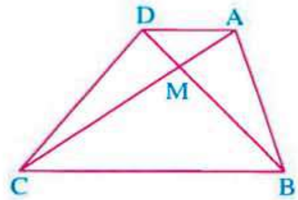
22 **In the opposite figure :**

ABCD is a quadrilateral.

M is the point of intersection of its diagonals.

If AM = $\frac{1}{2}$ MC and DM = $\frac{1}{2}$ MB

Prove that : $\overline{AD} \parallel \overline{BC}$



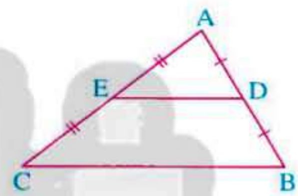
23 **In the opposite figure :**

D is the midpoint of $\overline{AB}$,

E is the midpoint of $\overline{AC}$

Using the theorems of equality of areas

Prove that : $\overline{DE} \parallel \overline{BC}$

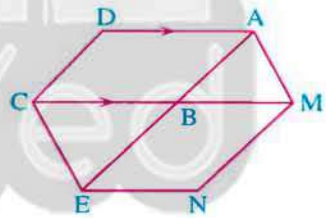


For excellent pupils

24 **In the opposite figure :**

ABCD and BMNE are two parallelograms having the same area where E ∈ $\overrightarrow{AB}$ and M ∈ $\overrightarrow{CB}$

Prove that : $\overline{AM} \parallel \overline{CE}$



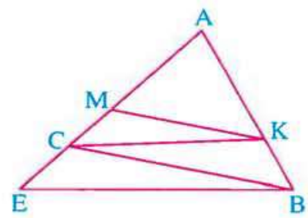
25 **In the opposite figure :**

ABC is a triangle.

K ∈ $\overline{AB}$, E ∈ $\overrightarrow{AC}$ and M is the midpoint of $\overline{AE}$,

the area of Δ ABC = twice the area of Δ AKM

Prove that : $\overline{KC} \parallel \overline{BE}$

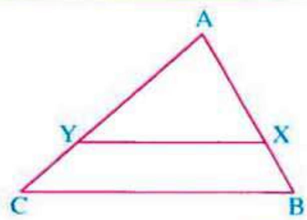


26 **In the opposite figure :**

ABC is a triangle and X ∈ $\overline{AB}$ such that : AX = 2 XB ,

Y ∈ $\overline{AC}$ such that : AY = 2 YC

Prove that : $\overline{XY} \parallel \overline{BC}$



Exercise 5

Areas of some geometric figures

From the school book

1 Complete the following :

1. The area of the rhombus = the side length × .... = $\frac{1}{2}$ of the product of ....
2. The area of the square = the square of the length of .... = $\frac{1}{2}$ ....
3. The length of the middle base of the trapezium equals ....
4. The area of the trapezium = half of the sum of lengths of the two parallel bases × ....
   = the length of .... × its height
5. The base angles of the isosceles trapezium are ....
6. 📖 The diagonals of an isosceles trapezium are ....

2 Find the area of the following figures :

1. A rhombus of side length 6 cm. and its height = 5 cm. « 30 cm^2 »
2. 📖 A rhombus whose side length 12 cm. and its height = 8 cm. « 96 cm^2 »
3. 📖 A rhombus whose diagonals lengths are 8 cm. and 10 cm. « 40 cm^2 »
4. A rhombus whose diagonal lengths are 24 cm. and 10 cm. « 120 cm^2 »
5. A square whose diagonal length = 10 cm. « 50 cm^2 »
6. 📖 A square whose diagonal length = 8 cm. « 32 cm^2 »
7. A trapezium whose bases lengths are 6 cm. and 8 cm. and its height = 12 cm. « 84 cm^2 »
8. A trapezium whose bases lengths are 8 cm. and 10 cm. and its height = 5 cm. « 45 cm^2 »
9. A trapezium whose middle base length is 7 cm. and its height = 6 cm. « 42 cm^2 »
10. A trapezium whose middle base length is 12 cm. and its height = 8 cm. « 96 cm^2 »

3 Choose the correct answer from those given :

1. The area of rhombus is 20 cm^2, the length of one of its diagonals is 5 cm., then the length of the other diagonal = ...

(a) 8 cm. (b) 4 cm. (c) 10 cm. (d) 15 cm.

2. If the area of a square is 50 cm^2, then the length of its diagonal = ...

(a) 25 cm. (b) 5 cm. (c) 10 cm. (d) 20 cm.

3. The area of the square whose side length is 6 cm. .... the area of the square whose diagonal length is 8 cm.

(a) > (b) < (c) = (d) ≡

4. If the perimeter of a rhombus is 24 cm. and its area = 30 cm.^{2}, then its height = ...

(a) 4 cm. (b) 5 cm. (c) 6 cm. (d) 12 cm.

5. If the product of the lengths of the diagonals of a rhombus = 96 cm^2 and its height is 6 cm., then its side length = ...

(a) 12 cm. (b) 8 cm. (c) 6 cm. (d) 4 cm.

6. The trapezium in which the lengths of its two parallel bases are 15 cm. and 11 cm. Its middle base is with length ....

(a) 26 cm. (b) 15 cm. (c) 13 cm. (d) 11 cm.

7. If the area of a trapezium is 32 cm^2 and its height is 4 cm., then the length of its middle base = ...

(a) 4 cm. (b) 8 cm. (c) 14 cm. (d) 16 cm.

8. If the area of the trapezium is 450 cm^2, and the lengths of its two parallel bases are 24 cm. and 12 cm. , then its height = ...

(a) 12.5 cm. (b) 25 cm. (c) 36 cm. (d) 52 cm.

9. The trapezium in which the length of one of its parallel bases is 15 cm., and its area is 108 cm^2 and its height is 8 cm., then the length of the other base is ....

(a) 15 cm. (b) 4 cm. (c) 12 cm. (d) 27 cm.

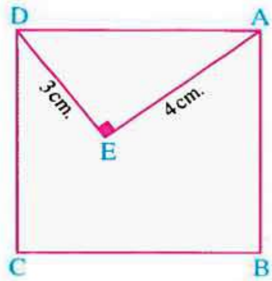
10. The trapezium whose middle base length is x cm. and its height = $\frac{1}{2}$ the length of the middle base, its area = ... cm^2

(a) x^2 (b) $\frac{x^2}{2}$ (c) $\frac{x^2}{4}$ (d) $\frac{x^2}{8}$

4 In the opposite figure :

ABCD is a square , E is a point inside it , where Δ AED is a right-angled triangle at E , AE = 4 cm. , ED = 3 cm.

Find the area of the shaded part.



« 19 cm^2 »

5 A square whose area equals the area of the rectangle whose dimensions are 2 cm. and 9 cm. Find the length of its diagonal. « 6 cm. »

6 Two land pieces are equal in area , the first is in the shape of a square and the second is in the shape of a rhombus whose diagonals lengths are 8 metres and 16 metres. Find the perimeter of the square-shaped piece. « 32 cm. »

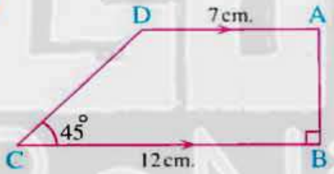
7 Two pieces of land have equal areas , one of them has the shape of a rhombus whose diagonals are 18 m. and 24 m. , and the other one has the shape of a trapezium whose height is 12 m. Find the length of its middle base. « 18 m. »

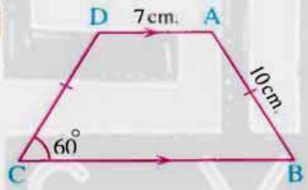
8 A rhombus whose diagonals are of lengths 12 cm. and 16 cm. Find its height. « 9.6 cm. »

9 Find the area of the rhombus whose perimeter is 52 cm. and the length of one of its diagonals is 10 cm. « 120 cm^2. »

10 The perimeter of a rhombus is 64 cm. and the measure of one of its angles is 60°
Find its area. « $128\sqrt{3}$ cm^2. »

11 **Find the area of each of the following figures by using the given data :**

1


2


« 47.5 cm^2. , $60\sqrt{3}$ cm^2. »

12 If the ratio between the two lengths of the diagonals of a rhombus is 3 : 4 and the length of the smallest diagonal is 9 cm. Find the area of the rhombus. « 54 cm^2. »

13 A rhombus , the ratio between the lengths of the two diagonals is 5 : 8 if its area = 2000 cm^2. Find the length of each of its diagonals. « 50 cm. , 80 cm. »

14 The length of the middle base of a trapezium is 30 cm. and the ratio between the lengths of its two parallel bases is 2 : 3 Find the length of each of them.
and if its height = 24 cm. , find its area. « 24 cm. , 36 cm. , 720 cm^2. »

15 The area of a trapezium is 180 cm^2. and its height is 12 cm. Find the lengths of its parallel bases if the ratio between their lengths is 3 : 2 « 18 cm. , 12 cm. »

16 A piece of land has the shape of a trapezium whose area is 4000 m^2.
The lengths of the two parallel bases and its height are of ratio 3 : 2 : 4, respectively.
Find the length of its middle base. « 50 cm. »

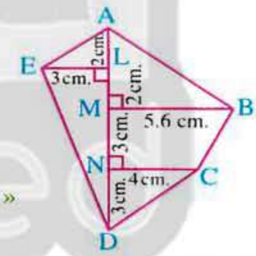
17 Two pieces of land , the first is in the shape of a trapezium in which the lengths of its two parallel bases are 76 metres and 64 metres and the perpendicular distance between them is 45 metres and the second is in the shape of a rhombus whose diagonals lengths are 74 metres and 90 metres. The two pieces are exchanged by a rectangular piece of land whose area equals to the sum of areas of the trapezium and rhombus pieces. The ratio between its length and its width is 5 : 4 Find its dimensions. « 90 m. , 72 m. »

18 ABCD is a trapezium in which $\overline{AD} // \overline{BC}$, X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{DC}$, if XY = 7 cm., BC = 10 cm. and the area of the trapezium = 35 cm^2.
Find the length of $\overline{AD}$ and the perpendicular distance between $\overline{AD}$ and $\overline{BC}$ « 4 cm. , 5 cm. »

19 ABCD is a trapezium in which $\overline{AD} // \overline{BC}$, AD = 27 cm. and BC = 45 cm.
If the area of Δ ABC = 225 cm^2. , find the area of the trapezium. « 360 cm^2. »

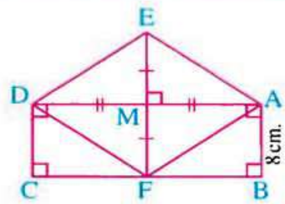
20 ABCD is a trapezium in which $\overline{AD} // \overline{BC}$, m ($\angle$ A) = 90°, BC = 4 cm., AD = 24 cm., BD = 30 cm. and $\overline{AF} \perp \overline{BD}$ to cut it at F , where AF = 14.4 cm.
Find the area of the trapezium ABCD « 252 cm^2. »

21 **In the opposite figure :**



Each of $\overline{BM}$, $\overline{CN}$ and $\overline{EL}$ is perpendicular to $\overline{AD}$
Using the shown lengths on the figure.
Find the area of the figure ABCDE « 46.6 cm^2. »

22 **In the opposite figure :**



ABCD is a rectangle of area 144 cm^2.
If AB = 8 cm., $\overline{AD} \perp \overline{EF}$
and M is the midpoint of each of $\overline{AD}$ and $\overline{EF}$
Find the area of the figure AFDE « 144 cm^2. »

23 ABCD is a rectangle with AB = 6 cm., BC = 8 cm., X , Y , L and M are the midpoints of the sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively.

1. **Prove that :** The figure XYLM is a rhombus and find its area.
2. Find the height of the rhombus XYLM « 24 cm^2. , 4.8 cm. »

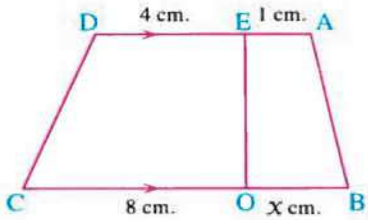
24 **In the opposite figure :**

ABCD is a trapezium , $E \in \overline{AD}$, $O \in \overline{BC}$

, where the area of the figure EOCD

= three times of the area of the figure ABOE

Find the value of : x



« 3 »

For excellent pupils

25 The area of an isosceles trapezium is 120 cm.^{2} , its perimeter is 60 cm. and the length of its middle base is 20 cm. Find the lengths of its bases. « 12 cm. , 28 cm. »

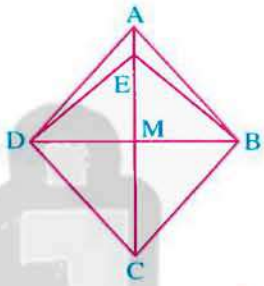
26 **In the opposite figure :**

ABCD is a rhombus , its diagonals intersect at M ,

AC + BD = 33 cm. , BD : AC = 5 : 6

and $E \in \overline{AM}$ such that ME = $\frac{2}{3}$ MA

Find the area of the shaded part.



« 90 cm.^{2} »

Summary of Unit 4

Theorem 1

Surfaces of two parallelograms with common base and between two parallel straight lines, one is carrying this base, are equal in area.

Corollary 1

The parallelogram and the rectangle with common base and between two parallel straight lines are equal in area.

Corollary 2

The area of the parallelogram = the length of the base × its corresponding height.

Corollary 3

The parallelograms with bases equal in length and lying on a straight line, while the opposite sides to these bases are on another straight line, are equal in area.

Corollary 4

Area of a triangle is equal to half of area of a parallelogram if they have a common base lying on one of two parallel straight lines including them.

Corollary 5

Area of the triangle = $\frac{1}{2}$ of the length of the base × its corresponding height

Theorem 2

Two triangles which have the same base and the vertices opposite to this base on a straight line parallel to the base have the same area.

Corollary 1

Triangles of bases equal in length and lying between two parallel straight lines are equal in area.

Corollary 2

The median of a triangle divides its surface into two triangular surfaces equal in area.

Corollary 3

Triangles with congruent bases on one straight line and have a common vertex are equal in area.

Theorem 3

If two triangles are equal in area and drawn on the same base and on one side of it, then their vertices lie on a straight line parallel to this base.

- If two triangles have the same area and they are included between two straight lines and their bases on these two straight lines are equal in length, then the two straight lines are parallel.

The area of the rhombus = its side length × its height.

$= \frac{1}{2}$ of the product of the lengths of its two diagonals.

- The area of the square = the square of its side length.

$= \frac{1}{2}$ of the square of the length of its diagonal.

- The two base angles of the isosceles trapezium are equal in measure.
- The two diagonals of the isosceles trapezium are equal in length.
- The isosceles trapezium has only one axis of symmetry which is the perpendicular bisector of its bases.
- The middle base of the trapezium is parallel to each of its two parallel bases and its length equals half of the sum of lengths of the two parallel bases.
- The area of the trapezium = half of the sum of lengths of the two parallel bases × height

= the length of the middle base × height

Exams on Unit Four

Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1. The length of the base of a triangle whose area 30 cm^2. and height 6 cm. is ....
   (a) 5 cm. (b) 10 cm. (c) 15 cm. (d) 20 cm.
2. The area of a parallelogram in which the lengths of two adjacent sides are 7 cm. , 5 cm. and the smaller height 4 cm. equals ....
   (a) 35 cm^2. (b) 25 cm^2. (c) 28 cm^2. (d) 49 cm^2.
3. The area of the rhombus whose diagonal lengths are 6 cm. , 8 cm. equals ....
   (a) 2 cm^2. (b) 14 cm^2. (c) 24 cm^2. (d) 48 cm^2.
4. The median of the triangle divides its surface into two triangles ....
   (a) congruent. (b) equal in area.
   (c) isosceles. (d) right-angled.
5. A rhombus of area 60 cm^2. and the length of one of its diagonals 10 cm. , then the length of the other diagonal equals ....
   (a) 4 cm. (b) 8 cm. (c) 10 cm. (d) 12 cm.
6. The ratio between area of parallelogram and area of triangle if they have a common base and included between two parallel lines equals ....
   (a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 2 : 3

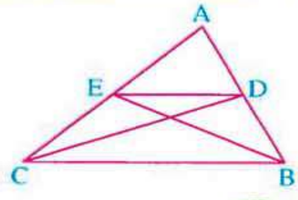
2 Complete :

1. The length of the middle base of a trapezium whose area = 30 cm^2. and height is 5 cm. equals ....
2. Surfaces of two parallelograms with common base and between two parallel straight lines ....
3. The length of the diagonal of a square of area 50 cm^2. equals .... cm.
4. If two triangles have equal areas and drawn on the same base and on one side of it , then ....
5. The measure of base angles of an isosceles trapezium are ....

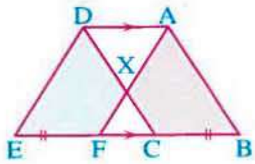
3 [a] In the opposite figure :

If the area of Δ ADC = the area of Δ AEB

Prove that : $\overline{DE} \parallel \overline{BC}$



[b] In the opposite figure :



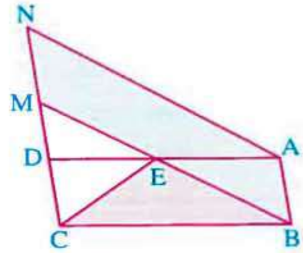
ABCD and AFED are two parallelograms

, $C \in \overline{BE}$, $F \in \overline{BE}$

, $\overline{AF} \cap \overline{DC} = \{X\}$

Prove that : The area of figure ABCX = the area of figure DEFX

4 **[a] In the opposite figure :**



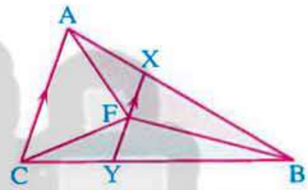
ABCD and ABMN are two parallelograms

and $M \in \overrightarrow{CD}$

Prove that :

The area of Δ EBC = $\frac{1}{2}$ the area of ▱ABMN

[b] In the opposite figure :

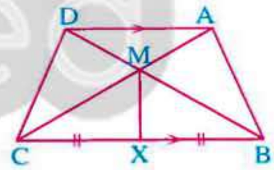


$\overline{AC}$ // $\overline{XY}$ and F is the midpoint of $\overline{XY}$

Prove that : The area of Δ ABF = the area of Δ CBF

5 **[a]** The area of a trapezium is 88 cm^2, its height is 8 cm. and the length of one of the two parallel bases 10 cm. Find the length of the other base.

[b] In the opposite figure :



$\overline{AD}$ // $\overline{BC}$, X is a midpoint of $\overline{BC}$

Prove that :

1 The area of Δ AMB = the area of Δ DMC

2 The area of shape ABXM = the area of shape DCXM

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The area of a parallelogram is 24 cm^2 and its base length is 6 cm. , then the corresponding height equals ....

(a) 144 cm. (b) 8 cm. (c) 4 cm. (d) 12 cm.

2 ABCD is a parallelogram, X ∈ $\overline{AD}$
, then the area of Δ X BC = ... the area of ▱ ABCD

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 2 (d) $\frac{1}{3}$

3 If the area of a trapezium is 40 cm^2 and the lengths of its two parallel bases are 7 cm. , 9 cm. , then its height is ....

(a) 10 cm. (b) 7 cm. (c) 8 cm. (d) 5 cm.

4 The area of the right-angled triangle whose right sides are 8 cm. , 9 cm. is .... cm^2

(a) 36 (b) 72 (c) 18 (d) 144

5 If ABCD is a parallelogram, AB = 3 cm. , BC = 8 cm. , the smaller height = 6 cm.
, then the greater height = ...

(a) 8 cm. (b) 16 cm. (c) 18 cm. (d) 2.25 cm.

6 A square of diagonal length 12 cm. , then its area = ...

(a) 24 cm^2 (b) 36 cm^2 (c) 144 cm^2 (d) 72 cm^2

2 Complete the following :

1 The area of the triangle equals .... the area of the parallelogram if they have a common base lying on one of two parallel straight lines including them.

2 The median of a triangle divides its surface into ....

3 The two diagonals of the isosceles trapezium are ....

4 A rhombus of perimeter 20 cm. and area 20 cm^2 , then its height equals ....

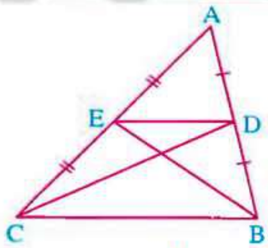
5 Triangles of bases equal in length and lying between two parallel straight lines are ....

3 **[a] In the opposite figure :**

D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

Prove that : The area of Δ BDE = the area of Δ CDE

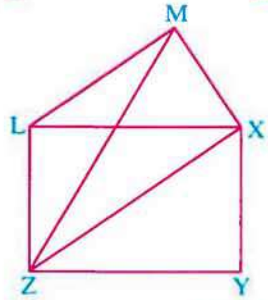


[b] In the opposite figure :

The area of the figure XYZM

= the area of the figure XYZL

Prove that : $\overline{XZ}$ // $\overline{LM}$

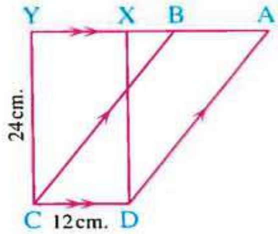


Unit 4

4 [a] **In the opposite figure :**

$\overleftrightarrow{AB} // \overleftrightarrow{CD}$, X ∈ $\overleftrightarrow{AB}$ and Y ∈ $\overleftrightarrow{AB}$

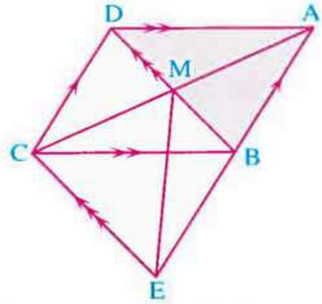
, X D C Y is a rectangle



1 Find the area of the figure ABCD

2 If AD = 30 cm. Find the length of the perpendicular from B to $\overline{AD}$

[b] **In the opposite figure :**



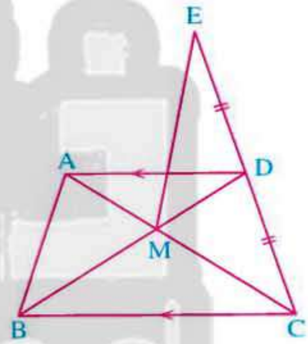
ABCD and BECD are two parallelograms,

where $\overline{AC} \cap \overline{BD} = \{M\}$

Prove that : The area of Δ ABD = the area of Δ MEC

5 [a] A rhombus of perimeter 40 cm. , the length of one of its diagonals is 16 cm. Find its area.

[b] **In the opposite figure :**



$\overline{AD} // \overline{BC}$

, $\overline{AC} \cap \overline{BD} = \{M\}$

, D is the midpoint of $\overline{CE}$

Prove that : The area of Δ MDE = the area of Δ AMB

UNIT 5

Similarity, converse of Pythagoras' theorem and Euclidean theorem.

Exercises of the unit :

6. Similarity.
7. Converse of Pythagoras' theorem.
8. Projections.
9. Euclidean theorem.
10. Classifying triangles according to their angles.

- ✪ Summary of unit five.
- ✪ Unit exams.

Exercise 6

Similarity

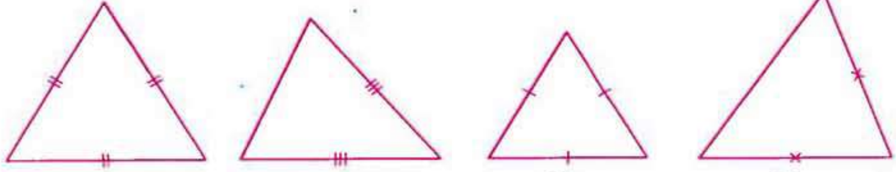
From the school book

1 Complete each of the following statements :

1. If two polygons are similar , then the corresponding .... are equal in measure.
2. If two polygons are similar , then the corresponding .... are proportional.
3. If each of two polygons is similar to a third , then they are ....
4. The two triangles are similar if the corresponding .... are proportional.
5. If the measures of the corresponding angles in the two triangles are equal , then the two triangles are ....
6. If we have two polygons , their corresponding angles are .... and their corresponding sides lengths are .... , then the two polygons are similar.
7. If the ratio between the lengths of two corresponding sides in two similar triangles is equal to 1 , then the two triangles are ....
8. If two polygons are similar and the ratio between the lengths of two corresponding sides is 3 : 4 , then the ratio between their perimeters is ....
9. In the right-angled triangle the perpendicular drawn from the vertex of the right angle to the hypotenuse divides the triangle into two .... triangles.

2 Choose the correct answer from those given ones :

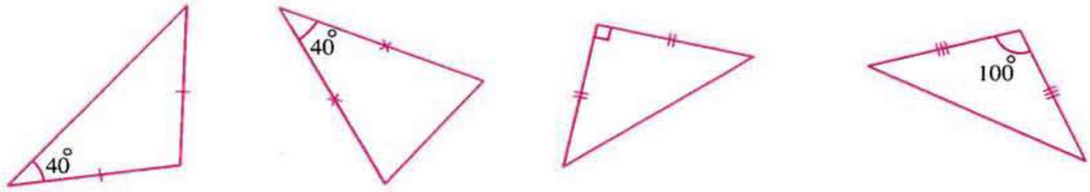
1. In the following figures , there are two similar triangles , they are ....



(1) (2) (3) (4)

(a) 1 , 2 (b) 1 , 3 (c) 1 , 4 (d) 2 , 4

2 In the following figures , there are two similar triangles , they are ....

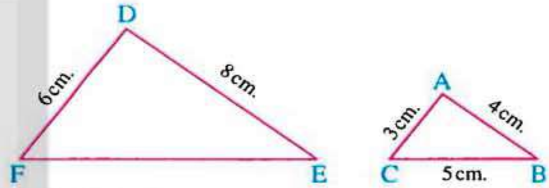


(1) (2) (3) (4)

(a) 1 , 2 (b) 1 , 3 (c) 2 , 4 (d) 1 , 4

3 In the opposite figure :

If Δ ABC ~ Δ DEF , then EF = ...

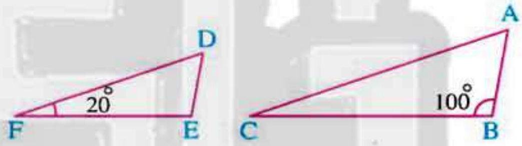


(a) 5 cm. (b) 6 cm.

(c) 8 cm. (d) 10 cm.

4 In the opposite figure :

If Δ ABC ~ Δ DEF , then m (∠ A) = ...



(a) 20° (b) 60°

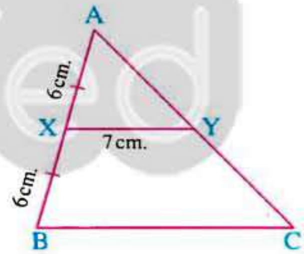
(c) 80° (d) 100°

5 In the opposite figure :

If Δ ABC ~ Δ AXY ,

AX = XB = 6 cm.

XY = 7 cm. , then BC = ...



(a) 6 cm. (b) 7 cm.

(c) 12 cm. (d) 14 cm.

6 If the ratio between the lengths of two corresponding sides of two squares is 1 and the perimeter of one of them is 20 cm. , then the area of the other square = ...

(a) 20 cm^2. (b) 25 cm^2. (c) 16 cm^2. (d) 25 cm.

7 If Δ ABC ~ Δ DEF and AB = $\frac{1}{5}$ DE , then perimeter of Δ ABC = ... perimeter of Δ DEF

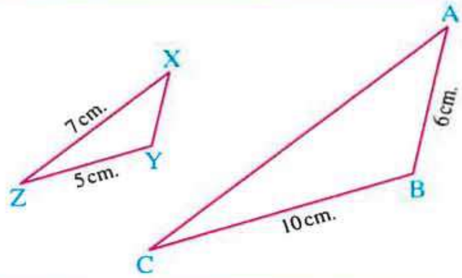
(a) 5 (b) 1 (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

Unit 5

3 **In the opposite figure :**

Δ ABC ~ Δ XYZ

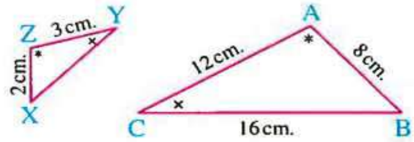
Find : AC and XY « 14 cm. , 3 cm. »



4 **Using the shown data in the figure , then prove that :**

Δ XYZ and Δ BCA are similar ,

then find the perimeter of Δ XYZ



« 9 cm. »

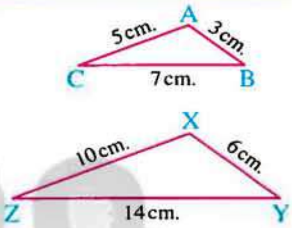
5 **In the opposite figure :**

1 **Prove that :**

Δ ABC and Δ XYZ are similar.

2 If m ($\angle$ B) + m ($\angle$ C) = 60°

Find : m ($\angle$ X) « 120° »

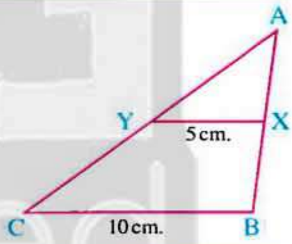


6 **In the opposite figure :**

If Δ AXY ~ Δ ABC

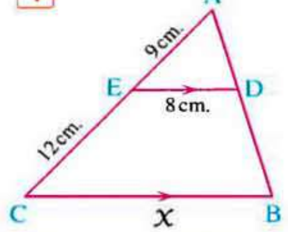
, XY = 5 cm. and BC = 10 cm.

Prove that : **1** $\overline{XY}$ // $\overline{BC}$ **2** Y is the midpoint of $\overline{AC}$

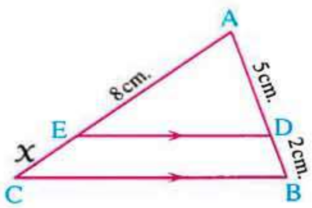


7 **In each of the following , find the numerical value of x (Given that lengths are in cm.) :**

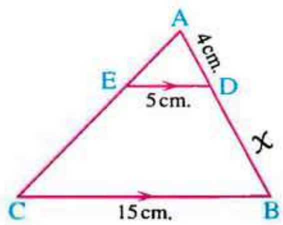
1



2



3



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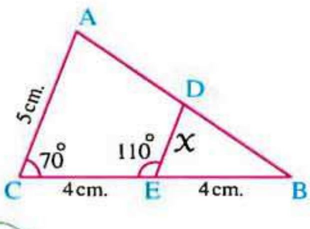
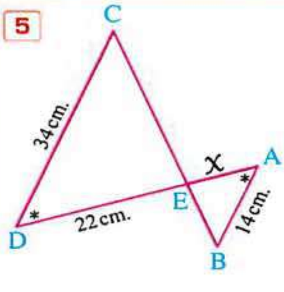
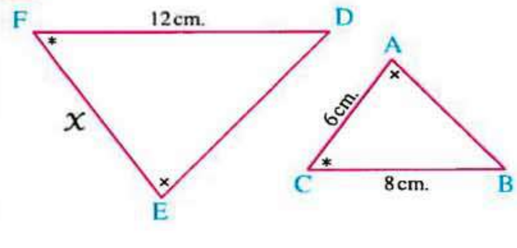


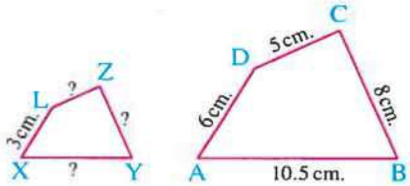
5



6



8 **In the opposite figure :**



The polygon ABCD ~ the polygon XYZL

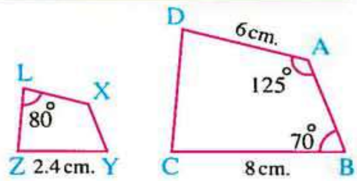
If AB = 10.5 cm. , BC = 8 cm. , CD = 5 cm. ,

DA = 6 cm. and LX = 3 cm.

Find the length of each of : $\overline{XY}$, $\overline{YZ}$ and $\overline{ZL}$

« 5.25 cm. , 4 cm. , 2.5 cm. »

9 **In the opposite figure :**

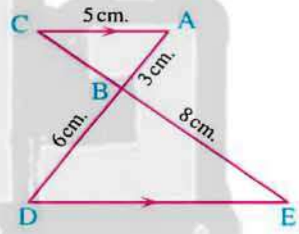


If the figure ABCD ~ the figure XYZL

1. Calculate m (∠ BCD)
2. Calculate the length of $\overline{XL}$ and determine the enlargement ratio.
3. If the perimeter of the figure ABCD = 26 cm. , what is the perimeter of the figure XYZL

« 85° , 1.8 cm. , $\frac{10}{3}$, 7.8 cm. »

10 **In the opposite figure :**



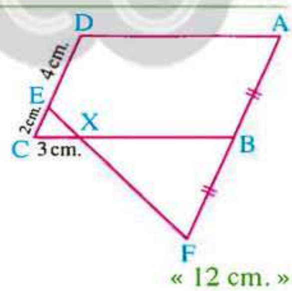
$\overline{AC} \parallel \overline{ED}$, $\overline{AD} \cap \overline{CE} = \{B\}$

, AC = 5 cm. , BE = 8 cm. , AB = 3 cm. and BD = 6 cm.

1. **Prove that :** Δ ABC ~ Δ DBE
2. **Find the length of each of :** $\overline{BC}$ and $\overline{ED}$
3. **Find :** The ratio of enlargement.

« 4 cm. , 10 cm. , 2 »

11 **In the opposite figure :**



ABCD is a parallelogram , B is the midpoint of $\overline{AF}$,

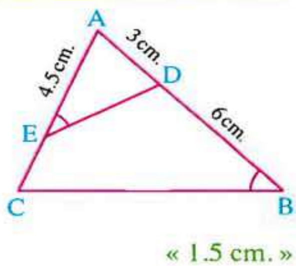
CE = 2 cm. , DE = 4 cm. and XC = 3 cm.

Prove that : Δ ECX ~ Δ FBX

, then find the length of : $\overline{AD}$

« 12 cm. »

12 **In the opposite figure :**



m (∠ AED) = m (∠ B) , AD = 3 cm. ,

AE = 4.5 cm. and BD = 6 cm.

1. **Prove that :** Δ ADE ~ Δ ACB
2. **Find the length of :** $\overline{EC}$

« 1.5 cm. »

Unit 5

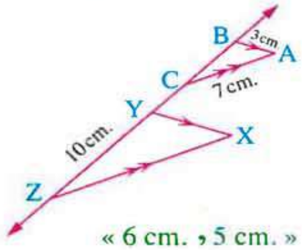
13 **In the opposite figure :**

$\overline{AB} \parallel \overline{XY}$, $\overline{AC} \parallel \overline{XZ}$

, AB = 3 cm. , AC = 7 cm. , YZ = 10 cm.

and XZ = 2 AC

Find : XY , BC



« 6 cm. , 5 cm. »

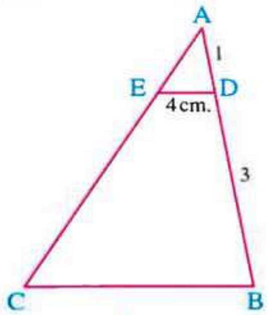
14 **In the opposite figure :**

Δ ADE ~ Δ ABC

, AD : DB = 1 : 3

1 **Find :** The ratio $\frac{AE}{AC}$

2 If DE = 4 cm. , **find the length of :** $\overline{BC}$



« $\frac{1}{4}$, 16 cm. »

15 **In the opposite figure :**

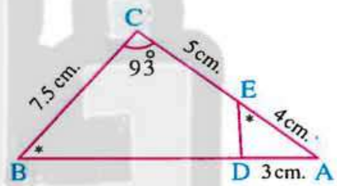
Δ ABC , D $\in \overline{AB}$, E $\in \overline{AC}$

, AE = 4 cm. , EC = 5 cm. , BC = 7.5 cm.

, AD = 3 cm. , m ($\angle$ AED) = m ($\angle$ B) and m ($\angle$ C) = 93°

1 **Prove that :** Δ AED ~ Δ ABC

2 **Find :** The length of each of $\overline{BD}$ and m ($\angle$ ADE)



« 9 cm. , 93° »

16 **In the opposite figure :**

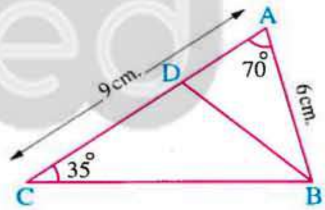
ABC is a triangle in which : m ($\angle$ A) = 70°

, m ($\angle$ C) = 35° , D $\in \overline{AC}$

If Δ ABD ~ Δ ACB

Find : m ($\angle$ DBC) and if : AB = 6 cm. , AC = 9 cm.

Find the length of : $\overline{CD}$



« 40° , 5 cm. »

17 **In the opposite figure :**

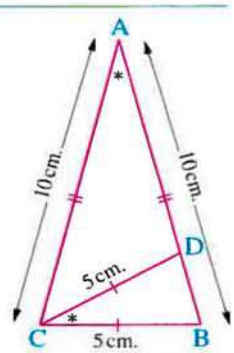
ABC is a triangle in which : D $\in \overline{AB}$

, AB = AC = 10 cm. , CD = CB = 5 cm.

and m ($\angle$ DCB) = m ($\angle$ BAC)

1 **Prove that :** Δ ABC ~ Δ CDB

2 **Find the length of :** $\overline{AD}$

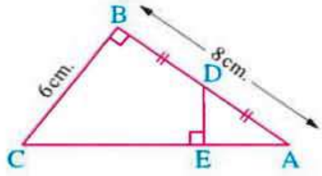


« 7.5 cm. »

18 **In the opposite figure :**

ABC is a right-angled triangle at B , D is the midpoint of $\overline{AB}$, $\overline{DE} \perp \overline{AC}$, AB = 8 cm. , BC = 6 cm.

Find the length of : $\overline{DE}$



« 2.4 cm. »

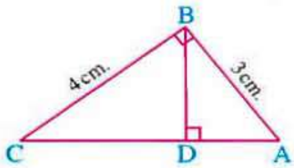
19 **In the opposite figure :**

ABC is a right-angled triangle at B in which :

AB = 3 cm. , BC = 4 cm. and $\overline{BD} \perp \overline{AC}$

1 **Prove that :** Δ BAC ~ Δ DAB

2 **Find the length of each of :** $\overline{AD}$ and $\overline{DC}$



« 1.8 cm. , 3.2 cm. »

20 ABC is a triangle. $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ are bisected at D , E and F respectively

Prove that : Δ ABC ~ Δ EFD

21 Two similar triangles , one of them has a perimeter of 74 cm. and the sides lengths of the other are 4.5 cm. , 6 cm. and 8 cm.

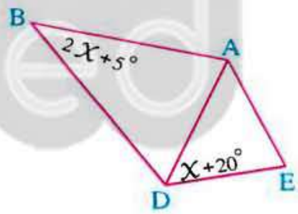
Find the length of the longest side in the first triangle. « 32 cm. »

22 **In the opposite figure :**

Δ AED ~ Δ ADB

, m (∠ ADE) = $x + 20°$ and m (∠ ABD) = $2x + 5°$

Find : m (∠ ADE)



« 35° »

23 **In the opposite figure :**

$\overline{BZ} \subset \overline{BC}$, $\overline{XZ} \parallel \overline{AC}$, $\overline{XY} \parallel \overline{AB}$

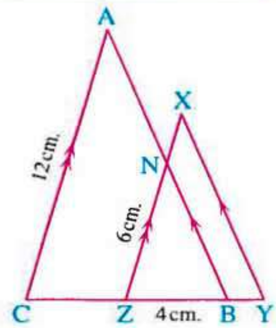
, $\overline{XZ} \cap \overline{AB} = \{N\}$, AC = 12 cm. ,

NZ = 3 NX = 6 cm. and BZ = 4 cm.

1 **Prove that :** Δ XYZ ~ Δ NBZ ~ Δ ABC

2 **Prove that :** Z is the midpoint of $\overline{BC}$

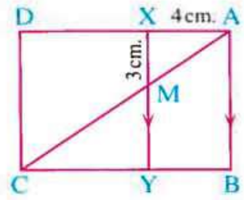
3 **Find the length of :** $\overline{YC}$



« $9\frac{1}{3}$ cm. »

Unit 5

24 **In the opposite figure :**



ABCD is a rectangle in which AD = 12 cm. and $X \in \overline{AD}$ where AX = 4 cm. , $\overline{XY} \parallel \overline{AB}$ and intersects $\overline{AC}$ at M and $\overline{BC}$ at Y , where MX = 3 cm.

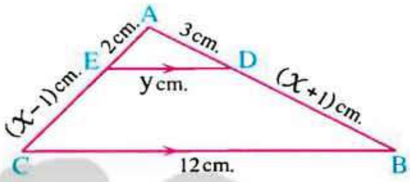
1. **Prove that : Δ AMX $\sim$ Δ CMY**
2. **Find the perimeter of : Δ YMC**
3. **Is the figure ABYM $\sim$ the figure CDXM ? Why ?**

« 24 cm. »

25 **In the opposite figure :**

ABC is a triangle in which : $D \in \overline{AB}$ and $E \in \overline{AC}$ such that : $\overline{DE} \parallel \overline{BC}$, AD = 3 cm. , AE = 2 cm. , BC = 12 cm. , BD = $(x + 1)$ cm. , EC = $(x - 1)$ cm. and DE = y cm.

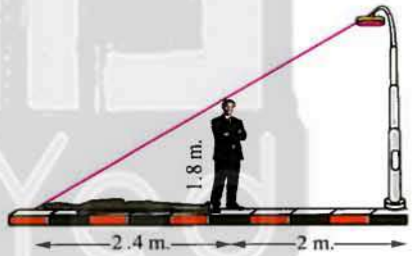
Find the length of each of : $\overline{AB}$, $\overline{EC}$ and $\overline{DE}$



« 9 cm. , 4 cm. , 4 cm. »

Life Applications

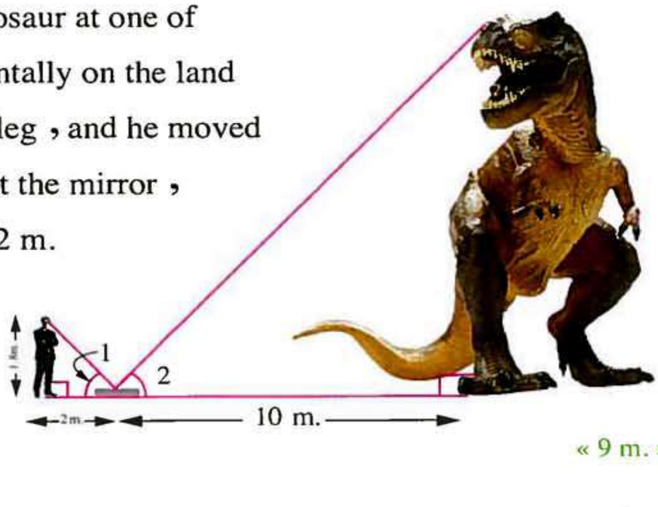
26 A man of height 1.8 m. stands in front of a lamppost at a distance of 2 m. from its base. If the length of the man's shade (when the lamppost is turned on) is 2.4 m. , find the height of the lamppost.



« 3.3 m. »

27 A man wanted to know the height of a dinosaur at one of the museums , then he put a mirror horizontally on the land at a distance of 10 m. from the dinosaur's leg , and he moved back till he could see the dinosaur's head at the mirror , then the distance that he moved back was 2 m.

If the height of the man is 1.8 m. , what is the height of the dinosaur , given that : m (∠ 1) = m (∠ 2) ?



« 9 m. »

For excellent pupils

28 ABC and DEF are two triangles in which , AB = AC and DE = DF

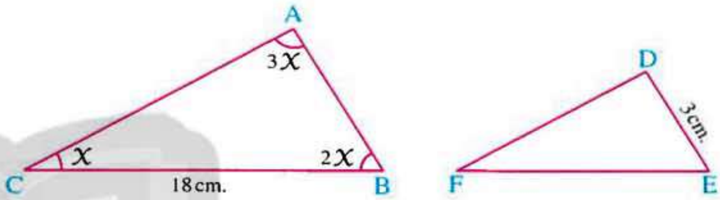
If $\frac{AB}{BC} = \frac{DE}{EF}$, prove that they are similar.

29 **In the opposite figure :**

If Δ ABC ~ Δ DEF

, BC = 18 cm. and DE = 3 cm.

Find the length of : $\overline{EF}$



« 6 cm. »

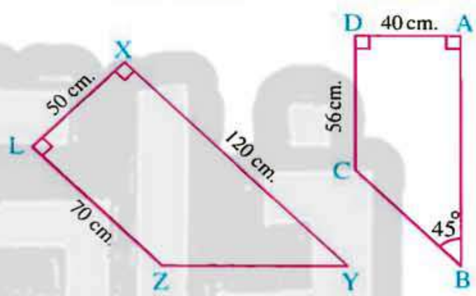
30 **In the opposite figure :**

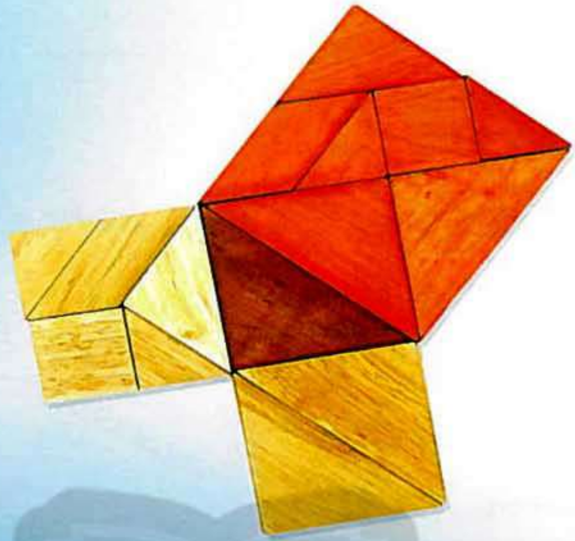
AD = 40 cm. , DC = 56 cm. , XL = 50 cm. ,

XY = 120 cm. , LZ = 70 cm. , m (∠B) = 45°

and m (∠ A) = m (∠ D) = m (∠ X) = m (∠ L) = 90°

Prove that : The polygon ABCD ~ the polygon XYZL





Exercise 7 Converse of Pythagoras' theorem

From the school book

1 **In each of the following figures , prove that : m (∠ B) = 90°**

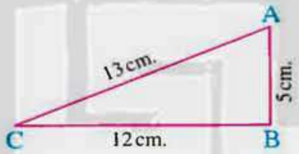


Fig. (1)

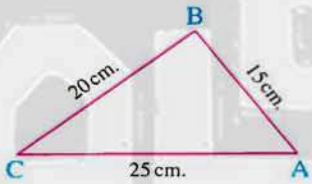


Fig. (2)

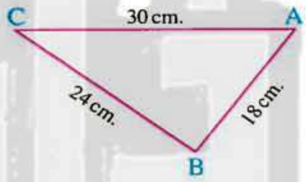
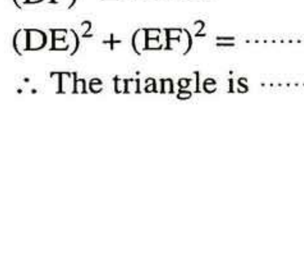


Fig. (3)

2 **Complete and show which of the following triangles is a right-angled triangle :**

1



$(DF)^2 =$ ....

$(DE)^2 + (EF)^2 =$ ....

∴ The triangle is ....

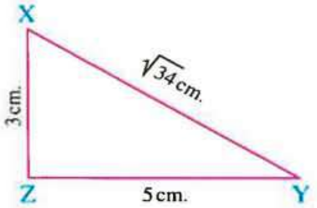
2

$(MN)^2 =$ ....

$(ML)^2 + (NL)^2 =$ ....

∴ The triangle is ....

3

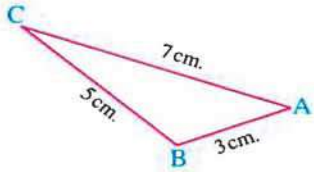


$(XY)^2 = \left(\sqrt{34}\right)^2 = \dots\dots\dots\dots$

$(YZ)^2 + (ZX)^2 = \dots\dots\dots\dots$

$\therefore$ The triangle is ....

4



$(AC)^2 = \dots\dots\dots\dots$

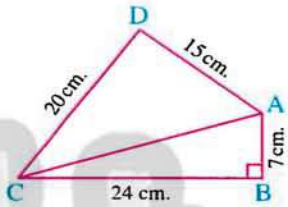
$(AB)^2 + (BC)^2 = \dots\dots\dots\dots$

$\therefore$ The triangle is ....

3 ABC is a triangle in which : AB = 4.5 cm. , BC = 7.5 cm. , AC = 6 cm.

Prove that : Δ ABC is right-angled.

4 **In the opposite figure :**

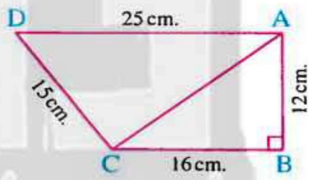


ABCD is a quadrilateral in which : m ($\angle$ ABC) = 90° ,

AB = 7 cm. , BC = 24 cm. , CD = 20 cm. and DA = 15 cm.

Prove that : m ($\angle$ ADC) = 90°

5 **In the opposite figure :**



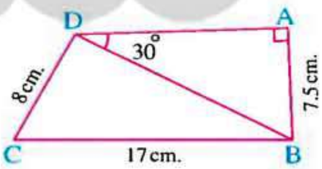
ABCD is a quadrilateral in which : m ($\angle$ B) = 90° ,

AB = 12 cm. , BC = 16 cm. , CD = 15 cm. and DA = 25 cm.

1 **Find :** The length of $\overline{AC}$

2 **Prove that :** m ($\angle$ ACD) = 90° « 20 cm. »

6 **In the opposite figure :**

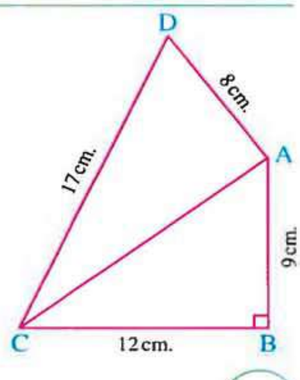


ABCD is a quadrilateral in which : m ($\angle$ BAD) = 90° ,

m ($\angle$ ADB) = 30° , AB = 7.5 cm. , BC = 17 cm. and CD = 8 cm.

Prove that : m ($\angle$ BDC) = 90°

7 **In the opposite figure :**



ABCD is a quadrilateral in which : m ($\angle$ B) = 90° ,

AB = 9 cm. , BC = 12 cm. ,

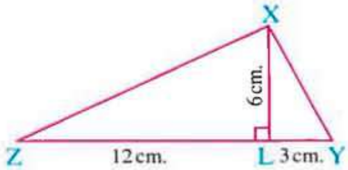
CD = 17 cm. and DA = 8 cm.

Prove that : m ($\angle$ DAC) = 90° ,

then find : The area of the figure ABCD « 114 cm^2 »

Unit 5

8 **In the opposite figure :**

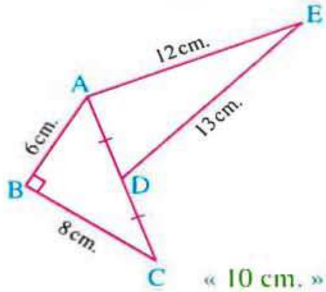


XYZ is a triangle in which : $\overrightarrow{XL} \perp \overline{YZ}$, LX = 6 cm. ,

LY = 3 cm. and LZ = 12 cm.

Prove that : m (∠ YXZ) = 90°

9 **In the opposite figure :**



m (∠ B) = 90° , D is the midpoint of $\overline{AC}$

, AB = 6 cm. , BC = 8 cm. ,

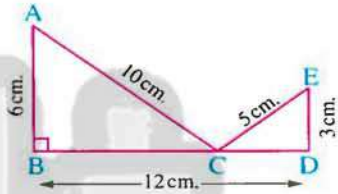
AE = 12 cm. and DE = 13 cm.

1 **Find :** The length of $\overline{AC}$

2 **Prove that :** m (∠ DAE) = 90°

« 10 cm. »

10 **In the opposite figure :**



m (∠ B) = 90° , AB = 6 cm. , BD = 12 cm.

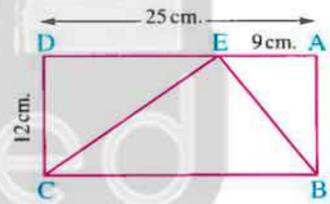
, AC = 10 cm. , CE = 5 cm. , DE = 3 cm. and C ∈ $\overline{DB}$

1 **Find :** The length of $\overline{BC}$

2 **Prove that :** m (∠ D) = 90°

« 8 cm. »

11 **In the opposite figure :**



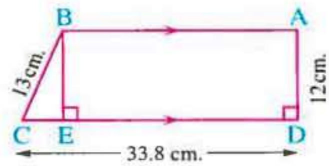
ABCD is a rectangle in which :

DC = 12 cm. , AD = 25 cm. and E ∈ $\overline{AD}$

such that : AE = 9 cm.

Prove that : $\overline{BE} \perp \overline{EC}$

12 **In the opposite figure :**



ABCD is a trapezium in which :

$\overline{AB}$ // $\overline{DC}$, $\overline{AD} \perp \overline{DC}$, AD = 12 cm. ,

BC = 13 cm. , DC = 33.8 cm. and $\overline{BE} \perp \overline{DC}$

First :

Find : 1 The length of each of $\overline{CE}$ and $\overline{AB}$

2 The length of $\overline{DB}$

3 The area of the trapezium ABCD

Second :

Prove that : m (∠ DBC) = 90°

« 5 cm. , 28.8 cm. , 31.2 cm. , 375.6 cm^2 »

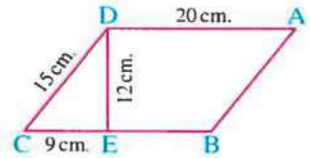
13 **In the opposite figure :**

ABCD is a parallelogram in which :

AD = 20 cm. , DC = 15 cm. and E $\in \overline{BC}$

such that : EC = 9 cm. and DE = 12 cm.

Find : The area of ▱ ABCD



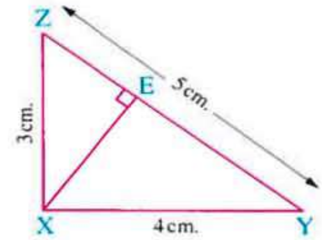
« 240 cm^2 »

14 **In the opposite figure :**

XYZ is a triangle in which $\overline{XE} \perp \overline{YZ}$, E $\in \overline{ZY}$,

YZ = 5 cm. , XZ = 3 cm. and XY = 4 cm.

Find : The area of Δ XYZ , then find the length of $\overline{XE}$



« 6 cm^2 , 2.4 cm. »

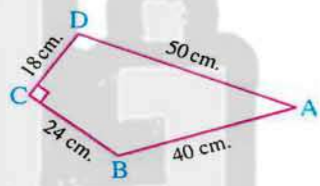
15 **In the opposite figure :**

ABCD is a quadrilateral in which :

AB = 40 cm. , BC = 24 cm. , CD = 18 cm.

, AD = 50 cm. and m (∠C) = 90°

Find : The area of the quadrilateral ABCD



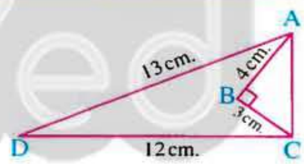
« 816 cm^2 »

16 **In the opposite figure :**

m (∠ B) = 90° , AB = 4 cm. , BC = 3 cm. ,

AD = 13 cm. and DC = 12 cm.

Find : The area of the figure ABCD



« 24 cm^2 »

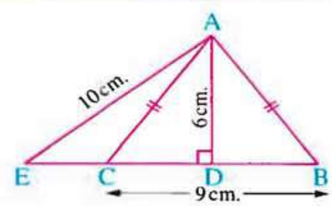
17 **In the opposite figure :**

ABC is an isosceles triangle in which : AB = AC , $\overline{AD} \perp \overline{BC}$

, E $\in \overrightarrow{BC}$, E $\notin \overline{BC}$, AD = 6 cm. ,

BC = 9 cm. and AE = 10 cm.

Prove that : m (∠ BAE) = 90°



18 ABC is a triangle. Draw $\overrightarrow{AD} \perp \overline{BC}$ to cut it at D , if AC = 10 cm. , AD = 6 cm. and AB = 7.5 cm. , calculate the length of $\overline{BC}$, then prove that : Δ ABC is right-angled. « 12.5 cm. »

19 ABC is a triangle. Draw $\overrightarrow{AD} \perp \overline{BC}$ to cut it at D , if AC = 20 cm. , AD = 12 cm. and BD = 9 cm.

Prove that : Δ ABC is right-angled at A

20 ABCD is a parallelogram in which : AB = 8 cm. , AC = 20 cm. , BD = 12 cm.

Prove that : m (∠ ABD) = 90° , then find the area of ▱ ABCD « 96 cm^2 »

Life Application

21 The opposite figure represents how the Ancient Egyptians were able to make a right angle by using a rope with knots on equal distances.

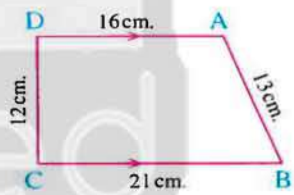
Show why this rope forms a right angle.

For excellent pupils

22 ABC is a triangle in which : AB = 24 cm. , BC = 70 cm. , $\overline{BD}$ is a median in the triangle where BD = 37 cm. **Prove that :** m (∠ ABC) = 90° , then find the length of $\overline{AC}$

(Hint. Draw $\overrightarrow{DE}$ // $\overline{BC}$ to cut $\overline{AB}$ at E) « 74 cm. »

23 **In the opposite figure :**

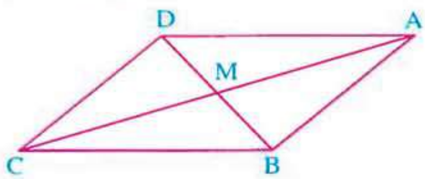


ABCD is a trapezium in which : $\overline{AD}$ // $\overline{BC}$, AB = 13 cm. ,

BC = 21 cm. , CD = 12 cm. and DA = 16 cm.

Prove that : m (∠ C) = 90°

24 **In the opposite figure :**



ABCD is a parallelogram in which : $\overline{AC} \cap \overline{BD} = \{M\}$

If $(AC)^2 = 4\,(AB)^2 + (BD)^2$

Prove that : m (∠ ABD) = 90°

Exercise 8 Projections

From the school book

1 Choose the correct answer from those given :

1. The projection of a point on a given straight line is ....
   (a) a point. (b) a line segment. (c) a ray. (d) a straight line.
2. The projection of a line segment on a straight line not perpendicular to it is ....
   (a) a ray. (b) a point. (c) a line segment. (d) a straight line.
3. The projection of a line segment on a straight line perpendicular to it is ....
   (a) a point. (b) a line segment. (c) a ray. (d) a straight line.
4. The projection of a ray on a straight line not perpendicular to it is ....
   (a) a point. (b) a line segment. (c) a ray. (d) a straight line.
5. The length of the projection of a line segment on a given straight line .... the length of the line segment itself.
   (a) $\leq$ (b) $>$ (c) $\geq$ (d) $=$
6. The length of the projection of a line segment on a straight line perpendicular to it is ....
   (a) greater than the length of the main line segment.
   (b) equal to the length of the main line segment.
   (c) greater than or equal to the length of the main line segment.
   (d) equal to zero.
7. The length of the projection of a line segment on a straight line parallel to it .... the length of the main line segment.
   (a) $<$ (b) $>$ (c) $=$ (d) $\neq$

Unit 5

2 **In each of the following figures , find :**

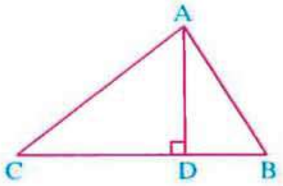


Fig. (1)

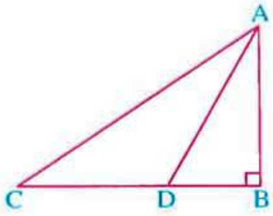


Fig. (2)

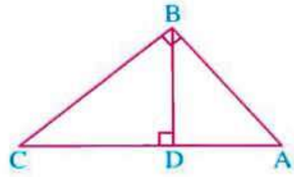
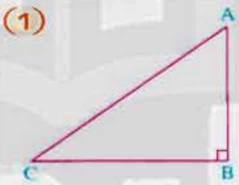
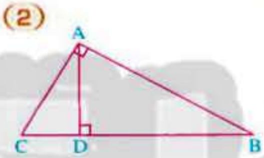
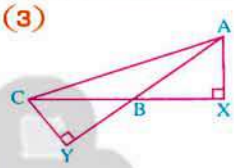


Fig. (3)

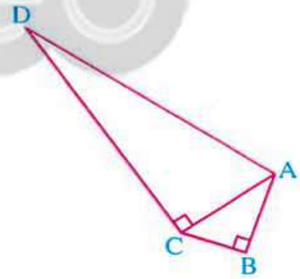
1. The projection of A on $\overleftrightarrow{BC}$
2. The projection of $\overline{AB}$ on $\overleftrightarrow{BC}$

3 **Complete the following table :**

The figure / The projections	(1) 	(2) 	(3) 
The projection of $\overline{AC}$ on $\overleftrightarrow{BC}$	....	....	....
The projection of $\overline{AB}$ on $\overleftrightarrow{BC}$	....	....	....
The projection of $\overline{AC}$ on $\overleftrightarrow{AB}$	....	....	....
The projection of $\overline{BC}$ on $\overleftrightarrow{AB}$	....	....	....

4 **In the opposite figure :**

m (∠ B) = m (∠ ACD) = 90°

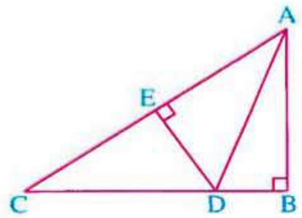


Complete :

1. The projection of $\overline{AD}$ on $\overleftrightarrow{CD}$ is ....
2. The projection of $\overline{AC}$ on $\overleftrightarrow{CD}$ is ....
3. The projection of $\overline{AC}$ on $\overleftrightarrow{AB}$ is ....

5 **In the opposite figure :**

Δ ABC is right-angled at B , D ∈ $\overline{BC}$ and $\overline{DE} \perp \overline{AC}$



Complete each of the following :

1. The projection of $\overline{AD}$ on $\overleftrightarrow{BC}$ = ...
2. The projection of $\overline{AD}$ on $\overleftrightarrow{AC}$ = ...

3 The projection of $\overline{DE}$ on $\overleftrightarrow{AC}$ = ...

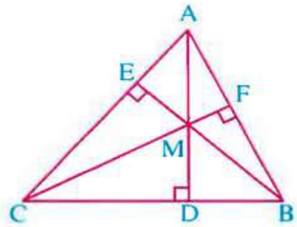
4 The projection of the point C on $\overleftrightarrow{AB}$ = ...

5 The projection of the point A on $\overleftrightarrow{DC}$ = ...

6 The projection of the point D on $\overleftrightarrow{AC}$ = ...

7 The projection of $\overline{AB}$ on $\overleftrightarrow{DC}$ = ...

6 In the opposite figure :



ABC is a triangle , $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ are three perpendicular line segments drawn from the vertices to the opposite sides and they are intersecting at M

Complete the following :

1 The projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is .... , the projection of $\overline{BC}$ on $\overleftrightarrow{AB}$ is ....

2 The projection of $\overline{AC}$ on $\overleftrightarrow{BC}$ is .... , the projection of $\overline{BC}$ on $\overleftrightarrow{AC}$ is ....

3 The projection of $\overline{AC}$ on $\overleftrightarrow{AB}$ is .... , the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$ is ....

4 The projection of $\overline{AM}$ on $\overleftrightarrow{AB}$ is .... , the projection of $\overline{BM}$ on $\overleftrightarrow{BC}$ is ....

5 The projection of $\overline{CM}$ on $\overleftrightarrow{AB}$ is .... , the projection of $\overline{BM}$ on $\overleftrightarrow{AC}$ is ....

7 Complete the following :

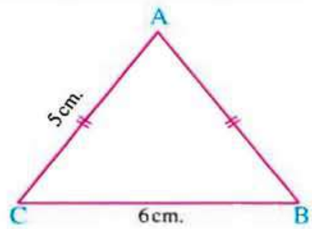
1 If $X \in \overleftrightarrow{AB}$, then the projection of X on $\overleftrightarrow{AB}$ is ....

2 If $\overline{AB} \perp \overline{BC}$, then the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is ....

3 In Δ ABC , if m ($\angle$B) = 90° , then the projection of C on $\overleftrightarrow{AB}$ is ....

4 ABC is a right-angled triangle at A , then the projection of $\overline{BA}$ on $\overleftrightarrow{AC}$ is ....

8 In the opposite figure :



ABC is a triangle in which : AB = AC = 5 cm. and BC = 6 cm.

Find :

1 The length of the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$

2 The area of the triangle ABC

« 3 cm. , 12 cm.^{2} »

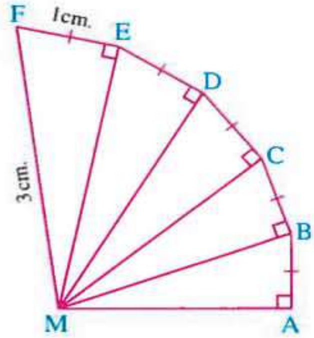
9 **In the opposite figure :**

$AB = BC = CD = DE = EF = 1$ cm.

and $MF = 3$ cm.

Find :

1. The length of the projection of $\overline{FM}$ on $\overleftrightarrow{EM}$
2. The length of the projection of $\overline{BM}$ on $\overleftrightarrow{AM}$



« $2\sqrt{2}$ cm. , 2 cm. »

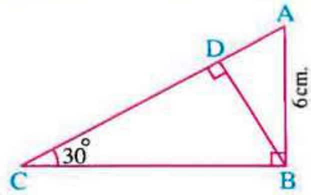
10 **In the opposite figure :**

ABC is a triangle in which : $m(\angle B) = 90°$, $m(\angle C) = 30°$,

$AB = 6$ cm. and $\overline{BD} \perp \overline{AC}$

Find :

1. The length of the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$
2. The length of the projection of $\overline{BC}$ on $\overleftrightarrow{AC}$



« 3 cm. , 9 cm. »

11 **In the opposite figure :**

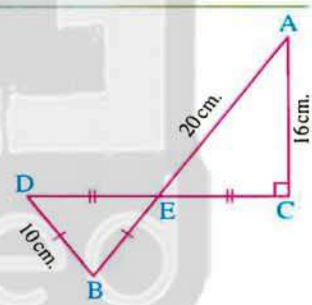
$\overline{AB} \cap \overline{CD} = \{E\}$, E is the midpoint of $\overline{CD}$,

$AC = 16$ cm. , $AE = 20$ cm.

and $BD = BE = 10$ cm.

Find :

1. The length of the projection of $\overline{BD}$ on $\overleftrightarrow{CD}$
2. The length of the projection of $\overline{AB}$ on $\overleftrightarrow{CD}$



« 6 cm. , 18 cm. »

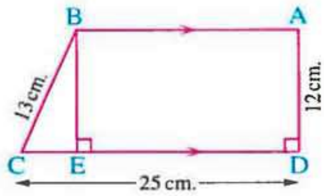
12 **In the opposite figure :**

ABCD is a trapezium in which : $\overline{AB} // \overline{DC}$, $\overline{AD} \perp \overline{DC}$

, $AD = 12$ cm. , $BC = 13$ cm. , and $DC = 25$ cm.

If $\overline{BE} \perp \overline{DC}$, **find :**

1. The length of the projection of $\overline{BC}$ on $\overleftrightarrow{DC}$
2. The length of the projection of $\overline{AB}$ on $\overleftrightarrow{DC}$
3. The length of the projection of $\overline{DC}$ on $\overleftrightarrow{AB}$
4. The area of the trapezium ABCD



« 5 cm. , 20 cm. , 25 cm. , 270 cm.^{2} »

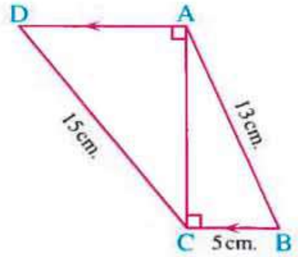
13 **In the opposite figure :**

$\overline{AD} \parallel \overline{BC}$, AB = 13 cm. , BC = 5 cm. ,

CD = 15 cm. and m (∠ ACB) = m (∠ DAC) = 90°

Find :

1. The length of the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$
2. The length of the projection of $\overline{CD}$ on $\overleftrightarrow{AD}$



« 12 cm. , 9 cm. »

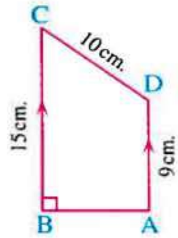
14 **In the opposite figure :**

ABCD is a trapezium in which : $\overline{AD} \parallel \overline{BC}$ and m (∠ ABC) = 90°

If AD = 9 cm. , DC = 10 cm. and CB = 15 cm.

Find :

1. The length of the projection of $\overline{DC}$ on $\overleftrightarrow{BC}$
2. The length of the projection of $\overline{DC}$ on $\overleftrightarrow{AB}$



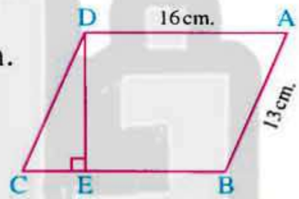
« 6 cm. , 8 cm. »

15 **In the opposite figure :**

ABCD is a parallelogram in which : AD = 16 cm. and AB = 13 cm.

If $\overline{DE} \perp \overline{BC}$ and the area of the parallelogram ABCD = 192 cm^2 ,

Find : The length of the projection of $\overline{DC}$ on $\overleftrightarrow{BC}$

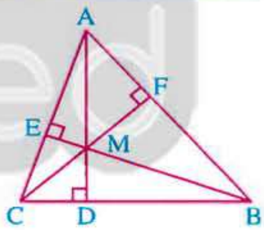


« 5 cm. »

16 **In the opposite figure :**

ABC is a triangle in which : $\overline{AD} \perp \overline{BC}$, $\overline{CF} \perp \overline{AB}$, $\overline{BE} \perp \overline{AC}$

and $\overline{AD} \cap \overline{CF} \cap \overline{BE} = \{M\}$



1. **Mention the following :**

   (a) The projection of $\overline{AD}$ on $\overleftrightarrow{BC}$
   (b) The projection of $\overline{BE}$ on $\overleftrightarrow{AC}$
   (c) The projection of $\overline{AE}$ on $\overleftrightarrow{AC}$
   (d) The projection of $\overline{AB}$ on $\overleftrightarrow{AD}$

2. If AC = 26 cm. , AB = 30 cm. , BC = 28 cm. and the area of Δ ABC = 336 cm^2

   Find : The length of the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$

« 18 cm. »

For excellent pupils

17 ABC is a triangle in which : m (∠ ABC) = 120° and AB = 12 cm.

Calculate the length of the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$

« 6 cm. »

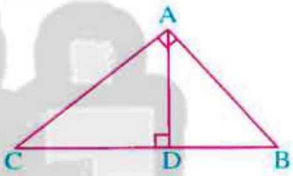
Exercise 9

Euclidean theorem

From the school book

1 **In the opposite figure :**

Δ ABC is right-angled at A , $\overline{AD} \perp \overline{BC}$

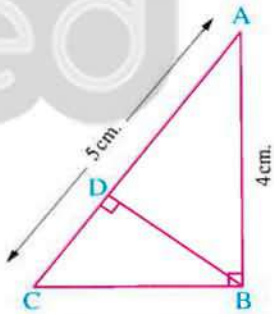


Complete each of the following :

1. $(AC)^2$ = ... + ...
2. $(AC)^2$ = ... − ....
3. $(AC)^2$ = ... × ....
4. $(AD)^2$ = ... × ....
5. AC × AB = ... × ....
6. Δ ABC ~ Δ .... ~ Δ ....

2 **In the opposite figure :**

ABC is a triangle in which : m ($\angle$ ABC) = 90° , AB = 4 cm. ,
AC = 5 cm. and $\overline{BD} \perp \overline{AC}$

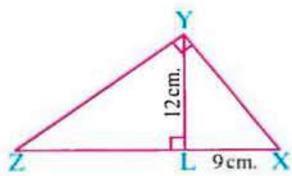


Complete :

1. BC = ... cm.
2. AD = ... cm.
3. BD = ... cm.
4. The area of Δ DBC = ... cm^2

3 **In the opposite figure :**

XYZ is a triangle in which : m ($\angle$ XYZ) = 90°
and L $\in \overline{XZ}$ such that $\overline{YL} \perp \overline{XZ}$,
XL = 9 cm. and YL = 12 cm. **Find :**



1. The length of $\overline{XY}$
2. The length of $\overline{LZ}$
3. The length of $\overline{ZY}$

« 15 cm. , 16 cm. , 20 cm. »

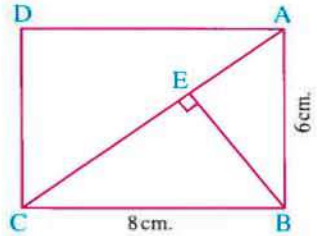
4 **In the opposite figure :**

ABCD is a rectangle in which : AB = 6 cm. , BC = 8 cm.

and E $\in \overline{AC}$ such that $\overline{BE} \perp \overline{AC}$

Find the length of each of :

1 $\overline{BE}$ 2 $\overline{EC}$



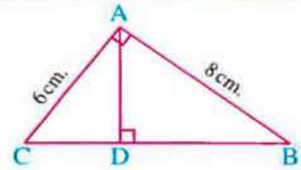
« 4.8 cm. , 6.4 cm. »

5 **In the opposite figure :**

ABC is a triangle in which : m (∠ BAC) = 90° , $\overline{AD} \perp \overline{BC}$

, AB = 8 cm. and AC = 6 cm.

Find : BD , CD and AD



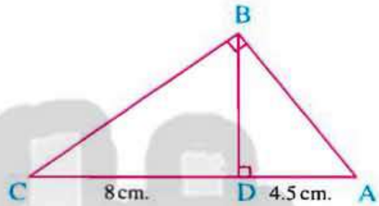
« 6.4 cm. , 3.6 cm. , 4.8 cm. »

6 **In the opposite figure :**

Δ ABC is right-angled at B and $\overline{BD} \perp \overline{AC}$

If AD = 4.5 cm. and DC = 8 cm.

Find : The length of each of $\overline{AB}$, $\overline{BC}$ and $\overline{BD}$



« 7.5 cm. , 10 cm. , 6 cm. »

7 **In the opposite figure :**

ABCD is a quadrilateral where :

m (∠ BCD) = m (∠ BAD) = 90° ,

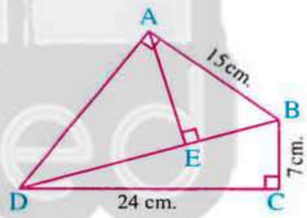
$\overline{AE} \perp \overline{BD}$, BC = 7 cm. , CD = 24 cm.

and AB = 15 cm.

Find : 1 The length of each of $\overline{BD}$ and $\overline{AD}$

2 The length of the projection of $\overline{AB}$ on $\overleftrightarrow{BD}$

3 The length of the projection of $\overline{AD}$ on $\overleftrightarrow{AE}$



« 25 cm. , 20 cm. , 9 cm. , 12 cm. »

8 **In the opposite figure :**

Δ AXE is right-angled at X and $\overline{XF} \perp \overline{AY}$

where Y $\in \overline{XE}$, F $\in \overline{AY}$,

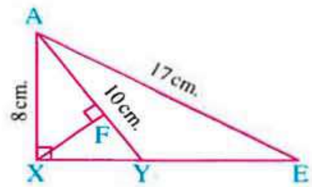
AX = 8 cm. , AY = 10 cm. and AE = 17 cm.

Find : 1 The length of the projection of $\overline{AY}$ on $\overleftrightarrow{XE}$

2 The length of $\overline{XF}$

3 The length of $\overline{AF}$

4 The area of Δ AXE



« 6 cm. , 4.8 cm. , 6.4 cm. , 60 cm.^{2} »

9 **In the opposite figure :**

XYZ is a triangle in which : m (∠ Y) = 90°

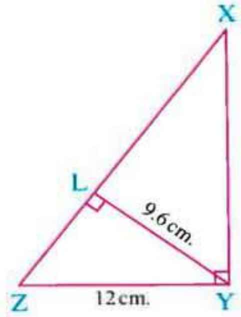
and $\overrightarrow{YL} \perp \overline{XZ}$ where L $\in \overline{XZ}$

If YZ = 12 cm. and YL = 9.6 cm.

Find : 1 The length of the projection of $\overline{YZ}$ on $\overleftrightarrow{XZ}$

2 The length of the projection of $\overline{XY}$ on $\overleftrightarrow{XZ}$

3 The length of the projection of $\overline{XZ}$ on $\overleftrightarrow{XY}$



« 7.2 cm. , 12.8 cm. , 16 cm. »

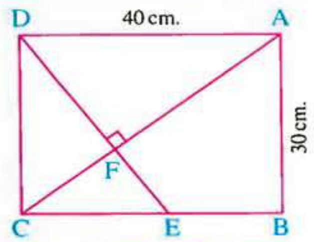
10 **In the opposite figure :**

ABCD is a rectangle in which : AB = 30 cm.

, AD = 40 cm. , $\overline{DE} \perp \overline{AC}$ intersects $\overline{AC}$ at F

and intersects $\overline{BC}$ at E

Find : The length of each of $\overline{AF}$, $\overline{DF}$ and $\overline{EC}$



« 32 cm. , 24 cm. , 22.5 cm. »

11 **In the opposite figure :**

Δ ABC is right-angled at B

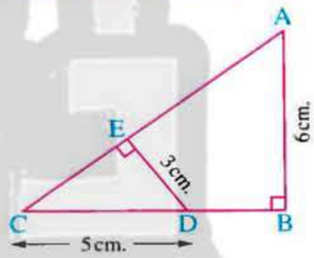
, $\overline{DE} \perp \overline{AC}$, AB = 6 cm.

, ED = 3 cm. and CD = 5 cm.

Prove that : Δ CED ~ Δ CBA

and find : The length of $\overline{AC}$

and the length of the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$



« 10 cm. , 3.6 cm. »

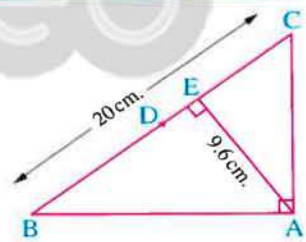
12 **In the opposite figure :**

Δ CAB is right-angled at A ,

E $\in \overline{BC}$ such that $\overline{AE} \perp \overline{BC}$,

D is the midpoint of $\overline{BC}$, AE = 9.6 cm. and BC = 20 cm.

Find : The length of each of $\overline{AB}$ and $\overline{AC}$



« 16 cm. , 12 cm. »

13 **In the opposite figure :**

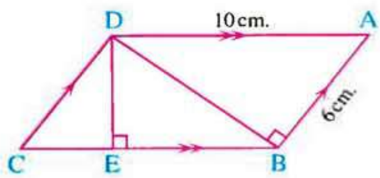
ABCD is a parallelogram , AB = 6 cm. , AD = 10 cm. ,

$\overline{BD} \perp \overline{AB}$ and $\overline{DE} \perp \overline{BC}$

Find : 1 The area of the parallelogram ABCD

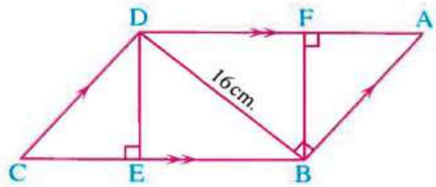
2 The length of the projection of $\overline{DB}$ on $\overleftrightarrow{BC}$

3 The length of $\overline{DE}$



« 48 cm.^{2} , 6.4 cm. , 4.8 cm. »

14 **In the opposite figure :**



ABCD is a parallelogram , m (∠ ABD) = 90°

, $\overline{DE} \perp \overline{CB}$ and $\overline{BF} \perp \overline{AD}$

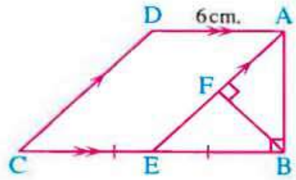
If the area of the parallelogram = 192 cm².

and BD = 16 cm.

Find : The area of the rectangle BEDF

« 122.88 cm². »

15 **In the opposite figure :**



ABCD is a trapezium whose area equals 72 cm². in which :

$\overline{AD}$ // $\overline{BC}$, m (∠ ABC) = 90° and AD = 6 cm. ,

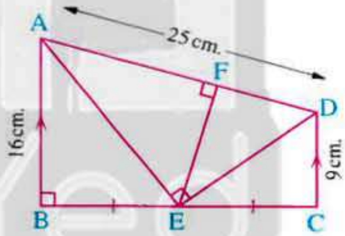
E is the midpoint of $\overline{BC}$ and F ∈ $\overline{AE}$ such that $\overline{BF} \perp \overline{AE}$

and $\overline{AE}$ // $\overline{DC}$

Find : The length of $\overline{BF}$

« 4.8 cm. »

16 **In the opposite figure :**



ABCD is a trapezium in which :

$\overline{AB}$ // $\overline{DC}$ and m (∠ ABC) = 90° ,

E is the midpoint of $\overline{BC}$

, AB = 16 cm. , AD = 25 cm. , DC = 9 cm. ,

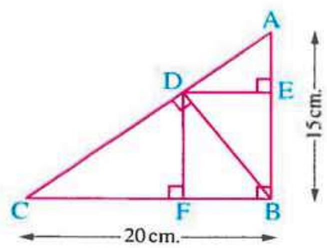
$\overline{AE} \perp \overline{ED}$ and $\overline{EF} \perp \overline{AD}$

Find : 1 The area of the trapezium ABCD

2 The length of the projection of $\overline{AE}$ on $\overleftrightarrow{AD}$

« 300 cm². , 16 cm. »

17 **In the opposite figure :**



Δ ABC is right-angled at B ,

$\overline{BD} \perp \overline{AC}$, $\overline{DE} \perp \overline{AB}$ and $\overline{DF} \perp \overline{BC}$

If AB = 15 cm. and BC = 20 cm.

Find : The length of each of $\overline{DF}$ and $\overline{DE}$

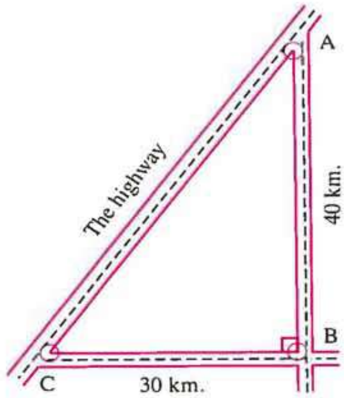
« 9.6 cm. , 7.2 cm. »

Unit 5

Life Applications

18 The opposite figure is a diagram of three agricultural villages A , B , C and a market for selling agricultural products should be built on the highway. Then a road should be made to connect the market with village B whereas the length of the road is the shortest.

Show how that can be done , then find the length of this road. « 24 km. »

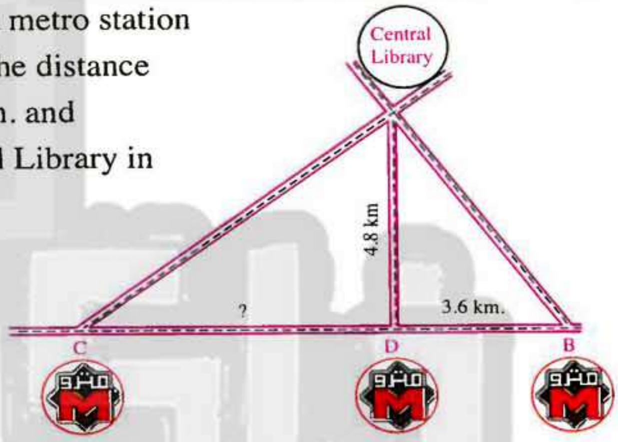


19 In one of the governorates , we want to build a metro station between two other stations B and C such that the distance between that station and the station B is 3.6 km. and the shortest distance between it and the Central Library in the governorate is 4.8 km.

If you know that the two ways between the Central Library and the two metro stations B and C are orthogonal.

Find in two different methods :

The distance between the metro station we want to build and the metro station C « 6.4 km. »



For excellent pupils

20 **In the opposite figure :**

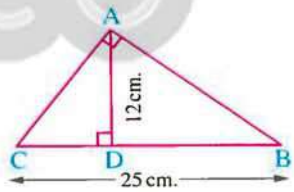
ΔABC is right-angled at A

and $\overline{AD} \perp \overline{BC}$ where $D \in \overline{BC}$

If AD = 12 cm. , BC = 25 cm. and CD < BD

Calculate the length of each of :

1. $\overline{AB}$ and its projection on $\overleftrightarrow{BC}$ « 20 cm. , 16 cm. »
2. $\overline{AC}$ and its projection on $\overleftrightarrow{BC}$ (Hint : Suppose CD = x cm.) « 15 cm. , 9 cm. »



21 ΔXYZ is right-angled at Y , $\overrightarrow{YL} \perp \overline{XZ}$

and $\overrightarrow{YL} \cap \overline{XZ} = \{L\}$ such that $XL = \frac{9}{25}$ XZ

Prove that : XY : YZ = 3 : 4

Exercise 10

Classifying triangles according to their angles

From the school book

1 **In each of the following , identify the type of Δ ABC according to its angles if :**

1. AB = 12 cm. , BC = 14 cm. and AC = 15 cm.
2. AB = 8 cm. , BC = 7 cm. and AC = 3 cm.
3. AB = 25 cm , BC = 15 cm. and AC = 20 cm.

2 Identify the type of $\angle$ Y in Δ XYZ if XY = 4 cm. , YZ = 5 cm. and XZ = 7 cm.

3 Identify the type of $\angle$ A in Δ ABC if AB = 6 cm. , BC = 10 cm. and AC = 8 cm.

4 Identify the type of $\angle$ B in Δ ABC if AB = 10 cm. , BC = 12 cm. and AC = 15 cm.

5 **Determine the type of the greatest angle in Δ ABC where :**

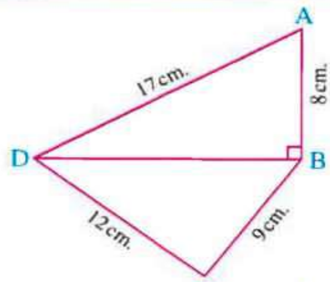
1. AB = 9 cm. , BC = 10 cm. and AC = 12 cm.
2. AB = 5 cm. , BC = 12 cm. and AC = 13 cm.
3. AB = 7 cm. , BC = 16 cm. and AC = 14 cm.

   Determine the type of the triangle according to its angles.

6 **In the opposite figure :**

ABCD is a quadrilateral in which : AB = 8 cm.,
BC = 9 cm. , CD = 12 cm. , AD = 17 cm.
and $\overline{DB} \perp \overline{AB}$

1. Find the length of the projection of $\overline{AD}$ on $\overleftrightarrow{BD}$
2. Determine the type of Δ BCD according to its angles.



« 15 cm. »

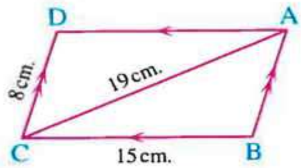
Unit 5

7 **In the opposite figure :**

ABCD is a parallelogram in which :

BC = 15 cm. , CD = 8 cm. and AC = 19 cm.

Prove that : ∠ ABC is an obtuse angle.



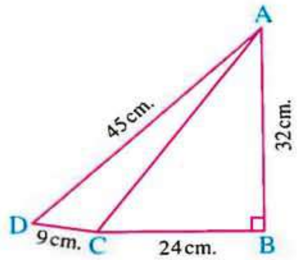
8 **In the opposite figure :**

ABCD is a quadrilateral in which :

m (∠ B) = 90° , AB = 32 cm. , BC = 24 cm. ,

CD = 9 cm. and AD = 45 cm.

Prove that : ACD is an obtuse-angled triangle.



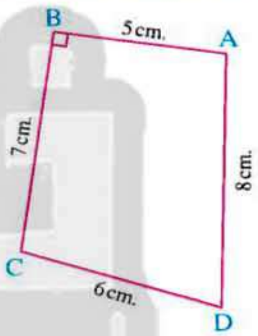
9 **In the opposite figure :**

ABCD is a quadrilateral in which :

m (∠ B) = 90° , AB = 5 cm. , BC = 7 cm.,

AD = 8 cm. and DC = 6 cm.

Prove that : ∠ D is an acute angle.

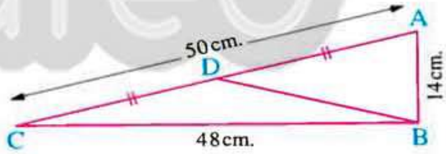


10 **In the opposite figure :**

$\overline{BD}$ is a median in Δ ABC , AB = 14 cm. ,

BC = 48 cm. and AC = 50 cm.

Prove that : ∠ BDC is an obtuse angle.



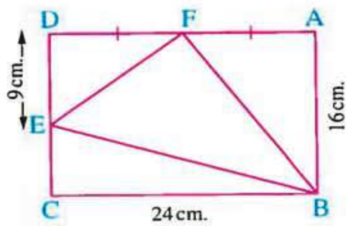
11 **In the opposite figure :**

ABCD is a rectangle in which :

AB = 16 cm. , BC = 24 cm. ,

E ∈ $\overline{CD}$ and DE = 9 cm.

Classify the triangle BFE according to the measures of its angles.



12 ABCD is a rhombus in which : AC = 16 cm. and BD = 12 cm.

Prove that : Δ ABD is acute-angled.

13 ABCD is a quadrilateral in which : AB = 8 cm. , BC = 9 cm. , CD = 12 cm. and DA = 17 cm. If m (∠ ABD) = 90° , find the projection length of $\overline{AD}$ on $\overleftrightarrow{BD}$, then classify the triangle BCD according to the measures of its angles. « 15 cm. »

14 Find the length of $\overline{BC}$ in Δ ABC in which : $(AB)^2 > (AC)^2 + (BC)^2$, AB = 15 cm. AC = 13 cm. , $\overline{AD} \perp \overleftrightarrow{BC}$ and intersects it at D and AD = 12 cm. « 4 cm. »

15 Choose the correct answer from those given :

1. A triangle whose side lengths are : 5 cm. , 12 cm. and 13 cm. its area = ... cm^2.
   (a) 30 (b) 32.5 (c) 78 (d) 60
2. ABC is an obtuse-angled triangle at A , if AB = 4 cm. , BC = 7 cm. , then AC can be equals .... cm.
   (a) 5 (b) 6 (c) 7 (d) 8
3. ABC is an obtuse-angled triangle at B if AB = 5 cm. , BC = 3 cm. , then AC can be equals .... cm.
   (a) 4 (b) 5 (c) 7 (d) 8
4. ABC is an acute-angled triangle in which : AB = 6 cm. , BC = 8 cm. , then the length of $\overline{AC}$ can be equals .... cm.
   (a) 2 (b) 6 (c) 10 (d) 14
5. ABC is a triangle in which : $(BC)^2 = (AB)^2 + (AC)^2$, m (∠ B) = 40° , then m (∠ C) = ...
   (a) 40° (b) 50° (c) 90° (d) 140°

16 Complete the following :

1. In Δ ABC , if $(AB)^2 = (BC)^2 + (AC)^2$, then : m (∠ ....) = 90°
2. In Δ ABC , if $(AB)^2 < (AC)^2 + (BC)^2$, then ∠ C is ....
3. In Δ ABC , if $(AB)^2 + (BC)^2 < (AC)^2$, then ∠ B is ....
4. In Δ XYZ , if $(XY)^2 = (YZ)^2 + (ZX)^2$, then ∠ Z is ....

5 In Δ XYZ , if $(YZ)^2 > (XZ)^2 - (XY)^2$, then $\angle$ Y is ....

6 In Δ ABC , if $(AB)^2 = (AC)^2 - (BC)^2$, then $\angle$ C is ....

7 In Δ ABC , if $(AC)^2 - (AB)^2 = (BC)^2 - 3$, then $\angle$ B is ....

8 In Δ ABC , if $(AB)^2 + (BC)^2 = 48$ cm.^{2} , AC = 7 cm. , then $\angle$ B is ....

9 In Δ XYZ , if $90° < m(\angle Y) < 180°$, then $(XZ)^2$ .... $(XY)^2 + (YZ)^2$

10 If $\angle$ A complements $\angle$ B in Δ ABC , then $(AB)^2$ .... $(AC)^2 + (BC)^2$

11 If the two lengths of two sides in a triangle are 3 cm. and 5 cm. , then the length of the third side is between .... , ...

12 ABC is a triangle whose sides lengths are 6 cm. , 8 cm. and 11 cm. Δ ABC is similar to the triangle XYZ , then Δ XYZ is .... according to its angles.

13 In Δ XYZ , if $(XZ - XY)(XZ + XY) < (ZY)^2$, then $\angle$ Y is ....

For excellent pupils

17 ABC is a triangle in which : AB = 13 cm. , BC = 11 cm. and AC = 20 cm.

1 **Prove that :** Δ ABC is obtuse-angled at B

2 **Find :** The length of the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ « 5 cm. »

3 **Find :** The area of Δ ABC « 66 cm.^{2} »

18 Calculate the measures of the greatest angle in Δ ABC

If AB = 7 cm. , BC = 3 cm. and AC = 5 cm. « 120° »

Summary of Unit 5

Similarity of two polygons

It is said that two polygons (of the same number of sides) are similar if the following two conditions are verified together :

1. Their corresponding angles are equal in measure.
2. Their corresponding side lengths are proportional.

- The congruent polygons are similar but it is not necessary that the similar polygons are congruent.
- All regular polygons of the same number of sides are similar.
- If each of two polygons is similar to a third polygon , then they are similar.
- The ratio between the perimeters of two similar polygons = the ratio between the lengths of two corresponding sides.

Similarity of two triangles

The two triangles are similar if ONE of the two following conditions is verified :

1. The measures of their corresponding angles are equal.
2. The lengths of their corresponding sides are proportional.

The converse of Pythagoras' theorem

In a triangle , if the sum of the areas of two squares on two sides is equal to the area of the square on the third side , then the angle opposite to this side is a right angle.

Projections

1 The projection of a point on a straight line.

- The projection of a point on a straight line is the point of intersection of the perpendicular segment from this point and the straight line.
- If the point lies on the straight line , its projection on it is the same point.

2 The projection of a line segment on a straight line.

- The projection of a line segment on a given straight line is the line segment whose two endpoints are the projections of the two endpoints of the main line segment on this straight line.
- The length of the projection of a line segment on a given straight line $\leq$ the length of the line segment.

3 The projection of a ray on a straight line.

- The projection of a ray on a straight line not perpendicular to it is a ray ⊂ this straight line.
- The projection of a ray on a straight line perpendicular to it is a point belonging to the straight line.

4 The projection of a straight line on another straight line.

- The projection of a straight line on another straight line not perpendicular to it is that another straight line.
- The projection of a straight line on another straight line perpendicular to it is the point of intersection of the two straight lines.

Eculidean theorem :

In the right-angled triangle , the area of the square on a side of the right angle is equal to the area of the rectangle whose dimensions are the length of the projection of this side on the hypotenuse and the length of the hypotenuse.

Determining the type of the triangle according to its angles in case of knowing the lengths of its three sides.

- If the square length of the longest side equals the sum of the squares lengths of the other two sides , then the triangle is right-angled.
- If the square length of the longest side is greater than the sum of squares lengths of the other two sides , then the triangle is obtuse-angled.
- If the square length of the longest side is less than the sum of squares lengths of the other two sides , then the triangle is acute-angled.

To determine the type of an angle in a triangle , we compare between the square length of the side opposite to it and the sum of squares lengths of the other two sides.

The greatest angle in measure in the triangle is opposite to the longest side.

Exams on Unit Five

Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1. If the ratio of enlargement between two similar triangles equals .... , then the two triangles are congruent.
   (a) 1 (b) 2 (c) 0.5 (d) 0.25
2. If $\overline{AB} \parallel \overleftrightarrow{XY}$, then the length of the projection of $\overline{AB}$ on $\overleftrightarrow{XY}$ .... the length of $\overline{AB}$
   (a) < (b) > (c) = (d) ≠
3. In Δ ABC : $(AC)^2 = (BC)^2 - (AB)^2$, then $\angle$ A is ....
   (a) acute. (b) right. (c) obtuse. (d) straight.
4. ABC is an obtuse-angled triangle at B , AB = 3 cm. , BC = 5 cm. , then AC = ...
   (a) 8 cm. (b) 7 cm. (c) 5 cm. (d) 4 cm.
5. If two polygons are similar and the ratio between the lengths of two corresponding sides is 2 : 3 , then the ratio between their perimeters is ....
   (a) 3 : 2 (b) 2 : 3 (c) 4 : 9 (d) 9 : 4
6. ABC is a triangle in which : $\overline{AD} \perp \overline{BC}$, then the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is ....
   (a) $\overline{DB}$ (b) $\overline{DC}$ (c) $\overline{AC}$ (d) $\overline{AB}$

2 Complete the following :

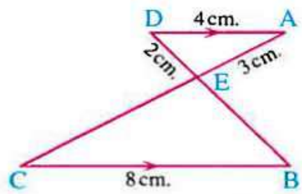
1. If each of two triangles is similar to a third triangle , then they are ....
2. If $\overline{AB} \perp \overline{BC}$, then the projection of $\overline{AC}$ on $\overleftrightarrow{BC}$ is ....
3. In Δ XYZ if $(XZ)^2 + (YZ)^2 > (XY)^2$, then $\angle$ Z is ....
4. The two triangles are similar if the lengths of their corresponding sides are ....
5. The projection of a point which belongs to a straight line on this line is ....

3 [a] In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, AD = 4 cm. , BC = 8 cm.

, AE = 3 cm. , ED = 2 cm.

1. **Prove that :** Δ AED ~ Δ CEB
2. **Find :** The perimeter of Δ EBC

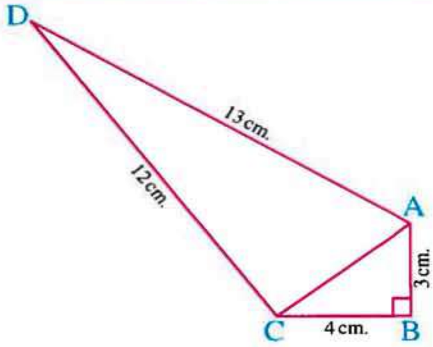


[b] In the opposite figure :

AB = 3 cm. , BC = 4 cm. , AD = 13 cm.

, CD = 12 cm. , m (∠ B) = 90°

Prove that : m (∠ ACD) = 90°



4 **[a]** Determine the type of Δ ABC according to its angles where :

AB = 7 cm. , BC = 8 cm. , AC = 10 cm.

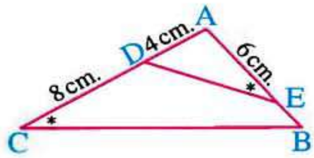
[b] In the opposite figure :

m (∠ AED) = m (∠ C) , AD = 4 cm.

, DC = 8 cm. , AE = 6 cm.

Prove that : Δ ADE ~ Δ ABC

, then find the length of $\overline{AB}$

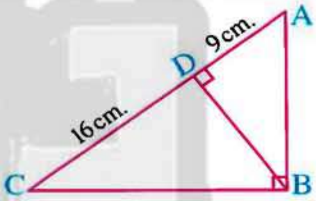


5 **[a] In the opposite figure :**

m (∠ B) = 90°

, $\overline{BD} \perp \overline{AC}$, AD = 9 cm. , CD = 16 cm.

Calculate : The length of each of $\overline{AB}$, $\overline{BC}$, $\overline{BD}$



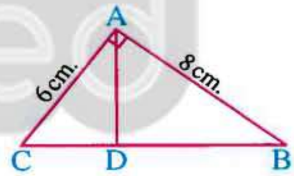
[b] In the opposite figure :

Δ DBA ~ Δ ABC

Prove that : $\overline{AD} \perp \overline{BC}$

, if AB = 8 cm. , AC = 6 cm.

Find : The length of $\overline{BD}$



Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1 If two triangles are similar and the ratio between the lengths of two corresponding sides is 3 : 4 and the perimeter of the smaller triangle is 36 cm. , then the perimeter of the greater triangle = ········· cm.

(a) 9 (b) 12 (c) 24 (d) 48

2 In Δ ABC if $(AC)^2 > (AB)^2 + (BC)^2$, then ∠ B is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

3 The length of the projection of a line segment on a given straight line .... the length of the line segment.

(a) > (b) ≥ (c) ≤ (d) =

4 In the right-angled triangle, the perpendicular from the vertex of the right angle to the hypotenuse divides it into two .... triangles

(a) congruent (b) similar (c) acute-angled (d) obtuse-angled

5 ABC is right-angled triangle at A, AB = AC = 4 cm., $\overline{AD} \perp \overline{BC}$ and cuts it at D, then AD = ... cm.

(a) $2\sqrt{2}$ (b) 5 (c) 2 (d) 3

6 The projection of a line segment perpendicular to a given straight line is ....

(a) a point. (b) a line segment. (c) a straight line. (d) a ray.

2 Complete the following :

1 Δ ABC is right at A, $\overline{AD} \perp \overline{BC}$, D $\in \overline{BC}$, then AB × AC = ...

2 The two similar polygons their corresponding angles are ....

3 In Δ ABC : if $(AB)^2 = (AC)^2 + (BC)^2$, m ($\angle$ B) = 30°, then m ($\angle$ A) = ...

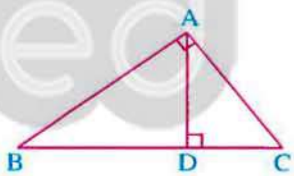
4 Two similar polygons, the ratio between the lengths of two corresponding sides is 1 : 3, If the measure of one angle in the smaller polygon is 30°, then the measure of the corresponding angle in the greater polygon is ....

5 **In the opposite figure :**

Δ ABC is right at A

, $\overline{AD} \perp \overline{BC}$, then

$(AC)^2$ = ... × ....

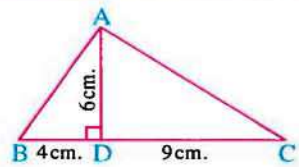


3 [a] In the opposite figure :

ABC is a triangle in which, $\overline{AD} \perp \overline{BC}$

, AD = 6 cm., BD = 4 cm., DC = 9 cm.

Prove that : m ($\angle$ BAC) = 90°

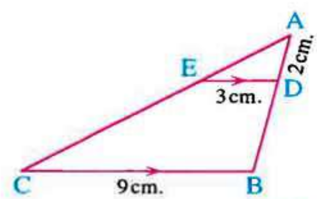


[b] In the opposite figure :

ABC is a triangle in which, $\overline{DE}$ // $\overline{BC}$

, DE = 3 cm., BC = 9 cm., AD = 2 cm.

Find : The length of $\overline{DB}$



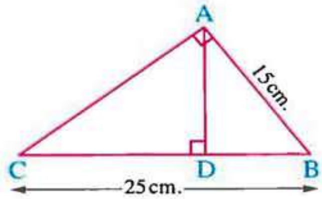
Unit 5

4 [a] **In the opposite figure :**

ABC is a triangle in which

m (∠ BAC) = 90° , $\overline{AD} \perp \overline{BC}$

Find the length of each of : $\overline{BD}$, $\overline{DC}$, $\overline{AD}$



[b] Determine the type of the greatest angle in Δ ABC where :

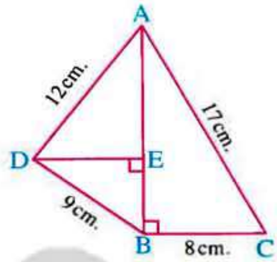
AB = 8 cm. , BC = 9 cm. , AC = 12 cm.

5 [a] **In the opposite figure :**

$\overline{DE} \perp \overline{AB}$, m (∠ ABC) = 90°

, AD = 12 cm. , AC = 17 cm.

, BC = 8 cm. , DB = 9 cm.



1 **Prove that :** m (∠ ADB) = 90°

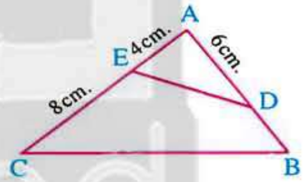
2 **Find :** The length of $\overline{DE}$

3 **Find :** The length of the projection of $\overline{AD}$ on $\overleftrightarrow{AB}$

[b] **In the opposite figure :**

If Δ ABC ~ Δ AED

Find : The length of $\overline{AB}$



TIMSS Problems

Accumulative basic skills

1 Choose the correct answer from the given ones :

1. If the side length of a square is $2\sqrt{2}$ cm. , then its area equals ....
   (a) 4 cm^2 (b) 8 cm^2 (c) $4\sqrt{2}$ cm^2 (d) $8\sqrt{2}$ cm^2

2. If the lengths of two sides of a triangle are 3 cm. and 7 cm. , then which of the following can not be the length of the third side ?
   (a) 7 cm. (b) 8 cm. (c) 9 cm. (d) 3 cm.

3. A circle , its area is 64π cm^2 , then its circumference equals ....
   (a) 8 cm. (b) 8π cm. (c) 16π cm. (d) 32π cm.

4. If ABC is a triangle in which m $(\angle A) = 3x°$, m $(\angle B) = 5x°$, m $(\angle C) = 4x°$, then m $(\angle B)$ = ...
   (a) 15° (b) 45° (c) 75° (d) 60°

5. If the sum of measures of the interior angles of a regular polygon is 720° and the length of one of its sides is 3 cm. , then the perimeter of this polygon = ...
   (a) 9 cm. (b) 12 cm. (c) 15 cm. (d) 18 cm.

6. If the height of a triangle equals half the length of its base , if the length of its base is ℓ cm. , then the area of this triangle = ...
   (a) $\frac{1}{2}\ell$ cm^2 (b) $\frac{1}{2}\ell^2$ cm^2 (c) $\frac{1}{4}\ell$ cm^2 (d) $\frac{1}{4}\ell^2$ cm^2

7. If the perimeter of a square equals $(3x - 4)$ cm. and the area of this square equals 25 cm^2 , then x = ...
   (a) 5 (b) 6 (c) 8 (d) 20

8. If the area of one face of a cube equals 9 cm^2 , then the volume of this cube equals ....
   (a) 9 cm^3 (b) 27 cm^3 (c) 36 cm^3 (d) 81 cm^3

9. In the parallelogram ABCD , if $\angle$ A is acute , then $\angle$ C is ....
   (a) acute. (b) obtuse. (c) right. (d) reflex.

Basic skills

10 The number of diagonals of the pentagon equals ....

(a) 3 (b) 5 (c) 7 (d) 9

11 The image of the point (– 1 , 3) by the translation (4 , – 2) is ....

(a) (3 , – 1) (b) (3 , 1) (c) (5 , 1) (d) (5 , – 5)

12 The measure of the angle of the regular octagon equals ....

(a) 108° (b) 120° (c) 135° (d) 144°

13 A rectangle , its length is 4 cm. and its width is 3 cm. , then the length of its diagonals equals ....

(a) 14 cm. (b) 12 cm. (c) 7 cm. (d) 5 cm.

14 The best unit for measuring the length of football playground is ....

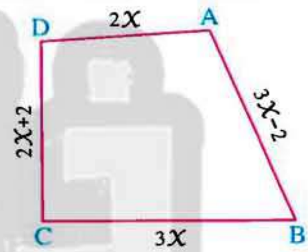
(a) metre. (b) squaremetre. (c) centimetre. (d) kilometre.

15 The ratio between the side length of a rhombus and its perimeter equal ....

(a) 1 : 1 (b) 1 : 2 (c) 1 : 4 (d) 4 : 1

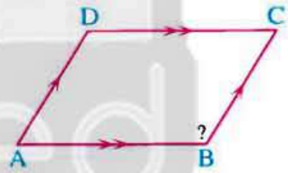
2 Complete the following :

1 If the perimeter of the opposite figure = 60 cm. , then the length of $\overline{AB}$ = ... cm.



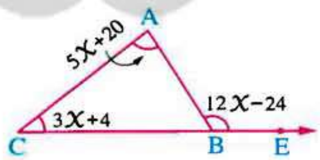
2 **In the opposite figure :**

If ABCD is parallelogram , m (reflex ∠ A) = 300° , then m (∠ B) = ...



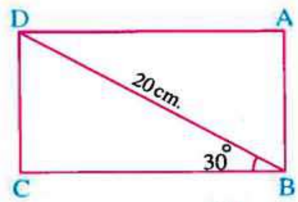
3 **In the opposite figure :**

If E $\in \overrightarrow{CB}$, m (∠ A) = $(5x + 20)°$, m (∠ C) = $(3x + 4)°$, m (∠ ABE) = $(12x - 24)°$, then the value of x = ...



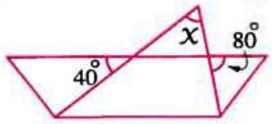
4 **In the opposite figure :**

ABCD is a rectangle , the length of its diagonal $\overline{BD}$ equals 20 cm. , m (∠ DBC) = 30° , then the perimeter of the rectangle ABCD = ... cm.



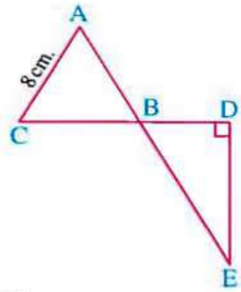
5 **In the opposite figure :**

The value of x = ...°



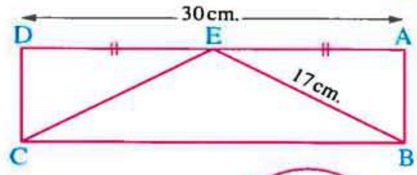
6 **In the opposite figure :**

If ABC is an equilateral triangle in which AC = 8 cm. , $\overline{AE} \cap \overline{DC} = \{B\}$, m (∠ D) = 90° , if the length of $\overline{AE}$ = 20 cm. , then the length of $\overline{CD}$ = ... cm.



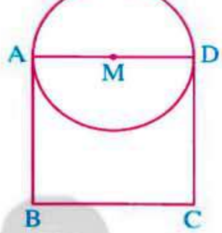
7 **In the opposite figure :**

IF ABCD is a rectangle in which AD = 30 cm. , E ∈ $\overline{AD}$ where AE = DE , if BE = 17 cm. , then the area of Δ EBC = ... cm^2



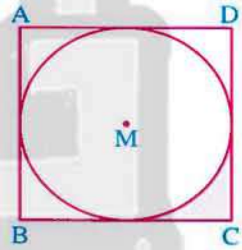
8 **In the opposite figure :**

ABCD is a square , M is a circle where $\overline{AD}$ is a diameter in the circle M , if the area of the circle is 49 π cm^2 , then the perimeter of the square equals .... cm.



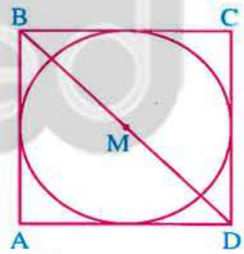
9 **In the opposite figure :**

If M is a circle touches the sides of the square ABCD , if the radius length of the circle equals 14 cm. , then the area of the shaded part equals .... cm^2. (Consider $\pi = \frac{22}{7}$)



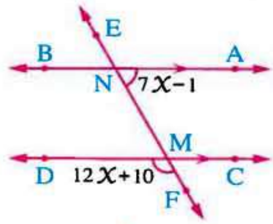
10 **In the opposite figure :**

A circle M is drawn inside the square ABCD , if the area of the circle is 25 π cm^2 , then the length of the diagonal $\overline{BD}$ of the square equals .... cm.



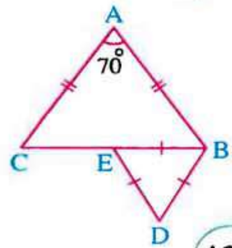
11 **In the opposite figure :**

If $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$, $\overleftrightarrow{EF}$ is a transversal to them , m (∠ANM) = $(7x - 1)°$, m (∠ DMF) = $(12x + 10)°$, then m (∠ ANE) = ... °



12 **In the opposite figure :**

Δ BDE is an equilateral , AB = AC and m (∠ A) = 70° , then m (∠ ABD) = ... °



Maths

By a group of supervisors

NOTEBOOK

- Quizzes
- Final Revision
- Final Examinations
- Research Projects

2nd PREP.

SECOND TERM

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CONTENTS

First Algebra and Statistics

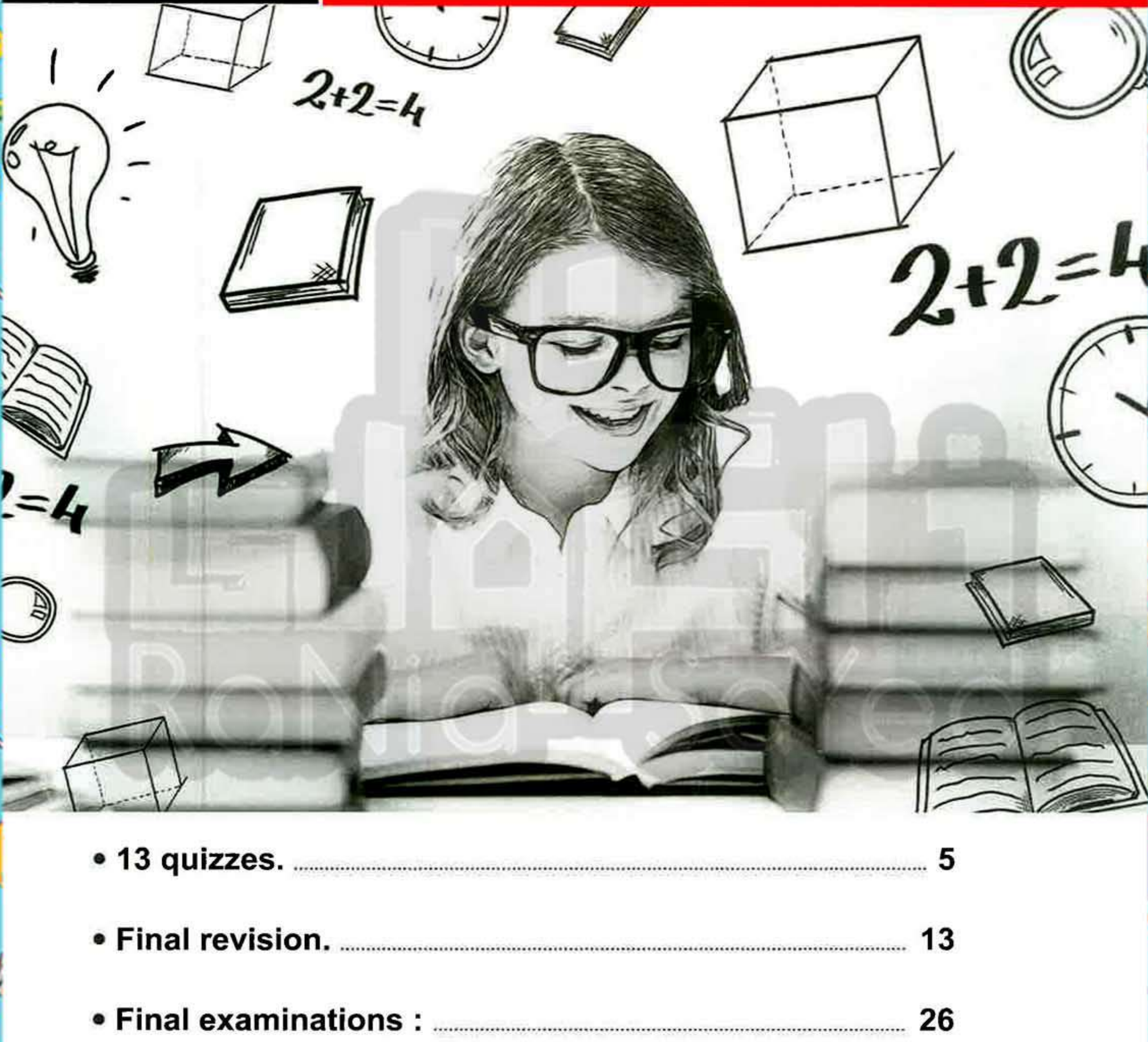
- **13 quizzes.**
- **Final revision**
- **Final examinations :**
  - School book examinations. (2 model examinations + model for the merge students)
  - 15 schools examinations of the previous years.

Second Geometry

- **10 quizzes.**
- **Final revision.**
- **Final examinations :**
  - School book examinations (2 model examinations + model for the merge students)
  - 15 schools examinations of the previous years.

Research projects on the second term

First Algebra and Statistics



Quizzes

on Algebra and Statistics

Quiz 1 on lesson 1 – unit 1

time 20 min.

1 Choose the correct answer from the given ones :

1 $x^2 - x - 12 =$ ....

(a) $(x-3)(x+4)$ (b) $(x-3)(x-4)$

(c) $(x+3)(x-4)$ (d) $(x-6)(x+2)$

2 If $x - y = 3$, $x - 2y = 5$, then $x^2 - 3xy + 2y^2 =$ ....

(a) 15 (b) 8 (c) 2 (d) – 2

3 The expression : $x^2 - x - a$ can be factorized if a = ...

(a) 3 (b) 4 (c) 5 (d) 6

2 [a] Factorize each of the following :

1 $x^2 - 5xy - 24y^2$ 2 $3x^2 - 15x + 18$

[b] Find the value of $k \in \mathbb{Z}$ such that each of the following expressions can be factorized :

1 $x^2 + kx - 12$ 2 $x^2 - 7x + k$

Quiz 2 till lesson 2 – unit 1

time 20 min.

1 Choose the correct answer from the given ones :

1 $6x^2 - 7x - 3 =$ ....

(a) $(3x-1)(2x-3)$ (b) $(3x+1)(2x-3)$

(c) $(3x+1)(2x+3)$ (d) $(3x-1)(2x+3)$

2 If $2x^2 + cx - 3 = (2x-1)(x+3)$, then c = ...

(a) 5 (b) – 5 (c) 7 (d) – 7

3 If $2x + 3y = 7$, $2x^2 + xy - 3y^2 = 28$, then $\frac{1}{2}x - \frac{1}{2}y =$ ....

(a) 4 (b) 2 (c) 1 (d) 8

2 [a] A rectangle is of area $(2x^2 + 19x + 35)$ cm².

Find the two possible dimensions of it in terms of x , then find its perimeter when $x = 1$

[b] Factorize each of the following :

1 $x^2 - 7x + 6$ 2 $2(x^2 - 2) - 7x$

Quiz 3 till lesson 3 – unit 1

time 20 min.

1 Choose the correct answer from the given ones :

1 If $x^2 + kx + 16$ is a perfect square , then k = ...

(a) 4 (b) ± 4 (c) ± 8 (d) 1

2 If $x^2 - 2xy + y^2 = 25$, then $x - y$ = ...

(a) 25 (b) – 5 (c) 5 (d) ± 5

3 If $(x + 1)$ is a factor of the expression : $5x^2 - 2x - 7$, then the other factor is ....

(a) $(5x - 7)$ (b) $(5x + 7)$ (c) $(x + 7)$ (d) $(x - 7)$

2 [a] Factorize each of the following expressions perfectly :

1 $x^2 - 7x + 10$

2 $2x^3 - 20x^2 + 50x$

[b] Use factorization to facilitate finding the value of : $(8.3)^2 + 2 \times 8.3 \times 1.7 + (1.7)^2$

Quiz 4 till lesson 4 – unit 1

time 20 min.

1 Choose the correct answer from the given ones :

1 If $x^2 - y^2 = 16$, $x + y = 8$, then $x - y$ = ...

(a) 2 (b) 1 (c) 128 (d) 64

2 If a + b = 8 , b – a = – 5 , then $a^2 - b^2$ = ...

(a) – 40 (b) 40 (c) 13 (d) – 13

3 If $x - y = 4$, $x^2 + 2xy + y^2 = 81$, then $x^2 - y^2$ = ...

(a) ± 9 (b) 18 (c) 81 (d) ± 36

2 [a] Factorize the following perfectly :

1 $2x^2 + x - 6$

2 $3x^2 - 27$

3 $9b^2(a - 5b) - 16(a - 5b)$

[b] Use factorization to facilitate getting the value of : $(99)^2 - 1$

Quiz 5 till lesson 5 – unit 1

time 20 min.

1 Choose the correct answer from the given ones :

1. $(x^3 + 64) = (x + 4)$ (........................)
   (a) $x^2 + 16$ (b) $x^2 - 4x + 16$
   (c) $x^2 + 4x + 16$ (d) $x^2 - 4x - 16$
2. If $a - b = 5$, then $a^2 - 2ab + b^2 =$ ....
   (a) 25 (b) 20 (c) 15 (d) 10
3. If $x + y = 2$, $x^2 - xy + y^2 = 8$, then $x^3 + y^3 =$ ....
   (a) 4 (b) 8 (c) 64 (d) 16

2 [a] Factorize each of the following perfectly :

1. $(x + 3)^2 - 25$
2. $x^3 - 27$
3. $2a^2 - 7ab + 6b^2$

[b] Factorize the following expression perfectly : $(x^3 - 9)(x^3 + 9) + 17$

Quiz 6 till lesson 6 – unit 1

time 20 min.

1 Choose the correct answer from the given ones :

1. If $a + 2b = 4$, $x + y = 8$, then the numerical value of the expression $ax + ay + 2bx + 2by =$ ....
   (a) 2 (b) 12 (c) 4 (d) 32
2. The expression : $x^2 - 6x + k$ is a perfect square when $k =$ ....
   (a) 36 (b) 9 (c) 3 (d) 12
3. $x^2 - y^2 - x - y = (x + y)$ (..............)
   (a) $x + y + 1$ (b) $x - y - 1$ (c) $x - y + 1$ (d) $x - y$

2 [a] Factorize each of the following perfectly :

1. $\frac{1}{2}x^3 - 4$
2. $4x^2 + 4xy + y^2 - 9$
3. $x^3 - x - y + y^3$

[b] Factorize the expression : $x^6 - y^6$ perfectly.

Quiz 7 till lesson 7 – unit 1

time 20 min.

1 Choose the correct answer from the given ones :

1. If $(x + y)^2 = 25$, $xy = 5$, then $x^2 + y^2 =$ ....

(a) 10 (b) 15 (c) 20 (d) 25

2. If $2x^2 - cx - 2 = (2x + 1)(x - 2)$, then c = ...

(a) 1 (b) 2 (c) 3 (d) 4

3. The side length of the square whose area is $(x^2 + 6x + 9)$ square units equals .... length units. (where $x > -3$)

(a) $x + 3$ (b) $x + 9$ (c) $x^2 + 3$ (d) $x + 6$

2 Factorize each of the following perfectly :

1. $20x^4 + 5y^4$
2. $x^2(x^2 - 5y^2) + 4y^4$
3. $3x^3 - 75x$
4. $xy - 5x - 6y + 30$

Quiz 8 till lesson 8 – unit 1

time 20 min.

1 Choose the correct answer from the given ones :

1. The solution set of the equation : $x(x - 3) = 0$ in $\mathbb{R}$ is ....

(a) $\{3\}$ (b) $\{0, 3\}$ (c) $\{0, -3\}$ (d) $\{0\}$

2. If the number 4 is a solution of the equation : $x^2 + x - 20 = 0$, then the other solution is ....

(a) 20 (b) 5 (c) -5 (d) -4

3. The solution set of the equation : $x^2 + 25 = 0$ in $\mathbb{R}$ is ....

(a) 5 (b) -5 (c) ± 5 (d) $\varnothing$

2 [a] Factorize each of the following perfectly :

1. $5x^2 - 4x - 12$
2. $a^3 - 9a + a^2 - 9$

[b] Find in $\mathbb{R}$ the solution set of each of the following two equations :

1. $x^2 - 7x - 30 = 0$
2. $x - \frac{3}{x} = 2$

Quiz 9 till lesson 9 – unit 1

time 20 min.

1 Choose the correct answer from the given ones :

1 If the age of Ayman 5 years ago was x years , then the square of his age now = ...

(a) $x^2 + 5$ (b) $x^2 + 25$ (c) $(x + 5)^2$ (d) $(x - 5)^2$

2 Twice the square of the number x is ....

(a) $2x$ (b) $2x^2$ (c) $(2x)^2$ (d) $4x$

3 If $x^2 + kxy - 25y^2 = (x + 5y)(x - 5y)$, then k = ...

(a) zero (b) 10 (c) -10 (d) -25

2 [a] Find the real number whose double exceeds its multiplicative inverse by one.

[b] A right-angled triangle, the lengths of the two sides of the right angle are $4x$ cm. and $x + 1$ cm. If the area of the triangle = 84 cm^2 , calculate the length of its hypotenuse.

Quiz 10 till lesson 1 – unit 2

time 20 min.

1 Choose the correct answer from the given ones :

1 If $2^x = 5$, then 2^{x+2} = ...

(a) 10 (b) 15 (c) 20 (d) 2

2 If $(x - 5)^0 = 1$, then $x \in$ ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{-5\}$ (c) $\mathbb{R} - \{5\}$ (d) $\mathbb{R}^*$

3 If $x - y = 3$, $x^2 + xy + y^2 = 6$, then $x^3 - y^3$ = ...

(a) 18 (b) 2 (c) 3 (d) 9

2 [a] **Prove that :** $\frac{(27)^{x-1} \times 8^x}{\left(2\sqrt{2}\right)^{2x} \times \left(3\sqrt{3}\right)^{2x}} = \frac{1}{27}$

[b] The length of a rectangle exceeds its width by 5 m. , if its area = 84 m^2.
Find the two dimensions of the rectangle and its perimeter.

Quiz 11 till lesson 2 – unit 2

time 20 min.

1 Choose the correct answer from the given ones :

1. If $3^x = 27$, then $2^x - 3 = \dots\dots$

(a) 3 (b) 9 (c) 8 (d) 5

2. If $2^x = 5$, then $8^x = \dots\dots$

(a) 8 (b) 125 (c) 8^3 (d) 25

3. If $3^{x-2} = 1$, then $x = \dots\dots$

(a) 2 (b) 3 (c) 0 (d) 1

2 [a] Factorize each of the following perfectly :

1. $2x^2 + x - 6$
2. $8x^3 - 27$

[b] Find the value of n if : $\frac{(14)^{2n} \times 4^{n+1}}{4 \times 7^n \times (16)^n} = 49$

Quiz 12 till lesson 3 – unit 2

time 20 min.

1 Choose the correct answer from the given ones :

1. If $6^x = 7$, then $6^{x+1} = \dots\dots$

(a) 8 (b) 13 (c) 36 (d) 42

2. If $3^x = 5$, $\frac{1}{3^y} = 7$, then $3^{x+y} = \dots\dots$

(a) $\frac{5}{7}$ (b) $\frac{7}{5}$ (c) 2 (d) 12

3. $\frac{3^x \times 3^x \times 3^x}{3^x + 3^x + 3^x} = \dots\dots$

(a) 3^{2x-1} (b) 3^{1-2x} (c) $3^{x^3 - 3x}$ (d) 3^3

2 [a] Find in ℝ the solution set of each of the following equation :

1. $x = \frac{1}{x}$
2. $3^{x-2} = 81$

[b] If $\frac{8^x \times 9^x}{(18)^x} = 64$, **find the value of :** 4^{-x}

Quiz 13 till lesson of probability - unit (3)

time 20 min.

1 Choose the correct answer from the given ones :

1 The S.S. of the equation : $2x(x-3)=0$ in $\mathbb{R}$ is ....

(a) $\{2, 3\}$ (b) $\{1, 3\}$ (c) $\{\frac{1}{2}, -3\}$ (d) $\{0, 3\}$

2 If $3^x = 5$, then $(27)^x =$ ....

(a) 9 (b) 25 (c) 125 (d) 729

3 As throwing a fair dice once, then the probability of appearing a number ≤ 4 is ....

(a) $\frac{3}{6}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{5}{6}$

2 [a] Factorize each of the following completely :

1 $27x^3 + 8y^3$

2 $6a^2 - 19ab - 7b^2$

[b] A club plays 30 games in the general league. If the probability of draw in one of the games is 0.3 and the probability of winning is 0.6

Find : 1 The number of games expected the club to draw.

2 The number of games expected the club to lose.

Final Revision

of Algebra and Statistics

Revision for the important rules of algebra and statistics

Remember — The steps of factorizing the algebraic expressions

1. Take out the H.C.F. from the terms of the algebraic expression (if it exists).
2. Determine the factorizing method according to the number of terms of the algebraic expression as shown in the following diagram :

Algebraic expression

- **(Binomial expression)** Formed from two terms
  - **Difference between two squares :** $a^2 - b^2 = (a + b)(a - b)$
  - **Sum or difference between two cubes :** $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
  - **Completing the square :** In this method we put the expression in the form : a perfect square trinomial – a perfect square monomial.
- **(Trinomial expression)** Formed from three terms
  - **Perfect square :** $a x^2 + b x + c = (\square x + \square)^2$
  - **In the form :** $x^2 + b x + c = (x + \square)(x + \square)$
  - **In the form :** $a x^2 + b x + c = (\square x + \square)(\square x + \square)$
  - **Completing the square :** In this method we put the expression in the form : a perfect square trinomial – a perfect square monomial.
- **Formed from four terms**
  - **Factorizing by grouping :** In this method we divide the expression into two expressions each of them formed from two terms or a perfect square trinomial and a perfect square monomial.

- And in the following , some examples which help you to remember the different factorizing methods :

1 Taking out the H.C.F : $ab + ac = a(b + c)$

- $6x^2y + 10xy^2 = 2xy(3x + 5y)$
- $2a(x + y) - b(x + y) = (x + y)(2a - b)$

Notice that : The H.C.F. an algebraic expression $(x + y)$

2 Difference between two squares : $a^2 - b^2 = (a + b)(a - b)$

- $x^2 - 9 = (x + 3)(x - 3)$
- $16x^4 - 81 = (4x^2 + 9)(4x^2 - 9) = (4x^2 + 9)(2x + 3)(2x - 3)$

  Notice : We continue in factorizing to be completely.
- $2x^3 - 72x = 2x(x^2 - 36) = 2x(x + 6)(x - 6)$

  Notice : We took out the H.C.F. firstly.

3 Sum of two cubes : $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Difference between two cubes : $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

- $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$
- $3x^4y - 81xy^4 = 3xy(x^3 - 27y^3) = 3xy\ ,(x - 3y)(x^2 + 3xy + 9y^2)$

4 Trinomial in the form : $(x^2 + bx + c)$

- $x^2 + 7x + 12 = (x + 3)(x + 4)$ (Their product = 12 , Their sum = 7)
- $x^2 + x - 12 = (x + 4)(x - 3)$ (Their product = −12 , Their sum = 1)

5 Trinomial in the form : $(ax^2 + bx + c)$

- $6x^2 + 7x + 2$

  $= (2x + 1)(3x + 2)$ (Their product = 6 , Their product = 2)
- $36x^3 - 84x^2 - 15x = 3x(12x^2 - 28x - 5)$

  $= 3x(6x + 1)(2x - 5)$

Method of scissors

$(2x + 1)$	$(6x + 1)$
$(3x + 2)$	$(2x - 5)$

6 Perfect square trinomial :

- $a^2 + 2ab + b^2 = (a + b)^2$
- $a^2 - 2ab + b^2 = (a - b)^2$

In the perfect square trinomial

- Each of the first term and the third term is a perfect square and its sign is positive.
- The middle term $= \pm 2 \times \sqrt{\text{the first term}} \times \sqrt{\text{the third term}}$

- $x^2 + 10x + 25 = (x + 5)^2$
- $9x^2 - 24xy + 16y^2 = (3x - 4y)^2$
- $4x^2 - 10x + 25$ is not a perfect square because : the middle term $\neq \pm 2 \times \sqrt{4x^2} \times \sqrt{25}$
- $16a^2 - 24a - 9$ is not a perfect square because the third term is negative.
- $12b^2 - 16b + 4$ is not a perfect square because the first term ($12b^2$) is not a perfect square.

7 Factorizing by grouping :

- $ax + ay + bx + by$
  $= a(x + y) + b(x + y)$
  $= (x + y)(a + b)$

Notice that : We divided the expression into two expressions , each one of them formed from two terms.

- $x^2 - y^2 + 2x - 2y = (x - y)(x + y) + 2(x - y) = (x - y)(x + y + 2)$
- $x^2 - 2xy + y^2 - 9$
  $= (x^2 - 2xy + y^2) - 9$
  $= (x - y)^2 - (3)^2$
  $= (x - y + 3)(x - y - 3)$

Notice that : We divided the expression into a perfect square trinomial and a perfect square monomial.

8 Completing the square :

- $x^4 + 4y^4$
  $= (x^4 + 4x^2y^2 + 4y^4) - 4x^2y^2$
  $= (x^2 + 2y^2)^2 - (2xy)^2$
  $= (x^2 + 2y^2 + 2xy)$
  $(x^2 + 2y^2 - 2xy)$

Notice that : We added : $2 \times \sqrt{x^4} \times \sqrt{4y^4}$
i.e. $4x^2y^2$ to the expression , then subtract it to make the expression in the form :
a perfect square trinomial – a perfect square monomial

- $x^4 - 19x^2y^2 + 9y^4$ by adding and subtracting : $2 \times \sqrt{x^4} \times \sqrt{9y^4}$

i.e. by adding and subtracting : $6x^2y^2$

$\therefore x^4 - 19x^2y^2 + 9y^4 + 6x^2y^2 - 6x^2y^2 = \underbrace{x^4 + 6x^2y^2 + 9y^4}_{\text{A perfect square trinomal}} - \underbrace{25x^2y^2}_{\text{A perfect square}}$

$= (x^2 + 3y^2)^2 - (5xy)^2 = (x^2 + 3y^2 - 5xy)(x^2 + 3y^2 + 5xy)$

Remember — Solving quadratic equations in one variable algebraically

1 If the equation in the form : $(x-\ell)(x-m)=0$ where ℓ and m are real numbers

, then : $x-\ell=0$ *i.e.* $x=\ell$ or $x-m=0$ *i.e.* $x=m$

$\therefore$ The solution set of the equation in $\mathbb{R}=\{\ell , m\}$

For example :

- If $(x+5)(x-3)=0$

, then : $x+5=0$ *i.e.* $x=-5$ or $x-3=0$ *i.e.* $x=3$

$\therefore$ The solution set in $\mathbb{R}=\{-5 , 3\}$

- If $x(2x+7)=0$

, then : $x=0$ or $2x+7=0$ *i.e.* $2x=-7$ $\therefore x=-\frac{7}{2}$

$\therefore$ The solution set in $\mathbb{R}=\left\{0 , -\frac{7}{2}\right\}$

2 If the equation in the form : $ax^2+bx+c=0$

, then we use the factorizing to put the equation in the form :

$(x-\ell)(x-m)=0$, then we find the solution set as in the previous.

For example :

Notice that : We used the factorizing of the quadratic trinomial to put the equation in the form : $(x-\ell)(x-m)=0$

- If $x^2-5x-6=0$

, then : $(x-6)(x+1)=0$

$\therefore x-6=0$ *i.e.* $x=6$ or $x+1=0$ *i.e.* $x=-1$

$\therefore$ The solution set in $\mathbb{R}=\{6 , -1\}$

- If $x(x-1)=6$

, then : $x^2-x=6$

Notice that : Before the factorizing , we should put the equation in the form : $ax^2+bx+c=0$

$\therefore x^2-x-6=0$

$\therefore (x-3)(x+2)=0$

$\therefore x-3=0$ *i.e.* $x=3$ or $x+2=0$ *i.e.* $x=-2$

$\therefore$ The solution set in $\mathbb{R}=\{3 , -2\}$

Remember that

We can use the quadratic equation in one variable in solving some word problems , and the following table helps you to form the quadratic equation using the given information in the problem :

If	Then
A number = x	• Its half = $\frac{1}{2}x$ • Its third = $\frac{1}{3}x$ • Its twice = $2x$ • Its three times = $3x$ • Its square = x^2 • Twice its square = $2x^2$ • Square its double = $(2x)^2 = 4x^2$ • Its additive inverse = $-x$ • Its multiplicative inverse = $\frac{1}{x}$, where $x \neq 0$
Two numbers , one of them exceeds the other by 5 or one of them is less than the other by 5 or their difference = 5	• The first number = x • The second number = $x + 5$
The sum of two numbers equals 5	• The first number = x • The second number = $5 - x$
Two numbers , one of them is more than twice the other by 5	• The first number = x • The second number = $2x + 5$
Three consecutive integers	• The first number = x • The second number = $x + 1$ • The third number = $x + 2$
Three even (or odd) consecutive numbers	• The first number = x • The second number = $x + 2$ • The third number = $x + 4$
Two numbers , the ratio between them is 2 : 3	• The first number = $2x$ • The second number = $3x$
The age of a man now is x years	• His age after 4 years = $x + 4$ • His age 3 years ago = $x - 3$ • The square of his age 6 years ago = $(x - 6)^2$
A rectangle whose length exceeds its width by 5 cm.	• The width = x cm. , the length = $(x + 5)$ cm. • Its perimeter = $(x + x + 5) \times 2 = (4x + 10)$ cm. • Its area = $x(x + 5) = (x^2 + 5x)$ cm^2.
A square of side length = x cm.	• Its perimeter = $4x$ cm. • Its area = x^2 cm^2.

Remember — The laws of integer powers in $\mathbb{R}$

If a and b are two real numbers , m and n are two integers , except the cases in which the denominator = 0 and the cases in which both the base and the power = 0 , then :

The law	Example	The explain
1 $a^m \times a^n = a^{m+n}$	$4^3 \times 4^2 = 4^{3+2} = 4^5$	When we multiply numbers of the same base , we add the powers.
2 $\frac{a^m}{a^n} = a^{m-n}$	$\frac{3^6}{3^2} = 3^{6-2} = 3^4$	When we divide numbers of the same base , we subtract the powers.
3 $(ab)^n = a^n b^n$	$(3 \times 4)^2 = 3^2 \times 4^2$	The power of a product of two numbers is distributed over the two factors.
4 $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3}$	The power of a quotient of two numbers is distributed over the numerator and the denominator.
5 $(a^m)^n = (a^n)^m = a^{mn}$	$(4^2)^3 = (4^3)^2 = 4^{3\times2} = 4^6$	When we raise a number , raised to a power , to another power , we multiply the two powers.

Note that :
- $a^0 = 1$ — *For example :* $(3)^0 = 1$
- $a^{-n} = \frac{1}{a^n}$ — *For example :* $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$
- $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ — *For example :* $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

Example 1

Find in the simplest form each of the following :

1. $(\sqrt{3})^7 \times (\sqrt{3})^{-9} \times (\sqrt{3})^4$
2. $\frac{\sqrt{2}}{(\sqrt{2})^{-2}}$
3. $(2\sqrt{5} \times \sqrt{3})^{-2}$
4. $\left(\frac{3\sqrt{2}}{\sqrt{3}}\right)^4$
5. $((\sqrt{3})^{-2})^2$

Solution

1. $(\sqrt{3})^7 \times (\sqrt{3})^{-9} \times (\sqrt{3})^4 = (\sqrt{3})^{7+(-9)+4} = (\sqrt{3})^2 = 3$

2. $\frac{\sqrt{2}}{(\sqrt{2})^{-2}} = (\sqrt{2})^{1-(-2)} = (\sqrt{2})^3 = 2\sqrt{2}$

3. $\left(2\sqrt{5}\times\sqrt{3}\right)^{-2} = 2^{-2}\times\left(\sqrt{5}\right)^{-2}\times\left(\sqrt{3}\right)^{-2}$

$= \frac{1}{2^2}\times\frac{1}{\left(\sqrt{5}\right)^2}\times\frac{1}{\left(\sqrt{3}\right)^2} = \frac{1}{4}\times\frac{1}{5}\times\frac{1}{3} = \frac{1}{60}$

4. $\left(\frac{3\sqrt{2}}{\sqrt{3}}\right)^4 = \frac{\left(3\sqrt{2}\right)^4}{\left(\sqrt{3}\right)^4} = \frac{3^4\times\left(\sqrt{2}\right)^4}{\left(\sqrt{3}\right)^4} = \frac{81\times 4}{9} = 36$

Another solution :

$\because 3 = \sqrt{3}\times\sqrt{3}$

$\therefore \left(\frac{3\sqrt{2}}{\sqrt{3}}\right)^4 = \left(\frac{\sqrt{3}\times\sqrt{3}\times\sqrt{2}}{\sqrt{3}}\right)^4 = \left(\sqrt{3}\right)^4\times\left(\sqrt{2}\right)^4 = 9\times 4 = 36$

5. $\left(\left(\sqrt{3}\right)^{-2}\right)^2 = \left(\sqrt{3}\right)^{-2\times 2} = \left(\sqrt{3}\right)^{-4} = \frac{1}{\left(\sqrt{3}\right)^4} = \frac{1}{9}$

Example 2

Find each of the following to the simplest form :

1. $\frac{\sqrt{3}\times\left(2\sqrt{3}\right)^2\times\left(-\sqrt{3}\right)^5}{\left(2\sqrt{3}\right)^4}$

2. $\frac{\left(\sqrt{18}\right)^5\times\left(\sqrt{2}\right)^3}{\left(\sqrt{12}\right)^4}$

Solution

1. $\frac{\sqrt{3}\times\left(2\sqrt{3}\right)^2\times\left(-\sqrt{3}\right)^5}{\left(2\sqrt{3}\right)^4} = \frac{\sqrt{3}\times 2^2\times\left(\sqrt{3}\right)^2\times -\left(\sqrt{3}\right)^5}{2^4\times\left(\sqrt{3}\right)^4}$

$= -\left(\sqrt{3}\right)^{1+2+5-4}\times 2^{2-4}$

$= -\left(\sqrt{3}\right)^4\times 2^{-2} = -9\times\frac{1}{2^2} = -\frac{9}{4}$

2. $\frac{\left(\sqrt{18}\right)^5\times\left(\sqrt{2}\right)^3}{\left(\sqrt{12}\right)^4} = \frac{\left(3\sqrt{2}\right)^5\times\left(\sqrt{2}\right)^3}{\left(2\sqrt{3}\right)^4}$

$= \frac{3^5\times\left(\sqrt{2}\right)^5\times\left(\sqrt{2}\right)^3}{2^4\times\left(\sqrt{3}\right)^4} = \frac{3^5\times\left(\sqrt{2}\right)^{5+3}}{2^4\times 3^2}$

$= \frac{3^5\times\left(\sqrt{2}\right)^8}{2^4\times 3^2} = \frac{3^5\times 2^4}{2^4\times 3^2} = 3^{5-2}\times 2^{4-4}$

$= 3^3\times 2^0 = 27\times 1 = 27$

Remember that

- $\sqrt{18} = \sqrt{9\times 2} = \sqrt{9}\times\sqrt{2} = 3\sqrt{2}$
- $\sqrt{12} = \sqrt{4\times 3} = \sqrt{4}\times\sqrt{3} = 2\sqrt{3}$
- $\left(\sqrt{3}\right)^4 = \sqrt{3^4} = 3^2$
- $\left(\sqrt{2}\right)^8 = \sqrt{2^8} = 2^4$

Example 3

Simplify to the simplest form : $\frac{2^{5n} \times 3^{2n+1}}{4^{-n} \times 6^{2n+1}}$, then find the value of the result when n = 1

Solution

$$\frac{2^{5n} \times 3^{2n+1}}{4^{-n} \times 6^{2n+1}} = \frac{2^{5n} \times 3^{2n+1}}{(2^2)^{-n} \times (2 \times 3)^{2n+1}} = \frac{2^{5n} \times 3^{2n+1}}{2^{-2n} \times 2^{2n+1} \times 3^{2n+1}}$$

$$= 2^{5n+2n-2n-1} \times 3^{2n+1-2n-1} = 2^{5n-1} \times 3^0 = 2^{5n-1}$$

When n = 1 :

∴ The expression $= 2^{5-1} = 2^4 = 16$

Remember **Solving the exponential equations in $\mathbb{R}$**

1. If a is a real number , m and n are two integers

   and $a^m = a^n$, then $m = n$ where : $a \neq 0$, $a \neq \pm 1$

   For example : If $3^n = 9$, then $3^n = 3^2$

   , ∵ the base = the base ∴ the power = the power ∴ $n = 2$

2. If a and b are two real numbers , m is an integer and $a^m = b^m$, then :

   - $a = b$ if m is an odd number. ***For example :*** If $n^5 = 3^5$, then : n = 3
   - $a = \pm b$ if m is an even number. ***For example :*** If $n^2 = 3^2$, then : $n = \pm 3$
   - $m = \text{zero}$ if $a \neq \pm b$

   For example : If $7^{n-2} = 5^{n-2}$, then : n − 2 = 0 ∴ n = 2

Example 1

Find the value of n in each of the following :

1. $2^{n+5} = 8$
2. $9^{n-1} = \frac{1}{81}$
3. $\left(\frac{3}{5}\right)^{n+2} = \left(2\frac{7}{9}\right)^{-2}$
4. $3^{3n-6} = 5^{3n-6}$
5. $7^{n(n-3)} = 1$
6. $3^{n+2} = n^{n+2}$

Solution

1. ∵ $2^{n+5} = 8$ ∴ $2^{n+5} = 2^3$

   ∵ The base = the base ∴ The power = the power

   ∴ n + 5 = 3 ∴ $n = -2$

2 $\because 9^{n-1} = \frac{1}{81}$ $\therefore 9^{n-1} = \frac{1}{9^2} = 9^{-2}$

$\because$ The base = the base $\therefore$ The power = the power

$\therefore n - 1 = -2$ $\therefore \boxed{n = -1}$

3 $\because \left(\frac{3}{5}\right)^{n+2} = \left(2\frac{7}{9}\right)^{-2}$ $\therefore \left(\frac{3}{5}\right)^{n+2} = \left(\frac{25}{9}\right)^{-2}$

$\therefore \left(\frac{3}{5}\right)^{n+2} = \left(\frac{9}{25}\right)^{2}$ $\therefore \left(\frac{3}{5}\right)^{n+2} = \left(\left(\frac{3}{5}\right)^{2}\right)^{2}$

$\therefore \left(\frac{3}{5}\right)^{n+2} = \left(\frac{3}{5}\right)^{4}$

, $\because$ The base = the base $\therefore$ The power = the power

$\therefore n + 2 = 4$ $\therefore \boxed{n = 2}$

4 $\because 3^{3n-6} = 5^{3n-6}$, $\because$ The power = the power

$\therefore$ Either the base = the base or the power = 0

, $\because 3 \neq 5$ $\therefore 3n - 6 = 0$

$\therefore 3n = 6$ $\therefore \boxed{n = 2}$

5 $\because 7^{n(n-3)} = 1$ $\therefore 7^{n(n-3)} = 7^0$

, $\because$ The base = the base

$\therefore$ The power = the power

$\therefore n(n-3) = 0$

$\therefore$ Either $\boxed{n = 0}$ or $n - 3 = 0$, then $\boxed{n = 3}$

> *Notice that :* If $\boxed{a^n = 1}$, then : $\boxed{n = 0}$
> where : $a \neq 0$, $a \neq \pm 1$

6 $\because 3^{n+2} = n^{n+2}$ $\because$ The power = the power

$\therefore$ Either the base = the base , then $\boxed{n = 3}$

or the power = 0 , then $n + 2 = 0$ $\therefore \boxed{n = -2}$

Example 2

Find the solution set of each of the following equations in $\mathbb{R}$:

1 $\frac{(18)^n}{8^n \times 9^n} = 16$ 2 $\left(\frac{3}{2}\right)^{x^2 - x} = 2\frac{1}{4}$

Solution

1 $\because \frac{(18)^n}{8^n \times 9^n} = 16$ $\therefore \frac{(3^2 \times 2)^n}{(2^3)^n \times (3^2)^n} = 2^4$

$\therefore \frac{3^{2n} \times 2^n}{2^{3n} \times 3^{2n}} = 2^4$ $\therefore \frac{2^n}{2^{3n}} = 2^4$

$\therefore 2^{n-3n} = 2^4$ $\therefore n - 3n = 4$

$\therefore -2n = 4$ $\therefore \boxed{n = -2}$

$\therefore$ The S.S. $= \{-2\}$

2 $\because \left(\frac{3}{2}\right)^{x^2-x} = 2\frac{1}{4}$ $\therefore \left(\frac{3}{2}\right)^{x^2-x} = \frac{9}{4}$

$\therefore \left(\frac{3}{2}\right)^{x^2-x} = \left(\frac{3}{2}\right)^2$ $\therefore x^2 - x = 2$

$\therefore x^2 - x - 2 = 0$

By factorizing : $\therefore (x-2)(x+1) = 0$

$\therefore$ Either $x - 2 = 0$, then $\boxed{x = 2}$

or $x + 1 = 0$, then $\boxed{x = -1}$ $\therefore$ The S.S. $= \{2, -1\}$

Remember **The probability**

At performing an experiment , then :

The probability of occurrence of a certain event $= \frac{\text{the number of times of repeating this outcome}}{\text{the number of all possible outcomes}}$

The expected number for occurrence of a certain event
= the probability of its occurrence × the total number of given individuals

For example :

If one of the factories produces 1500 electric lamps daily , as a sample of 100 electric lamps was examined randomly in one of the days , 5 defective units were found , then :

1 Probability of production a defective unit $= \frac{\text{Number of defective units in the sample}}{\text{Number of sample units}} = \frac{5}{100} = 0.05$

2 The expected number for the defective units in that day
= probability of production a defective unit
× number of produced units in that day
$= 0.05 \times 1500 = 75$ lamps.

> We can write the probability in the percentage form. *i.e.* 5 %

- The probability of occurrence of an event $A \subset S$ is denoted by P (A)

  It is found by using the relation :

  $$P(A) = \frac{\text{the number of elements of A}}{\text{the number of elements of the sample space}} = \frac{n(A)}{n(S)}$$

- The probability of the impossible event = 0
- The probability of the certain event = 1
- The probability of any event is not less than zero and it is not more than 1

  i.e. For any event A , $0 \le P(A) \le 1$ *i.e.* $P(A) \in [0 , 1]$

  , then it has no meaning that the probability of an event is 140 % or – 2

Example 1

If a fair die is thrown once and we observe the number on the upper face , find the probability of each of the following events :

1. A is the event of appearance of a number greater than 4
2. B is the event of appearance of an even number.
3. C is the event of appearance of the number 5
4. D is the event of appearance of the number 7
5. E is the event of appearance of a number less than 7

Solution

$S = \{1 , 2 , 3 , 4 , 5 , 6\}$, n (S) = 6

1. $A = \{5 , 6\}$, n (A) = 2 $\therefore P(A) = \frac{2}{6} = \frac{1}{3}$
2. $B = \{2 , 4 , 6\}$, n (B) = 3 $\therefore P(B) = \frac{3}{6} = \frac{1}{2}$
3. $C = \{5\}$, n (C) = 1 $\therefore P(C) = \frac{1}{6}$
4. $D = \{\ \}$ or $\varnothing$, n (D) = zero

   $\therefore P(D) = \frac{0}{6}$ = zero (the impossible event)
5. $E = \{1 , 2 , 3 , 4 , 5 , 6\}$, n (E) = 6

   $\therefore P(E) = \frac{6}{6} = 1$ (the certain event) or (the sure event)

Example 2

A bag contains an amount of marbles of the same size and softness. If 2 marbles are red , 3 marbles are blue and 5 marbles are white. A marble is drawn randomly. Calculate :

1. The probability that the drawn marble is red.
2. The probability that the drawn marble is blue.
3. The probability that the drawn marble is white.
4. The probability that the drawn marble is not blue.

Solution

The probability of getting a certain occurrent

$$= \frac{\text{the number of chances of getting this occurrent}}{\text{the total number of chances}}$$

$\because$ The total number of marbles = 2 + 3 + 5 = 10 marbles.

1. The probability that the drawn marble is red
$$= \frac{\text{number of red marbles}}{\text{total number of marbles}} = \frac{2}{10} = \frac{1}{5}$$

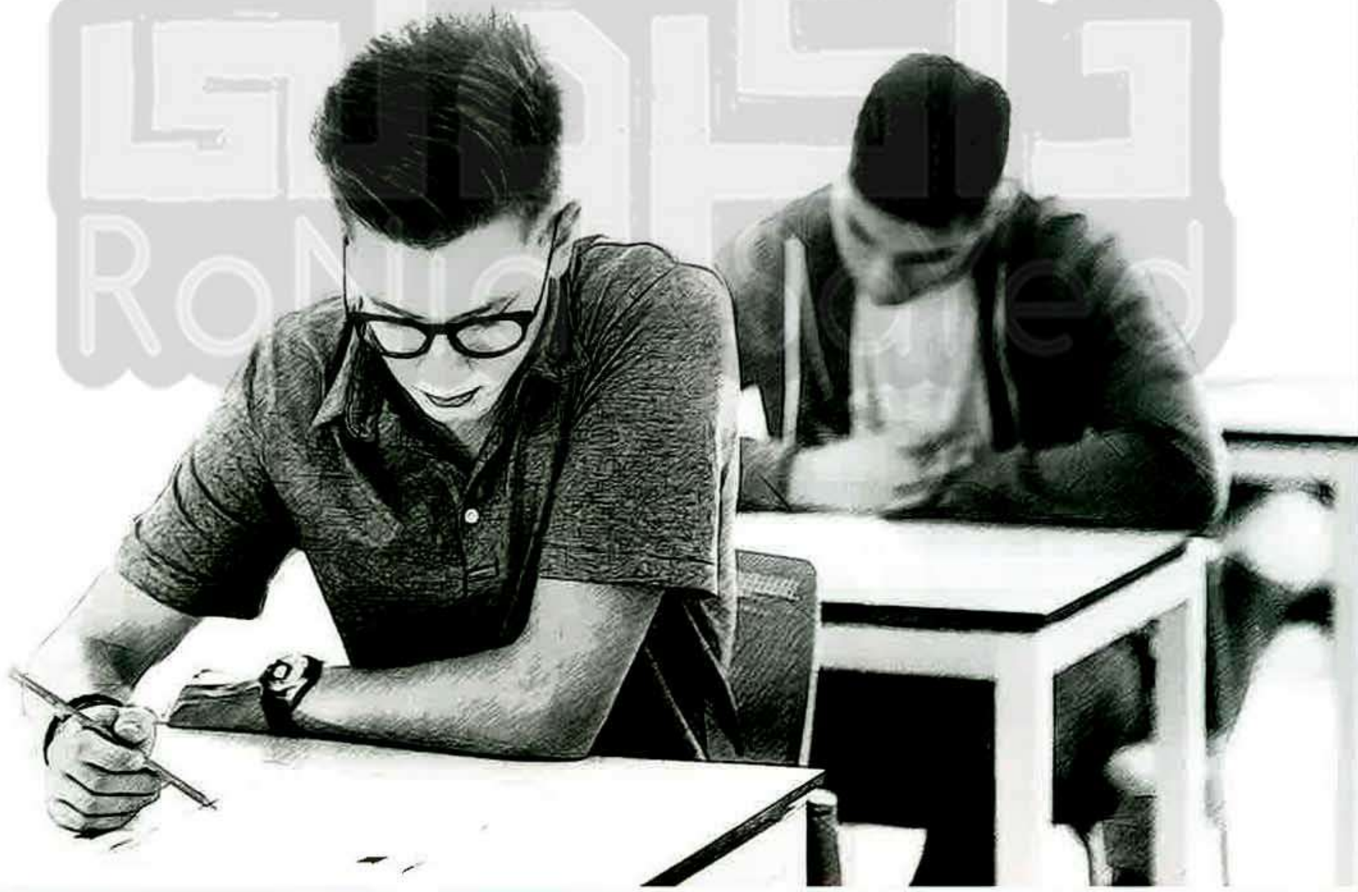
2. The probability that the drawn marble is blue
$$= \frac{\text{number of blue marbles}}{\text{total number of marbles}} = \frac{3}{10}$$

3. The probability that the drawn marble is white
$$= \frac{\text{number of white marbles}}{\text{total number of marbles}} = \frac{5}{10} = \frac{1}{2}$$

4. The probability that the drawn marble is not blue
$$= \frac{\text{number of the marbles which are not blue}}{\text{total number of marbles}} = \frac{10-3}{10} = \frac{7}{10}$$

Final Examinations

on Algebra and Statistics



Model Examinations of the School Book on Algebra and Statistics

Model 1

Answer the following questions :

1 Complete the following :

1. If $2^{x+3} = 1$, then $x =$ ....
2. If $x + y = 4$, $x - y = 2$, then $x^2 - y^2 =$ ....
3. The solution set of the equation : $x^2 - 1 = 8$, where $x \in \mathbb{Z}^+$ is ....
4. If $2^x = 3$, then $8^{-x} =$ ....
5. $1 - \frac{3}{4} =$ .... %

2 Choose the correct answer :

1. $\frac{5^{-2} \times \sqrt{5}}{5\sqrt{5}} =$ ....
   (a) $\frac{1}{125}$ (b) $\frac{1}{25}$ (c) 25 (d) 125
2. $\mathbb{Z} - \mathbb{Z}^- =$ ....
   (a) $\mathbb{Z}^+$ (b) $\mathbb{N}$ (c) $\varnothing$ (d) $\{0\}$
3. The volume of a cube of side length 3 cm. equals .... cm^3.
   (a) 9 (b) 12 (c) 27 (d) 81
4. The expression : $x^2 + k x + 36$ is a perfect square when k equals ....
   (a) ± 6 (b) ± 8 (c) ± 12 (d) ± 18
5. A regular die is thrown and observed the upper face, then the probability of appearance a number divisible by 3 is ....
   (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$
6. If $\left(\frac{5}{3}\right)^x = \frac{27}{125}$, then $x =$ ....
   (a) -5 (b) -3 (c) 3 (d) 5

3 Factorize each of the following expressions :

1. $x^2 + 8x + 15$
2. $2x^2 + 7x + 3$
3. $x^3 - 1$
4. $ax - 7a + 3x - 21$

4 **[a] Simplify to the simplest form :** $\frac{4^n \times 6^{2n}}{2^{4n} \times 3^{2n}}$

[b] Find the S.S. for the following equation where $x \in \mathbb{R}$: $x^2 - 8x + 12 = 0$

5 [a] A bag contains a number of similar balls , 5 of them are white and the rest are red. If the probability of drawing a red ball is $\frac{2}{3}$, find the number of all the balls.

[b] If $3^x = 27$, $4^{x+y} = 1$, **find the values of :** x and y

Model 2

Answer the following questions :

1 Complete the following :

1. If $7^{x-1} = 3^{x-1}$, then $x =$ ....
2. $x^3 -$ .... $= (x-2)$ (.............. $+ 2x + 4)$
3. $(5x - 2y)(25x^2 + 10xy + 4y^2) =$ ....
4. If $\frac{2x}{5} = 6$, then $x =$ ....
5. A bag contains 9 cards labeled by numbers from 1 to 9 , a card is drawn randomly , then the probability that the card carries an odd number is ....

2 Choose the correct answer :

1. If $x^3 y^{-3} = 8$, then $\frac{y}{x} =$ ....
   (a) 8 (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) 2
2. The expression : $x^2 + 4x + a$ is a perfect square when a equals ....
   (a) 3 (b) 4 (c) 8 (d) 16
3. The S.S. of the equation : $x^2 - x = 0$ is .... where $x \in \mathbb{R}$
   (a) $\{0\}$ (b) $\varnothing$ (c) $\{0 , 1\}$ (d) $\{1\}$
4. In the figure opposite the shaded region represents .... the circle.
   (a) $\frac{1}{8}$ (b) $\frac{1}{6}$
   (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

5. If $3^x + 3^x + 3^x = 1$, then $x =$ ....
   (a) -1 (b) 0 (c) $\frac{1}{3}$ (d) 1
6. If $6^x = 11$, then $6^{x+1} =$ ....
   (a) 12 (b) 22 (c) 66 (d) 72

3 Factorize each of the following :

1. $4x^2 - 9$
2. $x^3 + 8$
3. $x^2 - 5x$
4. $x^2 - x - 6$

4 **[a] Find in $\mathbb{R}$ the S.S. of the following equation :** $x^2 - x - 6 = 0$

[b] Simplify to the simplest form : $\dfrac{(\sqrt{2})^5 \times 3^{-2}}{3 \times (\sqrt{2})^9}$

5 **[a]** If $\frac{2^x \times 3^x}{(12)^x} = \frac{1}{2}$ find the value of x

[b] A bag contains a number of similar balls. Some of them are red, 2 greens, 4 blues. If the probability of drawing a ball with green colour is $\frac{1}{6}$, find the number of red balls.

Model for the merge students

Answer the following questions :

1 **Choose the correct answer from those given :**

1. The solution set of the equation : $x^2 + 25 = 0$ in $\mathbb{R}$ is ....
   (a) $\{-5, 5\}$ (b) $\{5\}$ (c) $\{-5\}$ (d) $\varnothing$
2. If the expression : $x^2 + a x + 9$ is a perfect square, then a = ...
   (a) 3 (b) 6 (c) 9 (d) 18
3. If $(x - 1)$ is one factor of expression : $x^2 - 4x + 3$, then the other factor is ....
   (a) $x + 3$ (b) $x + 1$ (c) $x - 3$ (d) $x - y$
4. If $\left(\frac{5}{3}\right)^x = \left(\frac{3}{5}\right)^2$, then x = ...
   (a) -2 (b) 2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$
5. The probability of the sure event = ...
   (a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

2 **Join from the column (A) to the suitable in the column (B) :**

Column (A)	Column (B)
1 If $a^2 - b^2 = 15$, $a + b = 3$, then $a - b$ = ...	• 5
2 If one digit of the number 37450 is chosen at random, then the probability of the chosen number is even = ...	• 6
	• $\frac{2}{5}$
3 If $(x + 3y)^2 = x^2 + k x y + 9y^2$, then k = ...	• 0
4 $4^3 + 4^3 + 4^3 + 4^3$ = ...	• 4^4
5 The probability of the impossible event = ...	

3 Complete each of the following :

1. $x^2 - y^2 = (\dots\dots - \dots\dots)(\dots\dots + \dots\dots)$
2. $x^3 - 8 = (\dots\dots - \dots\dots)(x^2 + 2x + \dots\dots)$
3. $x^2 - 5x + 6 = (x - \dots\dots)(\dots\dots - 3)$
4. $(a + b)x + (a + b)y = (a + \dots\dots)(\dots\dots + \dots\dots)$

4 Put (✔) for the correct statements and (✘) for the incorrect ones :

1. A school has 320 pupils , if the probability of the chosen pupil is a boy , is 0.6 , then the number of girls = 120 ( )
2. If $3^x = 27$, then $x = \frac{1}{3}$ ( )
3. A card is drawn at random , from cards numbered from 1 to 10 , then the probability that the card carries an odd number greater than 3 is $\frac{3}{10}$ ( )
4. The positive real number which if its square is added to its three times , the result will be 28 is 4 ( )
5. The solution set of the equation : $x(x-3)(x+5) = 0$ in $\mathbb{R}$ is $\{0 , 3 , -5\}$ ( )

5 Complete the solution in which the expression : $\frac{4^n \times 6^{2n}}{2^{4n} \times 3^{2n}}$ in its simplest form :

$$\frac{(2^{\dots\dots})^n \times (\dots\dots \times 3)^{2n}}{2^{4n} \times 3^{2n}} = \frac{2^{\dots\dots} \times \dots\dots^{2n} \times 3^{2n}}{2^{4n} \times 3^{2n}}$$

$$= 2^{\dots\dots + 2n - \dots\dots} \times 3^{2n \dots\dots}$$

$$= 2^{\dots\dots} \times 3^{\dots\dots}$$

$$= \dots\dots$$

Schools Examinations of the previous years on Algebra and Statistics

1 Cairo Governorate

Heliopolis Educational Zone

Answer the following questions :

1 Choose the correct answer :

1. If $x^2 + kx + 25$ is a perfect square , then k = ...
   (a) 5 (b) 10 (c) ± 10 (d) ± 5
2. If $5^{x+2} = 1$, then x = ...
   (a) 1 (b) – 2 (c) 2 (d) 5
3. If $x^2 - a = (x - 3)(x + 3)$, then a = ...
   (a) 2 (b) – 2 (c) 9 (d) – 9
4. The half of the number 2^8 is ....
   (a) 2^4 (b) 2^7 (c) 4 (d) – 4
5. If $\left(\frac{2}{3}\right)^x = \frac{8}{27}$, then x = ...
   (a) 2 (b) 1 (c) 8 (d) 3
6. If $x^3 + 8 = (x + 2)(x^2 + k + 4)$, then k = ...
   (a) $-2x$ (b) $4x$ (c) $2x$ (d) $-4x$

2 Complete the following :

1. The S.S. of $x^2 + 9 = 0$ in $\mathbb{R}$ is ....
2. The multiplicative inverse of the number $\left(\sqrt{3}\right)^4$ is ....
3. If $(x - 4)$ is a factor of the expression : $x^2 - 5x + 4$, then the other factor is ....
4. The probability of any event A ∈ ....
5. $\left(\sqrt{5}\right)^3 \div 5\sqrt{5}$ = ...

3 [a] Factorize each of the following :

1. $ax + bx + 5a + 5b$
2. $x^3 - 1$
3. $x^4 + 4$

[b] Find in $\mathbb{R}$ the S.S. of the equation : $x^2 + 9x + 18 = 0$

4 [a] If $3^{x-1} = 27$, **find the value of :** x

[b] Simplify to the simplest form : $\dfrac{\left(\sqrt{5}\right)^7 \times \left(\sqrt{5}\right)^3}{\left(\sqrt{5}\right)^9 \times \left(\sqrt{2}\right)^{-3}}$

5 [a] If $x = 3$, $y = \sqrt{3}$, **find the value of :** $\left(\frac{y}{x}\right)^{-2}$

[b] **Simplify the following to the simplest form :** $\frac{4^{x} \times 6^{2x}}{2^{4x} \times 3^{2x}}$

2 Cairo Governorate

East Nasr City Zone
Manaret Heliopolis School

Answer the following questions :

1 Complete each of the following :

1. The probability of the impossible event is ....
2. $a x + b y + b x + a y =$ ....
3. Fifth the number 5^{20} is ....
4. If $3^{x} = 5$, then $(27)^{x} =$ ....
5. The solution set of the equation : $x^2 + 1 = 0$ in $\mathbb{R}$ is ....

2 Choose the correct answer :

1. If the probability that a student succeeds in a subject is 0.8 , then the probability of his failure is ....
   (a) 0 (b) 1 (c) 0.2 (d) 0.8
2. If $6^{x} = 7$, then $6^{x+1} =$ ....
   (a) 42 (b) $\frac{7}{6}$ (c) 1 (d) 6
3. $4^3 + 4^3 + 4^3 + 4^3 =$ ....
   (a) 4^{12} (b) 4^9 (c) 4^4 (d) 4^{81}
4. The solution set of equation : $x^2 - 5x + 4 = 0$ in $\mathbb{R}$ is ....
   (a) $\{1 , 4\}$ (b) $\{2 , -2\}$ (c) $\varnothing$ (d) $\{1\}$
5. A die is thrown then the probability of appearance number 7 is ....
   (a) 0 (b) 1 (c) $\frac{2}{5}$ (d) $\frac{1}{6}$
6. If $x^2 + k x + 25$ is a perfect square , then $k =$ ....
   (a) 5 (b) 10 (c) ± 10 (d) ± 5

3 [a] **Factorize each of the following completely :**

1. $3a^2 + 7a + 2$
2. $5\ell + 10m + a\ell + 2am$

[b] **Find the value of the x in each of the following :**

1. $(x - 3)^7 = 128$
2. $4^{2x-1} = 1024$
3. $5^{x-7} = 1$

4 **[a] Simplify each of the following :**

1) $\dfrac{(\sqrt{3})^{-4} \times (\sqrt{2})^{-5} \times (\sqrt{3})^{-3}}{(\sqrt{3})^{-9} \times (\sqrt{2})^{-7}}$

2) $\left(\dfrac{2\sqrt{3}}{3\sqrt{2}}\right)^4$

[b] A bag contains balls labeled by the numbers from 1 to 15 if a ball is drawn at random **Find the probability that the drawn ball carries each of the following :**

1) An even number.
2) A number divisible by 3
3) A prime number.

5 **[a]** In producing 600 electric lamps , if the probability of the defected lamps is 0.05 , then find the number of the good lamps and also the number if the defected.

[b] Find in ℝ the solution set of each of the following :

1) $x^2 - 9 = 0$
2) $x^2 = 5x$
3) $3x = -x^2 - 2$

3 Cairo Governorate

Zietoun Educational Administration
Gomhouria Language School

Answer the following questions :

1 **Choose the correct answer :**

1) If $6^{x} = 7$, then $6^{x+1} =$ ....
(a) 8 (b) 13 (c) 36 (d) 42

2) If the expression : $a x^2 + 12x + 9$ is a perfect square , then a = ...
(a) 3 (b) 4 (c) 9 (d) 16

3) If $xy = 3$, $(x+y)^2 = 16$, then $x^2 + y^2 =$ ....
(a) 4 (b) 10 (c) 13 (d) 8

4) If a regular die is tossed once , then the probability of appearing the number 7 is ....
(a) $\frac{1}{7}$ (b) $\frac{1}{6}$ (c) 1 (d) 0

5) $3^{\text{zero}} + 3^{-1} - \left(\dfrac{1}{\sqrt{3}}\right)^2 =$ ....
(a) 3 (b) 1 (c) $\frac{1}{3}$ (d) 0

6) If $x + y = 3$, $x^2 - xy + y^2 = 5$, then $x^3 + y^3 =$ ....
(a) 15 (b) 25 (c) 8 (d) 7

2 **Complete each of the following :**

1) If three times a number is 3^3 , then $\frac{2}{3}$ this number = ...

2) If $x + y = 7$ and $a - 2b = 4$, then the numerical value of the expression :
$a(x+y) - 2b(x+y) =$ ....

3 If $\left(\frac{2}{3}\right)^{x} = \frac{27}{8}$, then $x =$ ....

4 A class has 50 students (boys and girls) , if the probability of choosing a girl randomly is 0.6 , then the number of boys = ...

5 If $x^3 y^{-3} = 8$, then $\frac{y}{x} =$ ....

3 [a] **Factorize each of the following completely :**

1 $9 - y^2$ 2 $4x^4 + 81y^4$

[b] If $2^{x-2} = \left(\frac{1}{2\sqrt{2}}\right)^2$ **Find the value of :** x

4 [a] **Find in $\mathbb{R}$ the S.S. of the equation :** $3x^2 + 15x - 18 = 0$

[b] **Simplify to the simplest form :** $(3^{x-1} \times 2^{x+1}) \div 6^{x-1}$

5 [a] A positive real number , if its square is added to three times of it then the result equals 28 **Find this number.**

[b] A box has 15 regular balls , 3 of them are white , 9 of them are black , a ball is choosing randomly.

Find the probability of the drawn ball is :

1 Black. 2 Not white and not black.

4 Giza Governorate

Omranea Direction
Baraem Misr Language School

Answer the following questions :

1 **Choose the correct answer :**

1 The S.S. of the equation : $x^2 - 1 = 8$ in $\mathbb{R}$ is ....

(a) $\varnothing$ (b) $\{3\}$ (c) $\{-3\}$ (d) $\{-3 , 3\}$

2 If $6^{x} = 7$, then $6^{x+1} =$ ....

(a) 8 (b) 13 (c) 36 (d) 42

3 If a die is thrown once , then the probability that the number 5 appears is ....

(a) $\frac{5}{6}$ (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $\frac{0}{6}$

4 If $7^{x-3} = 5^{x-3}$, then $x =$ ....

(a) 5 (b) 7 (c) 3 (d) 0

5 $2^{12} \times 3^{12} =$ ....

(a) 6^2 (b) 6^4 (c) 6^{12} (d) 6^{24}

6 If the expression : $x^2 + 14x + b$ is a perfect square , then b = ...

(a) 2 (b) 7 (c) 14 (d) 49

2 Complete each of the following :

1 $\left(\frac{3}{5}\right)^x = \frac{27}{125}$, then x = ...

2 The solution set of the equation : $x^2 + 9 = 0$ in $\mathbb{R}$ is ....

3 If the probability that a student failed is 7% , then the probability that this student succeeded is ....

4 If $3^x = 81$, then x = ...

5 The age of a man now x years, then his age 7 years ago is .... years.

3 [a] Factorize each of the following :

1 $8x^2 - 50$

2 $x^4 + 4y^4$

[b] If a real number is added to its square the result will be 12 , find this number.

4 [a] Find in $\mathbb{Q}$ the solution set of :

1 $x^2 - x = 12$

2 $4x^2 - 25 = 0$

[b] If $\frac{8^x \times 9^x}{18^x} = 64$, **find :** x

5 [a] A box contains a similar balls , 8 white balls , 5 red balls and 7 black balls , if we choose a ball , **then find the probability that the ball is :**

1 White.

2 Black or red.

[b] Find the value of x if : $2^{x-2} = 16$

5 Giza Governorate

Dokki District
Modern Narmer Language School

Answer the following questions :

1 Complete each of the following :

1 If $x = 3$ is a solution of the equation : $x^2 + 2x + k = 0$, then k = ...

2 The solution set of the equation : $x^2 + 4 = 0$ in $\mathbb{R}$ is ....

3 The quadratic equation : $(x + \text{.......})(3x - 2) = 0$ is equivalent to .... + ... $- 10 = 0$

4 If $3^{x-2} = 27$, then x = ...

5 There are 21 boys and 15 girls in a classroom , if a student is chosen at random , then the probability that the student is a boy =..............

2 Choose the correct answer :

1. The solution set in $\mathbb{R}$ of the equation : $(x-1)^2 = 0$ is
   (a) $\{0\}$ (b) $\{-1\}$ (c) $\{1, -1\}$ (d) $\{1\}$
2. If $3^x + 3^x + 3^x = 1$, then $x =$ ....
   (a) -1 (b) 0 (c) 1 (d) 2
3. 3^{-2} equals ....
   (a) 9 (b) $\frac{1}{9}$ (c) $-\frac{1}{9}$ (d) -9
4. $2^{12} \times 3^{12} =$ ....
   (a) 6^2 (b) 6^4 (c) 6^{12} (d) 6^{24}
5. A die is thrown once, then the probability that 5 appears is ....
   (a) $-\frac{5}{6}$ (b) zero (c) $\frac{1}{6}$ (d) $\frac{5}{6}$
6. The expression : $x^2 + a\,x + 2$ can be factorized, then a = ...
   (a) 1 (b) 2 (c) 3 (d) 4

3 [a] Solve in $\mathbb{R}$ the equations :

1. $x^2 - x - 12 = 0$
2. $x(x-2) - 2(2-x) = 0$

[b] The length of a rectangle is more than its width by 5 cm. If its area is 36 cm^2
, then find its dimensions and its perimeter.

4 [a] Simplify : $\frac{4^{x+1} \times 9^{2-x}}{6^{2x}}$, then find the value of the answer when $x = 2$

[b] If the sum of the square of a positive number and three times this number is 28
, then find the value of this number.

5 [a] Find the value of x if : $3^{2x-3} = 243$

[b] A bag contains 20 balls numbered from 1 to 20, if one ball is drawn at random
, then find the probability that :

1. The number is a multiple of 4
2. The number is less than or equal 7

6 Giza Governorate

6 October Directorate

Answer the following questions :

1 Choose the correct answer :

1. $(x-2)^2 =$ ....
   (a) $x^2 - 4$ (b) $(2+x)^2$ (c) $x^2 + 4$ (d) $x^2 - 4x + 4$

2. $4^3 + 4^3 + 4^3 + 4^3 = \dots\dots$

(a) 4^3 (b) 4^4 (c) 4^{12} (d) 4^{81}

3. If $k x^2 - 12x + 4$ is a perfect square , then $k = \dots\dots$

(a) -6 (b) -4 (c) -2 (d) 9

4. If $\frac{a}{b} = 1$, then $4a - 4b = \dots\dots$

(a) 8 (b) 4 (c) 1 (d) 0

5. If $x + y = 3$, $x^2 - xy + y^2 = 5$, then $x^3 + y^3 = \dots\dots$

(a) 15 (b) 25 (c) 8 (d) 7

6. If $3^x = 2$, then $27^x = \dots\dots$

(a) 9 (b) 4 (c) 8 (d) 1

2 Complete the following :

1. If $x^3 y^{-3} = 8$, then $\frac{y}{x} = \dots\dots$
2. A bag contains 9 cards labeled by numbers from 1 to 9 , a card is drawn randomly , then the probability that this card carries an odd number = $\dots\dots$
3. The S.S. of the equation : $x^2 + 1 = 0$ in $\mathbb{R}$ is $\dots\dots$
4. If $\frac{2x}{5} = 6$, then $x - 5 = \dots\dots$
5. $(a - 2)(2a - 3) = \dots\dots - 7a + \dots\dots$

3 Factorize each of the following :

1. $3x^2 - 48$
2. $x^2 - 7x + 10$
3. $x^3 + 2x^2 - 4x - 8$
4. $2x^3 - 16y^3$

4 [a] Find the S.S. in $\mathbb{R}$:

1. $3^{2n-5} = 1$
2. $\left(\frac{2}{3}\right)^{2n} = \frac{81}{16}$

[b] A bag contains cards numbered from 1 to 20 and card drawn randomly

Find the probability of :

1. Getting a number divisible by 4
2. Getting a number multiple of 7

5 [a] Find the real number which if we added its square to its three times , it becomes 28

[b] Simplify : $\frac{4^n \times 6^{2n}}{2^{4n} \times 3^{2n}}$

7 Alexandria Governorate

Eastern Educational Zone
Taymour English School

Answer the following questions :

1 Complete the following :

1. If $a = \sqrt{3}$, $b = \sqrt{2}$, then the value of $\frac{a^4}{b^4} =$ ....
2. $\frac{(10)^2 \times (10)^{-7}}{(0.1)^2 \times 0.001} =$ ....
3. A numbered card is selected at random from a set of similar cards numbered from 1 to 24 , the probability of getting a card carries a multiple of 4 is ....
4. $(9a^2 - 4b^2) = (3a -$ ....$)($.............. $+ 2b)$
5. $(x + 3y)^2 = x^2 +$ .... $+ 9y^2$

2 Choose the correct answer :

1. If $(x + 3)$ is a factor of the expression : $x^2 + x - 6$, then the other factor is ....
   (a) $(x - 2)$ (b) $(x - 3)$ (c) $(x + 2)$ (d) $(x + 6)$
2. If $3^x = 27$, $4^{x+y} = 1$, then y = ...
   (a) 0 (b) 3 (c) – 3 (d) 1
3. The S.S. of the equation : $x^2 - 3 = 0$ in $\mathbb{R}$ is ....
   (a) $\{3 , -3\}$ (b) $\{\sqrt{3}\}$ (c) 9 (d) $\{-\sqrt{3} , \sqrt{3}\}$
4. $\left(\sqrt{3} + \sqrt{2}\right)^9 \left(\sqrt{3} - \sqrt{2}\right)^9 =$ ....
   (a) 1 (b) $\sqrt{5}$ (c) $\sqrt{6}$ (d) 5
5. Which of the following may be equal the probability of an event ....
   (a) – 0.73 (b) 1.23 (c) 79 % (d) $\frac{4}{3}$
6. If $x^3 + 27 = (x + 3)(x^2 + k + 9)$, then k = ...
   (a) $-6x$ (b) $-3x$ (c) $3x$ (d) $6x$

3 [a] Simplify : $\frac{4^n \times 6^{2n}}{2^{4n} \times 3^{2n}}$

[b] Find the value of x : $\left(\frac{2}{5}\right)^{2x+1} = \frac{8}{125}$

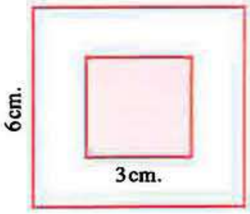
4 Factorize each of the following :

1. $(x + 2)^3 - 4x - 8$
2. $a^2 + 2ab + b^2 - c^2$
3. $5a^2 - 18a + 16$
4. $xy + 5y + 7x + 35$

5 [a] **Find the S.S. in $\mathbb{R}$:** $2x^3 = 18x$

[b] **In the opposite figure :**

Two squares, if a person shoot at a picture in the figure then find the probability of hitting the shaded part.



8 Alexandria Governorate

Mid Educational Zone
Maths Supervision

Answer the following questions :

1 Complete each of the following :

1. The simplest form of : $(\sqrt{3})^3 \times (\sqrt{3})^5 =$ ....
2. If $x + y = 5$, $x - y = 3$, then $x^2 - y^2 =$ ....
3. If $x - 6 = 0$, then $x =$ ....
4. $y^3 -$ .... $= (y - 2)(y^2 +$ .... $+ 4)$
5. $(\sqrt{7}+\sqrt{6})^8 (\sqrt{7}-\sqrt{6})^8 =$ ....

2 Choose the correct answer :

1. The expression : $x^2 + 8x + a$ is a perfect square when a = ...
   (a) -4 (b) 4 (c) 8 (d) 16
2. A die is thrown once , then the probability of appearance 7 on the upper face is ....
   (a) $\frac{-5}{6}$ (b) $\frac{1}{6}$ (c) 0 (d) $\frac{5}{6}$
3. If $(x + 3)^{zero} = 1$, then $x \in$ ....
   (a) $\mathbb{R} - \{3\}$ (b) $\mathbb{R} - \{-3\}$ (c) $\{3\}$ (d) $\mathbb{R}$
4. If the age of Kamal now is x years , then his age 3 years ago was .... years.
   (a) $x + 3$ (b) $3x$ (c) $x - 3$ (d) $6x$
5. The multiplicative inverse of 1 is ....
   (a) 0 (b) 1 (c) 2 (d) 3
6. $3^3 + 3^3 + 3^3 =$ ....
   (a) 3^3 (b) 3^4 (c) 3^{12} (d) 3^{81}

3 [a] **Factorize :**

1. $9x^2 - 4$
2. $10y^2 - 7y - 12$
3. $4x^4 + 1$

[b] **Find the solution set in $\mathbb{R}$ for :** $2x^2 - 2x - 12 = 0$

4 **[a] Find in the simplest form :** $\frac{x^6 \times x^2}{x^3}$ where $x \neq 0$

[b] Factorize : $3x - 21 + ax - 7a$

5 **[a]** A numbered card is selected randomly from a set of similar cards numbered from 1 to 15

Find the probability of getting a card carries :

1 A prime number.

2 A number divisible by 3

[b] If $2^{x-2} = 32$ **, then find the value of :** x

9 El-Kalyoubia Governorate

Maths Supervision

Answer the following questions :

1 **Choose the correct answer :**

1 $\mathbb{Z} - \mathbb{Z}^- =$ ....

(a) $\mathbb{Z}^+$ (b) $\mathbb{N}$ (c) $\varnothing$ (d) $\{0\}$

2 The volume of the cube of side length 3 cm. equals .... cm^3

(a) 9 (b) 12 (c) 27 (d) 81

3 The expression : $x^2 + 4x + a$ is a perfect square when a = ...

(a) 3 (b) 4 (c) 8 (d) 16

4 The S.S. of the equation : $x^2 - x = 0$ is .... in $\mathbb{R}$

(a) $\{0\}$ (b) $\varnothing$ (c) $\{0, 1\}$ (d) $\{1\}$

5 If $(x - 1)$ is one factor of the expression : $x^2 - 4x + 3$, then the other factor is ....

(a) $x + 3$ (b) $x + 1$ (c) $x - 3$ (d) $x - 4$

6 If $\left(\frac{5}{3}\right)^x = \left(\frac{3}{5}\right)^2$, then $x =$ ....

(a) -2 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

2 **Complete :**

1 If $7^{x-1} = 3^{x-1}$, then $x =$ ....

2 A bag contains 9 cards labled by numbers from 1 to 9 , a card is drawn randomly , then the probability that the card carries an odd number is ....

3 $a^{-4} + 1 = a^{-4}$ (.............. + ...) where $a \neq 0$

4 $1 - \frac{3}{4} =$ ....

5 $4^3 + 4^3 + 4^3 + 4^3 = 4^{\cdots\cdots}$

3 **[a] Factorize :**

1 $x^2 - y^2$ 2 $y^3 + 8$

[b] Find the S.S. of the following equation in $\mathbb{R}$: $x^2 - x - 6 = 0$

4 **[a] Factorize :**

1 $a x - 7 a + 3 x - 21$ 2 $x^2 - 5 x$

[b] If $3^x = 27$, $4^{x+y} = 1$, **find the value of :** x , y

5 **[a] Find in the simplest form :** $\frac{4^{x+1} \times 9^{2-x}}{(6)^{2x}}$, then calculate the result when $x = 1$

[b] If one digit of the number 37450 chosen at random , find the probability that the chosen digit is an even number.

10 El-Monofia Governorate

Kwesna Educational Directorate
Mathematics Supervision

Answer the following questions : *(Calculator is premitted)*

1 **Choose the correct answer from those given :**

1 $0.002 \times 0.05 =$ ....

(a) 10^{-5} (b) 10^{-4} (c) 10^4 (d) 10^5

2 The expression : $(x - 2y)(x^2 + 2xy + 4y^2)$ equals ....

(a) $x^3 - 2y^3$ (b) $x^3 - 8y^3$ (c) $x^3 + 8y^3$ (d) $x^3 + 18y^3$

3 The value of the expression : $5^{20} + 5^{21}$ equals ....

(a) 5×5^{40} (b) 5×5^{41} (c) 6×5^{20} (d) 6×5^{21}

4 The value of the expression : $2^5 + (\sqrt{2})^{10}$ equals ....

(a) 2^6 (b) 2^{10} (c) $(\sqrt{2})^{15}$ (d) $(\sqrt{2})^{20}$

5 If the probability of choosing a boy from a class of 40 students is 0.375 , then the number of girls is .... girls.

(a) 35 (b) 25 (c) 20 (d) 15

6 The solution set of the equation : $(x - 1)^2 = 0$ in $\mathbb{R}$ is ....

(a) $\{-1\}$ (b) $\{1\}$ (c) $\{-1 , 1\}$ (d) $\{2\}$

2 **Complete :**

1 The expression : $x^2 - 2x + k$ is perfect square when k = ...

2 If $3^x \times 2^{-x} = 1.5$, then $x =$ ....

3 If $a^2 + b^2 = 7$, $a b = 3$, then $(a - b)^2 =$ ....

4 $x(y - z) + L(y - z) = (y - z)$ (...............)

5 $\left(\frac{\sqrt{3}}{9}\right)^{-1} = (\sqrt{3})^{\cdots\cdots}$

3 [a] An integer is added to its multiplicative inverse the result equals 2 Find the number.

[b] **Factorize each of the following :**

1 $3x^2 - 15x + 12$

2 $\frac{1}{3}x^3 - 9$

3 $x^4 + 9x^2 + 81$

4 [a] **Simplify :** $\frac{4^{x+1} \times 9^{2-x}}{(6)^{2x}}$, then calculate its value at $x = 1$

[b] If $x = \frac{\sqrt{3}}{2}$, $y = \frac{1}{\sqrt{3}}$, $z = \frac{\sqrt{2}}{2}$, **find the value :** $x^2 + (x z)^2 \times y^2$

5 [a] A set of cards numbered from 0 to 10 , if a card is drawn randomly

, find the probability of each :

1 Drawing a card carries odd number.

2 Drawing a card carries a number divisible by 5

[b] **Factorize each of the following :**

1 $a x - 7a + 3x - 21$

2 $9x^2 - 25$

11 El-Dakahlia Governorate

Maths Supervision

Answer the following questions :

1 Complete each of the following :

1 If $3^{x-1} = 27$, then $x =$ ....

2 If $(x - 5)^0 = 1$, then $x \in$ ....

3 $a + b = 2(x + y) = 14$, then $a(x + y) + b(x + y) =$ ....

4 The probability of impossible event = ...

5 If the perimeter of square x cm. , then its area = ...

2 Choose the correct answer :

1 If $6^x = 7$, then $6^{x+1} =$ ....

(a) 8 (b) 13 (c) 36 (d) 42

2 If the product of multiplying four by a number equals 48 , then the third of this number = ...

(a) 4 (b) 8 (c) 12 (d) 16

3 The value of $2^5 + (\sqrt{2})^{10}$ = ...

(a) 2^6 (b) 2^{10} (c) $(\sqrt{2})^{15}$ (d) $(\sqrt{2})^{20}$

4 The S.S. of the equation : $x^3 + 9x = 0$ in $\mathbb{R}$ is ....

(a) $\{0 , 3\}$ (b) $\{0\}$ (c) $\{0 , -3\}$ (d) $\{0 , 3 , -3\}$

5 If $2^x = 5$, then 8^x = ...

(a) $\frac{5}{8}$ (b) 25 (c) 125 (d) $\frac{64}{125}$

6 If $y^3 - a = (y - 2)(y^2 + 2y + 4)$, then a = ...

(a) 2 (b) 4 (c) 8 (d) – 8

3 Factorize :

1 $x^4 + y^4 - 11x^2y^2$

2 $9x^2 - 4a^2 + y^2 + 6xy$

3 $3x^3 - 2x^2 + 12x - 8$

4 $25x^2 - 30x + 9$

4 [a] If the length of a rectangle 5 cm. more than its width and its area 36 cm^2. **Find its perimeter.**

[b] If $\left(\sqrt{\frac{2}{3}}\right)^x = \frac{4}{9}$ **Find the value of :** $\left(\frac{2}{3}\right)^{x-1}$

5 [a] **Prove that :** $\frac{(27)^{x-1} \times 8^x}{(2\sqrt{3})^{2x} \times (3\sqrt{2})^{2x}} = \frac{1}{27}$

[b] A team plays 30 matches in national league , its drawn probability is 0.3 and its win probability is 0.6 **Calculate the number of loss matches.**

12 Ismailia Governorate

Directorate of Education
Elmanar Language School

Answer the following questions :

1 Complete each of the following :

1 $4a(x + y) - 3b(x + y) = (x + y)$ (.............. – ....)

2 The S.S. of the equation : $x^2 + 3x = 0$ in $\mathbb{R}$ is ....

3 If $3^x = 27$, then x = ...

4 The probability of impossible event is ....

5 If the probability of absent pupils in a school is $\frac{2}{19}$, then the probability of present pupils is ....

2 Choose the correct answer :

1 If $(x-5)^{zero} = 1$, then $x \in$ ....

(a) $\mathbb{R} - \{5\}$ (b) $\mathbb{R} - \{-5\}$ (c) $\{5\}$ (d) $\mathbb{R}$

2 The S.S. in $\mathbb{R}$ of the equation : $x^2 + 25 = 0$ is ....

(a) $\{5\}$ (b) $\{5, -5\}$ (c) $\varnothing$ (d) $\{-5\}$

3 If $5^x = 2$, then $5^{x+2} =$ ....

(a) 25 (b) 2 (c) 50 (d) 100

4 A bag contains 20 balls , 8 of them are white and the rest are black , then the probability of the drawn ball is black is ....

(a) 1 (b) 0.6 (c) 0 (d) $\frac{8}{20}$

5 Which of the following can be probability of an event ?

(a) 1.2 (b) $\frac{4}{3}$ (c) -0.2 (d) 37 %

6 If $x^2 - a = (x-3)(x+3)$, then a = ...

(a) 3 (b) -3 (c) 9 (d) -9

3

[a] Factorize : 1 $x^3 - 3x^2 + 6x - 18$ 2 $3x^3 - 81$

[b] If $\left(\frac{2}{5}\right)^{2x-1} = \frac{8}{125}$ **Find the value of :** x

4

[a] A positive real number if you add its square to its three times the result will be 28 find the number.

[b] Find in $\mathbb{R}$ the S.S. of : $x^2 - 8x = -15$

5

[a] If a card is chosen randomly from 10 cards numbered from 1 to 10

, then the probability that the number on the chosen card is :

1 Even. 2 Divisible by 3

3 Even prime.

[b] Prove that : $\frac{(27)^{x-1} \times 8^x}{(2\sqrt{2})^{2x} \times (3\sqrt{3})^{2x}} = \frac{1}{27}$

13 Damietta Governorate

Inspection of mathematics

Answer the following questions :

1 Choose the correct answer from those given :

1. $3^{-2} = \dots\dots$
   (a) -9 (b) $\frac{1}{9}$ (c) $-\frac{1}{9}$ (d) 9
2. $\sqrt{100-64} = 10 - \dots\dots$
   (a) 4 (b) 6 (c) 8 (d) -6
3. If a coin thrown once , then the probability of appearing tail = ……
   (a) 1 (b) 0.3 (c) 0.5 (d) 0
4. The solution set of the equation : $x^2 + 9 = 0$ in $\mathbb{R}$ is ……
   (a) $\{3\}$ (b) $\{-3\}$ (c) $\varnothing$ (d) $\{3, -3\}$
5. $4^3 + 4^3 + 4^3 + 4^3 = \dots\dots$
   (a) 4^3 (b) 4^4 (c) 4^{12} (d) 4^{81}
6. The expression : $a x^2 - 40 x + 25$ is a perfect square when a = ……
   (a) 2 (b) 4 (c) 9 (d) 16

2 Complete each of the following :

1. If the probability that a pupil succeed is 0.8 , then probability of his failure = ……
2. If $7^{x} = 1$, then $x = \dots\dots$
3. $2 \times 6 - 8 \div 4 = \dots\dots$
4. If $2^{x} = 5$, then $2^{-x} = \dots\dots$
5. If $x - y = 3$ and $x + y = 4$, then $x^2 - y^2 = \dots\dots$

3 **[a] Simplify :** $\dfrac{(\sqrt{3})^{8} \times (\sqrt{3})^{-14}}{(\sqrt{3})^{-4}}$

[b] Find the solution set of the following equation in $\mathbb{R}$: $x^2 - 8x = -15$

4 **[a] Factorize each of the following expressions :**

1. $x^2 - 4y^2$
2. $x^4 + 4y^4$

[b] Find the solution set of the following equation in $\mathbb{R}$: $3^{x-4} = 9$

5 **[a]** If $a = \sqrt{2}$, $b = \sqrt{3}$, **find the numerical value of :** $\frac{b^4 - a^4}{b^2 + a^2}$

[b] A box contains 5 white , 2 red , 3 green balls , a ball is drawn randomly from the box

Calculate the probabilities of the following events :

1 The ball is white. 2 The ball yellow. 3 The ball is not red.

14 El-Fayoum Governorate

Directorate of Education
Supervision of Mathematics

Answer the following questions :

1 **Choose the correct answer :**

1 If $\frac{a}{b} = 1$, then $4a - 4b =$ ....

(a) 8 (b) 4 (c) 1 (d) 0

2 If the probability that a pupil succeeds is 0.7 , then the probability of his failure is ....

(a) 0.7 (b) 0.07 (c) 0.3 (d) 0.03

3 If the age of Ahmed now is x years , then the square of his age is .... years.

(a) x^2 (b) $2x$ (c) $2x^2$ (d) $x + 2$

4 $(-1)^3 + (-1)^5 =$ ....

(a) 0 (b) -2 (c) 2 (d) 201

5 $(5a)^0 =$ .... , $a \neq 0$

(a) 5 (b) a (c) 5 a (d) 1

6 If $x - 2y = 3$, $x^2 - 4y^2 = 21$, then $x + 2y =$ ....

(a) 14 (b) 9 (c) 7 (d) 6

2 **Complete each of the following :**

1 $\frac{3}{4} =$ .... %

2 If $a = 7^x$, $b = 7^{-x}$, then $a \times b =$ ....

3 $2^{-3} \times 2^{-2} \div 4^{-3} =$ ....

4 The solution set of the equation : $x^2 - 6x = 0$ in $\mathbb{R}$ is ....

5 If a fair coin is tossed once , then the probability of appearance of a head = ...

3 **[a]** **Factorize each of the following completely :**

1 $36 - 60k + 25k^2$ 2 $x^4 + 64$

[b] **Find in $\mathbb{R}$ the S.S. of the following equation :** $x^2 + x = 6$

4 **[a] Simplify to the simplest form :** $\frac{(\sqrt{5})^{10} \times (-\sqrt{5})^{5}}{(\sqrt{5})^{11}}$

[b] A regular die is thrown once **Find the probability of the appearance a number is :**

1 Even. 2 Between 0 and 6 3 Prime.

5 **[a] Find in $\mathbb{R}$ the S.S. of the following equation :** $2^{n-3} = \frac{1}{4}$

[b] Simplify to the simplest form : $\frac{4^{x+1} \times 9^{2-x}}{6^{2x}}$, then calculate its value at $x = 1$

15 El-Menia Governorate

El-Menia Educational Directorate
Minia Kawmia Language School

Answer the following questions :

1 **Complete the following :**

1 The solution set of the equation : $x^2 - 1 = 8$, where $x \in \mathbb{Z}^+$ is ....

2 If $3^{x-4} = 1$, then $x =$ ....

3 The S.S. of the equation : $x^2 - 25 = 0$ in $\mathbb{R}$ is ....

4 If $\left(\frac{2}{3}\right)^x = \frac{3}{2}$, then $x =$ ....

5 The volume of a cube of side length 3 cm. equals .... cm^3

2 **Choose the correct answer :**

1 The S.S. of the equation : $x(x-2) = 0$ in $\mathbb{R}$ is ....

(a) $\{0\}$ (b) $\{2\}$ (c) $\{0, 2\}$ (d) $\{0, -2\}$

2 If $x^3 y^{-3} = 8$, then $\frac{x}{y} =$ ....

(a) $\frac{1}{512}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) 2

3 The expression : $x^2 + k x + 36$ is a perfect square when k equals ....

(a) ± 6 (b) ± 8 (c) ± 12 (d) ± 18

4 $4^3 + 4^3 + 4^3 + 4^3 =$ ....

(a) 4^{12} (b) 16^{12} (c) 16^2 (d) 16^3

5 If the probability of success of a student is 0.75 , then the probability of his failure is ....

(a) 0.20 (b) 0.25 (c) 0.30 (d) 0.35

6 If $(x-1)$ is one factor of expression : $x - 4x + 3$, then the other factor is ....

(a) $x+3$ (b) $x+1$ (c) $x-3$ (d) $x-y$

3 [a] If $\frac{8^x \times 9^x}{18^x} = 64$ **find :** x

[b] Find the S.S. of the equation in $\mathbb{R}$ **:** $x^2 - 1 = 8$

4 Factorize each of the following expresions :

1. $a x - 7a + 3x - 21$
2. $x^3 + 8$
3. $x^2 - x - 6$
4. $4x^2 - 9$
5. $x^4 + 324$

5 [a] If $(3)^{x-2} = 9$, then find the value of : x

[b] A colored marble is drawn randomly out of a box containing 12 red marbles , 18 white marbles and 20 blue marbles.

Find the probability of selecting :

1. A white marble.
2. A yellow marble.
3. A red or blue marble.
4. A non red marble.

Final Examinations

of Algebra and Statistics

Model Examinations of the School Book on Algebra and Statistics

Model 1

Answer the following questions :

1 Complete the following :

1. If $2^{x+3} = 1$, then $x =$ ....
2. If $x + y = 4$, $x - y = 2$, then $x^2 - y^2 =$ ....
3. The solution set of the equation : $x^2 - 1 = 8$, where $x \in \mathbb{Z}^+$ is ....
4. If $2^x = 3$, then $8^{-x} =$ ....
5. $1 - \frac{3}{4} =$ .... %

2 Choose the correct answer :

1. $\frac{5^{-2} \times \sqrt{5}}{5\sqrt{5}} =$ ....
   (a) $\frac{1}{125}$ (b) $\frac{1}{25}$ (c) 25 (d) 125
2. $\mathbb{Z} - \mathbb{Z}^- =$ ....
   (a) $\mathbb{Z}^+$ (b) $\mathbb{N}$ (c) $\varnothing$ (d) $\{0\}$
3. The volume of a cube of side length 3 cm. equals .... cm^3.
   (a) 9 (b) 12 (c) 27 (d) 81
4. The expression : $x^2 + k x + 36$ is a perfect square when k equals ....
   (a) ± 6 (b) ± 8 (c) ± 12 (d) ± 18
5. A regular die is thrown and observed the upper face, then the probability of appearance a number divisible by 3 is ....
   (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$
6. If $\left(\frac{5}{3}\right)^x = \frac{27}{125}$, then $x =$ ....
   (a) – 5 (b) – 3 (c) 3 (d) 5

3 Factorize each of the following expressions :

1. $x^2 + 8x + 15$
2. $2x^2 + 7x + 3$
3. $x^3 - 1$
4. $a x - 7 a + 3 x - 21$

4 **[a] Simplify to the simplest form :** $\frac{4^n \times 6^{2n}}{2^{4n} \times 3^{2n}}$

[b] Find the S.S. for the following equation where $x \in \mathbb{R}$: $x^2 - 8x + 12 = 0$

5 **[a]** A bag contains a number of similar balls , 5 of them are white and the rest are red. If the probability of drawing a red ball is $\frac{2}{3}$, find the number of all the balls.

[b] If $3^x = 27$, $4^{x+y} = 1$, **find the values of :** x and y

Model 2

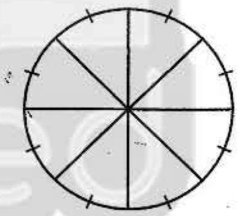
Answer the following questions :

1 Complete the following :

1. If $7^{x-1} = 3^{x-1}$, then $x =$ ....
2. $x^3 -$ .... $= (x - 2)$ (.............. $+ 2x + 4)$
3. $(5x - 2y)(25x^2 + 10xy + 4y^2) =$ ....
4. If $\frac{2x}{5} = 6$, then $x =$ ....
5. A bag contains 9 cards labeled by numbers from 1 to 9 , a card is drawn randomly , then the probability that the card carries an odd number is ....

2 Choose the correct answer :

1. If $x^3 y^{-3} = 8$, then $\frac{y}{x} =$ ....
   (a) 8 (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) 2
2. The expression : $x^2 + 4x + a$ is a perfect square when a equals ....
   (a) 3 (b) 4 (c) 8 (d) 16
3. The S.S. of the equation : $x^2 - x = 0$ is .... where $x \in \mathbb{R}$
   (a) $\{0\}$ (b) $\varnothing$ (c) $\{0 , 1\}$ (d) $\{1\}$
4. In the figure opposite the shaded region represents .... the circle.
   (a) $\frac{1}{8}$ (b) $\frac{1}{6}$
   (c) $\frac{1}{4}$ (d) $\frac{1}{3}$



5. If $3^x + 3^x + 3^x = 1$, then $x =$ ....
   (a) -1 (b) 0 (c) $\frac{1}{3}$ (d) 1
6. If $6^x = 11$, then $6^{x+1} =$ ....
   (a) 12 (b) 22 (c) 66 (d) 72

3 Factorize each of the following :

1. $4x^2 - 9$
2. $x^3 + 8$
3. $x^2 - 5x$
4. $x^2 - x - 6$

4

[a] Find in $\mathbb{R}$ the S.S. of the following equation : $x^2 - x - 6 = 0$

[b] Simplify to the simplest form : $\dfrac{(\sqrt{2})^5 \times 3^{-2}}{3 \times (\sqrt{2})^9}$

5 [a] If $\frac{2^x \times 3^x}{(12)^x} = \frac{1}{2}$ find the value of x

[b] A bag contains a number of similar balls. Some of them are red, 2 greens, 4 blues.
If the probability of drawing a ball with green colour is $\frac{1}{6}$, find the number of red balls.

Model for the merge students

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The solution set of the equation : $x^2 + 25 = 0$ in $\mathbb{R}$ is ....

(a) $\{-5, 5\}$ (b) $\{5\}$ (c) $\{-5\}$ (d) $\varnothing$

2 If the expression : $x^2 + a x + 9$ is a perfect square, then a = ...

(a) 3 (b) 6 (c) 9 (d) 18

3 If $(x - 1)$ is one factor of expression : $x^2 - 4x + 3$, then the other factor is ....

(a) $x + 3$ (b) $x + 1$ (c) $x - 3$ (d) $x - y$

4 If $\left(\frac{5}{3}\right)^x = \left(\frac{3}{5}\right)^2$, then x = ...

(a) -2 (b) 2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

5 The probability of the sure event = ...

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

2 **Join from the column (A) to the suitable in the column (B) :**

Column (A)	Column (B)
1 If $a^2 - b^2 = 15$, $a + b = 3$, then $a - b$ = ...	• 5
2 If one digit of the number 37450 is chosen at random, then the probability of the chosen number is even = ...	• 6
	• $\frac{2}{5}$
3 If $(x + 3y)^2 = x^2 + k x y + 9y^2$, then k = ...	• 0
4 $4^3 + 4^3 + 4^3 + 4^3$ = ...	• 4^4
5 The probability of the impossible event = ...	

3 **Complete each of the following :**

1. $x^2 - y^2 = (\cdots\cdots - \cdots\cdots)(\cdots\cdots + \cdots\cdots)$
2. $x^3 - 8 = (\cdots\cdots - \cdots\cdots)(x^2 + 2x + \cdots\cdots)$
3. $x^2 - 5x + 6 = (x - \cdots\cdots)(\cdots\cdots - 3)$
4. $(a + b)x + (a + b)y = (a + \cdots\cdots)(\cdots\cdots + \cdots\cdots)$

4 **Put (✔) for the correct statements and (✘) for the incorrect ones :**

1. A school has 320 pupils , if the probability of the chosen pupil is a boy , is 0.6 , then the number of girls = 120 ( )
2. If $3^x = 27$, then $x = \frac{1}{3}$ ( )
3. A card is drawn at random , from cards numbered from 1 to 10 , then the probability that the card carries an odd number greater than 3 is $\frac{3}{10}$ ( )
4. The positive real number which if its square is added to its three times , the result will be 28 is 4 ( )
5. The solution set of the equation : $x(x - 3)(x + 5) = 0$ in $\mathbb{R}$ is $\{0 , 3 , -5\}$ ( )

5 **Complete the solution in which the expression : $\frac{4^n \times 6^{2n}}{2^{4n} \times 3^{2n}}$ in its simplest form :**

$$\frac{(2^{\cdots\cdots})^n \times (\cdots\cdots \times 3)^{2n}}{2^{4n} \times 3^{2n}} = \frac{2^{\cdots\cdots} \times \cdots\cdots^{2n} \times 3^{2n}}{2^{4n} \times 3^{2n}}$$

$$= 2^{\cdots\cdots + 2n - \cdots\cdots} \times 3^{2n \cdots\cdots}$$

$$= 2^{\cdots\cdots} \times 3^{\cdots\cdots}$$

$$= \cdots\cdots$$

Schools Examinations on Algebra and Statistics

1 Cairo Governorate

East Nasr City Adminstration
Al Raya Language School

Remark: Some school exams are modified to include what was canceled last year

Answer the following questions :

1 Complete :

1 If $x + y = 4$, $x - y = 2$, then $x^2 - y^2 =$ ....

2 A bag contain a number of similar balls , 4 of them are white and the rest is red , if the probability of drawing a white ball is $\frac{2}{3}$, then the number of red balls equals ....

3 If $3^x = 81$, then $x + 1 =$ ....

4 The simplest form of $\left(\sqrt{5}\right)^{-4} =$ ....

5 If $(x + 3)$ one factor of the expression : $x^2 + x - 6$, then the other factor is ....

2 Choose the correct answer from the given ones :

1 $4^5 + 4^5 + 4^5 + 4^5 =$ ....

(a) 4^{12} (b) 4^{20} (c) 4^{81} (d) 4^6

2 A man's age now is x years , then his age 7 years ago was .... years.

(a) $x - 7$ (b) $7x$ (c) $7 - x$ (d) $x + 7$

3 If the expression : $x^2 - 12x + k$ is a perfect square , then $k =$ ....

(a) 3 (b) 36 (c) 9 (d) 6

4 The S.S. of $x^2 + 4 = 0$ in $\mathbb{R}$ is ....

(a) $\{0\}$ (b) $\varnothing$ (c) $\{0 , 1\}$ (d) $\{1\}$

5 If $a^3 b^{-3} = 8$, then $\frac{a}{b} =$ ....

(a) $\frac{1}{512}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) 2

6 If $a \in \mathbb{R}^*$, m , n are two non negative integers , then $a^m \times a^{-n} = a^{\cdots\cdots}$

(a) $m - n$ (b) $m + n$ (c) $m\,n$ (d) $n - m$

3 Factorize each of the following :

1 $x^3 + 8$

2 $x^2 - 5x + 6$

3 $3x - 21 + ax - 7a$

4 $9x^2 - 4$

5 * $x^4 + 64$

4 **[a] Find in ℝ the S.S. of the following equation :** $x^3 - 8x^2 + 12x = 0$

[b] 1 Find the positive real number which is if we add its square to three times it , the result will be 28

2 If $\left(\frac{2}{5}\right)^{2x+1} = \frac{8}{125}$, then find the value of x

5 **[a] 1 Simplify to the simplest form :** $\frac{6^{2n} \times 4^n}{2^{4n} \times 3^{2n}}$

2 If $x = 3$, $y = \sqrt{2}$, **find in the simplest form the value of :** $\left(\frac{x}{y}\right)^{-2}$

[b] Selecting randomly a card out of cards numbered from 1 to 20 ,
Find the probability of getting a card carries :

1 A perfect square number. 2 A prime number.

2 Cairo Governorate

Heliopolis Educational Zone

Answer the following questions :

1 **Choose the correct answer :**

1 If $x^2 + kx + 25$ is a perfect square , then k = ...

(a) 5 (b) 10 (c) ± 10 (d) ± 5

2 If $5^{x+2} = 1$, then x = ...

(a) 1 (b) – 2 (c) 2 (d) 5

3 If $x^2 - a = (x - 3)(x + 3)$, then a = ...

(a) 2 (b) – 2 (c) 9 (d) – 9

4 The half of the number 2^8 is = ...

(a) 2^4 (b) 2^7 (c) 4 (d) – 4

5 If $\left(\frac{2}{3}\right)^x = \frac{8}{27}$, then x = ...

(a) 2 (b) 1 (c) 8 (d) 3

6 If $x^3 + 8 = (x + 2)(x^2 + k + 4)$, then k = ...

(a) $-2x$ (b) $4x$ (c) $2x$ (d) $-4x$

2 **Complete the following :**

1 The S.S. of $x^2 + 9 = 0$ in ℝ is ....

2 The multiplicative inverse of the number $(\sqrt{3})^4$ is ....

3 If $(x - 4)$ is a factor of the expression : $x^2 - 5x + 4$, then the other factor is ....

4 The probability of any event A ∈ ....

5 $(\sqrt{5})^3 \div 5\sqrt{5}$ = ...

3 **[a] Factorize each of the following :**

1 $a x + b x + 5 a + 5 b$ 2 $x^3 - 1$ 3 ✻ $x^4 + 4$

[b] Find in ℝ the S.S. of the equation : $x^2 + 9x + 18 = 0$

4 **[a] If** $3^{x-1} = 27$ **, find the value of :** x

[b] Simplify to the simplest form : $\frac{(\sqrt{5})^7 \times (\sqrt{5})^3}{(\sqrt{5})^9 \times (\sqrt{2})^{-3}}$

5 **[a] If** $x = 3$, $y = \sqrt{3}$ **, find the value of :** $\left(\frac{y}{x}\right)^{-2}$

[b] Simplify the following to the simplest form : $\frac{4^{x} \times 6^{2x}}{2^{4x} \times 3^{2x}}$

3 Cairo Governorate

El Wailly Directorate School
Mostakbal G.D.L.4

Answer the following questions :

1 **Choose the correct answer :**

1 If the expression : $x^2 + k x + 36$ is a perfect square , then k = ...

(a) ± 6 (b) ± 8 (c) ± 12 (d) ± 18

2 If $x - y = 5$ and $x^2 + x y + y^2 = 7$, then $x^3 - y^3$ = ...

(a) 2 (b) 7 (c) 12 (d) 35

3 If $3^{x} = 2$, then $(27)^{x}$ = ...

(a) 8 (b) 9 (c) 27 (d) 54

4 If 2 is a solution for the equation : $x^2 - 5x + k = 0$, then k = ...

(a) – 3 (b) 3 (c) 6 (d) – 6

5 $3^4 + 3^4 + 3^4$ = ...

(a) 3^3 (b) 3^4 (c) 3^{12} (d) 3^5

6 $\sqrt{100 - (-6)^2}$ = ...

(a) 4 (b) ± 8 (c) 8 (d) 16

2 **Complete each of the following :**

1 $9x^2 - 3x = 3x(3x -$ ....$)$

2 If $3^{x-2} = 1$, then x = ...

3 If $x^2 - y^2 = 35$ and $x - y = 5$, then $x + y$ = ...

4 $\left(\frac{x}{y}\right)^{-3} = (\text{..............})^3$

5 The S.S. of the equation : $x^2 + 4 = 0$ in $\mathbb{R}$ is ....

3 Factorize completely each of the following :

1 $9x^2 - 16$ 2 $x^3 - 125$

3 $2x^2 + 7x - 4$ 4 $x^3 + x^2 + x + 1$

4 [a] Find in $\mathbb{R}$ the S.S. of the equation : $2x^3 = 18x$

[b] Simplify : $\frac{4^{x+1} \times 9^{2-x}}{6^{2x}}$, then find its value at : $x = 1$

5 [a] If $3^{x-4} = 27$ **, then find the value of :** x

[b] A rectangle whose area is 32 cm^2 and its length is twice its width find its length , width and perimeter.

4 Giza Governorate

National Institutes
Al-Horreya Language School

Answer the following questions :

1 Choose the correct answer :

1 The expression : $x^2 + kx + 36$ is a perfect square when k equals ....

(a) ± 6 (b) ± 8 (c) ± 12 (d) ± 18

2 The solution set of the equation : $x^2 - x = 0$ in $\mathbb{R}$ is ....

(a) $\{0\}$ (b) $\{0 , 1\}$ (c) $\varnothing$ (d) $\{1\}$

3 $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})$ is ....

(a) 1 (b) $\sqrt{5}$ (c) $\sqrt{6}$ (d) 5

4 A bag contains 10 similar cards labeled from 1 to 10 , a card is drawn at random , then the probability that this card carries a number divisible by 5 is ....

(a) zero (b) $\frac{1}{5}$ (c) $\frac{2}{5}$ (d) $\frac{1}{2}$

5 If $x + y = 3$ and $x^2 - xy + y^2 = 5$, then $x^3 + y^3 =$ ....

(a) 15 (b) 25 (c) 8 (d) 7

6 The volume of a cube of side length 3 cm. equals .... cm^3

(a) 12 (b) 9 (c) 27 (d) 81

2 **Complete :**

1. $1 - \frac{1}{4} = \dots\dots\dots\dots \%$
2. The solution set of the equation : $x^2 - 1 = 8$ where $x \in \mathbb{Z}^+$ is $\dots\dots\dots\dots$
3. If $7^{x-1} = 3^{x-1}$, then $x = \dots\dots\dots\dots$
4. The probability of a certain event is $\dots\dots\dots\dots$
5. If $x + y = 4$, $x - y = 2$, then $x^2 - y^2 = \dots\dots\dots\dots$

3 **Factorize :**

1. $x^3 - 1$
2. $4x^2 - 9$
3. $a x - 7a + 3x - 21$
4. $2x^2 - 7x + 3$
5. ✱ $x^4 + 4y^4$

4 **[a] Simplify to the simplest form :** $\frac{(\sqrt{2})^5 \times (3)^{-2}}{3 \times (\sqrt{2})^9}$

[b] Find the S.S. for the following equation where $x \in \mathbb{R}$: $x^2 - 8x + 12 = 0$

5 **[a]** If $2^x = 16$, $3^{x+y} = 1$, **find the values of :** x , y

[b] A bag contains a number of similar balls some of them are red , 2 greens and 4 blues. If the probability of drawing a ball with green color is $\frac{1}{6}$, find the number of red balls.

5 Giza Governorate

6 October Directorate

Answer the following questions :

1 **Choose the correct answer :**

1. $(x - 2)^2 = \dots\dots\dots\dots$
   (a) $x^2 - 4$ (b) $(2 + x)^2$ (c) $x^2 + 4$ (d) $x^2 - 4x + 4$
2. $4^3 + 4^3 + 4^3 + 4^3 = \dots\dots\dots\dots$
   (a) 4^3 (b) 4^4 (c) 4^{12} (d) 4^{81}
3. If $k x^2 - 12x + 4$ is a perfect square , then $k = \dots\dots\dots\dots$
   (a) -6 (b) -4 (c) -2 (d) 9
4. If $\frac{a}{b} = 1$, then $4a - 4b = \dots\dots\dots\dots$
   (a) 8 (b) 4 (c) 1 (d) 0

5 If $x + y = 3$, $x^2 - xy + y^2 = 5$, then $x^3 + y^3 =$ ....

(a) 15 (b) 25 (c) 8 (d) 7

6 If $3^x = 2$, then $27^x =$ ....

(a) 9 (b) 4 (c) 8 (d) 1

2 Complete the following :

1 If $x^3 y^{-3} = 8$, then $\frac{y}{x} =$ ....

2 A bag contains 9 cards labeled by numbers from 1 to 9 , a card is drawn randomly , then the probability that this card carries an odd number = ...

3 The S.S. of the equation : $x^2 + 1 = 0$ in $\mathbb{R}$ is ....

4 If $\frac{2x}{5} = 6$, then $x - 5 =$ ....

5 $(a - 2)(2a - 3) =$ .... $- 7a +$ ....

3 Factorize each of the following :

1 $3x^2 - 48$

2 $x^2 - 7x + 10$

3 $x^3 + 2x^2 - 4x - 8$

4 $2x^3 - 16y^3$

4 [a] Find the S.S. in $\mathbb{R}$:

1 $3^{2n-5} = 1$

2 $\left(\frac{2}{3}\right)^{2n} = \frac{81}{16}$

[b] A bag contains cards numbered from 1 to 20 and card drawn randomly

Find the probability of :

1 Getting a number divisible by 4

2 Getting a number multiple of 7

5 [a] Find the real number which if we added its square to its three times , it becomes 28

[b] Simplify : $\frac{4^n \times 6^{2n}}{2^{4n} \times 3^{2n}}$

6 Alexandria Governorate

Mid Educational Zone
Maths Supervision

Answer the following questions :

1 Complete each of the following :

1 The simplest form of : $(\sqrt{3})^3 \times (\sqrt{3})^5 =$ ....

2 If $x + y = 5$, $x - y = 3$, then $x^2 - y^2 =$ ....

3 If $x - 6 = 0$, then $x =$ ....

4 $y^3 -$ .... $= (y - 2)(y^2 +$ .... $+ 4)$

5 $\left(\sqrt{7}+\sqrt{6}\right)^8 \left(\sqrt{7}-\sqrt{6}\right)^8 =$ ....

2 Choose the correct answer :

1 The expression : $x^2 + 8x + a$ is a perfect square when a = ...

(a) -4 (b) 4 (c) 8 (d) 16

2 A die is thrown once , then the probability of appearance 7 on the upper face is ....

(a) $\frac{-5}{6}$ (b) $\frac{1}{6}$ (c) 0 (d) $\frac{5}{6}$

3 If $(x + 3)^{zero} = 1$, then $x \in$ ....

(a) $\mathbb{R} - \{3\}$ (b) $\mathbb{R} - \{-3\}$ (c) $\{3\}$ (d) $\mathbb{R}$

4 If the age of Kamal now is x years , then his age 3 years ago was .... years.

(a) $x + 3$ (b) $3x$ (c) $x - 3$ (d) $6x$

5 The multiplicative inverse of 1 is ....

(a) 0 (b) 1 (c) 2 (d) 3

6 $3^3 + 3^3 + 3^3 =$ ....

(a) 3^3 (b) 3^4 (c) 3^{12} (d) 3^{81}

3 [a] Factorize :

1 $9x^2 - 4$

2 $10y^2 - 7y - 12$

3 ✱ $4x^4 + 1$

[b] Find the solution set in $\mathbb{R}$ for : $2x^2 - 2x - 12 = 0$

4 [a] Find in the simplest form : $\frac{x^6 \times x^2}{x^3}$ where $x \neq 0$

[b] Factorize : $3x - 21 + ax - 7a$

5 [a] A numbered card is selected randomly from a set of similar cards numbered from 1 to 15

Find the probability of getting a card carries :

1 A prime number.

2 A number divisible by 3

[b] If $2^{x-2} = 32$, **then find the value of :** x

7 Alexandria Governorate

East Educational Zone
Math's Supervision

Answer the following questions :

1 Complete the following :

1. If $x : 49 = 2 : 7$, then $x =$ ....
2. If $2^x = 8$, then $x =$ ....
3. The value of the expression : $3^5 + \left(\sqrt{3}\right)^{10} - 2\,(3)^5 =$ ....
4. $6x^2 - 11x - 10 = (2x -$ .... $)$ $($.............. $+ 2)$
5. 1 , 4 , 9 , 16 , ... (in the same pattern)

2 Choose the correct answer from those given :

1. If $5x = 35$, then $2x + 1 =$ ....
   (a) 7 (b) 8 (c) 15 (d) 71
2. If $5^x = 4$, then $5^{x-1} =$ ....
   (a) 1.25 (b) 0.8 (c) 0.125 (d) 0.08
3. If $x^2 - y^2 = 16$, $y - x = 2$, then $x + y =$ ....
   (a) 4 (b) 8 (c) -8 (d) 2
4. A regular die is thrown and observed the upper face , then the probability of appearance a number divisible by 3 is ....
   (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$
5. If the expression : $x^2 + 14x + b$ is a perfect square , then b = ...
   (a) 2 (b) 7 (c) 14 (d) 49
6. If $3^x + 3^x + 3^x = 1$, then $x =$ ....
   (a) -1 (b) 0 (c) $\frac{1}{3}$ (d) 1

3 [a] A bag contain a number of similar balls , some of them are red , 2 greens , 4 blues. If the probability of drawing a ball with green color is $\frac{1}{6}$
Find the number of red balls.

[b] Factorize each of the following expression :

1. $2x^2 + 7x + 3$
2. $x^2 - 5x$

4 **[a] Find the S.S. for the following equation where $x \in \mathbb{R}$: $x^2 - 8x + 12 = 0$**

[b] Simplify to the simplest form : $\dfrac{\left(\sqrt{2}\right)^5 \times (3)^{-2}}{3 \times \left(\sqrt{2}\right)^9}$

5 **[a]** Find the dimensions of a rectangle whose length is 4 cm. more than its width and whose area is 21 cm.^{2}

[b] If $\frac{8^x \times 9^x}{18^x} = 64$ find the value of 4^{-x}

8 El-Kalyoubia Governorate

Maths Supervision

Answer the following questions :

1 **Choose the correct answer :**

1. $\mathbb{Z} - \mathbb{Z}^- =$ ....
   (a) $\mathbb{Z}^+$ (b) $\mathbb{N}$ (c) $\varnothing$ (d) $\{0\}$
2. The volume of the cube of side length 3 cm. equals .... cm.^{3}
   (a) 9 (b) 12 (c) 27 (d) 81
3. The expression : $x^2 + 4x + a$ is a perfect square when a = ...
   (a) 3 (b) 4 (c) 8 (d) 16
4. The S.S. of the equation : $x^2 - x = 0$ is .... in $\mathbb{R}$
   (a) $\{0\}$ (b) $\varnothing$ (c) $\{0, 1\}$ (d) $\{1\}$
5. If $(x - 1)$ is one factor of the expression : $x^2 - 4x + 3$, then the other factor is ....
   (a) $x + 3$ (b) $x + 1$ (c) $x - 3$ (d) $x - y$
6. If $\left(\frac{5}{3}\right)^x = \left(\frac{3}{5}\right)^2$, then $x =$ ....
   (a) -2 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

2 **Complete :**

1. If $7^{x-1} = 3^{x-1}$, then $x =$ ....
2. A bag contains 9 cards labled by numbers from 1 to 9, a card is drawn randomly, then the probability that the card carries an odd number is ....
3. $a^{-4} + 1 = a^{-4}$ (.............. + ...) where $a \neq 0$
4. $1 - \frac{3}{4} =$ ....
5. $4^3 + 4^3 + 4^3 + 4^3 = 4^{\cdots\cdots}$

3 **[a] Factorize :**

1. $x^2 - y^2$
2. $y^3 + 8$

[b] Find the S.S. of the following equation in $\mathbb{R}$: $x^2 - x - 6 = 0$

4 **[a] Factorize :**

1 $a x - 7a + 3x - 21$ 2 $x^2 - 5x$

[b] If $3^x = 27$, $4^{x+y} = 1$, **find the value of :** x , y

5 **[a] Find in the simplest form :** $\frac{4^{x+1} \times 9^{2-x}}{(6)^{2x}}$, then calculate the result when $x = 1$

[b] If one digit of the number 37450 chosen at random , find the probability that the chosen digit is an even number.

9 El-Monofia Governorate

Kwesna Educational Directorate
Mathematics Supervision

Answer the following questions : ***(Calculator is premitted)***

1 **Choose the correct answer from those given :**

1 $0.002 \times 0.05 =$ ....

(a) 10^{-5} (b) 10^{-4} (c) 10^4 (d) 10^5

2 The expression : $(x - 2y)(x^2 + 2xy + 4y^2)$ equals ....

(a) $x^3 - 2y^3$ (b) $x^3 - 8y^3$ (c) $x^3 + 8y^3$ (d) $x^3 + 18y^3$

3 The value of the expression : $5^{20} + 5^{21}$ equals ....

(a) 5×5^{40} (b) 5×5^{41} (c) 6×5^{20} (d) 6×5^{21}

4 The value of the expression : $2^5 + \left(\sqrt{2}\right)^{10}$ equals ....

(a) 2^6 (b) 2^{10} (c) $\left(\sqrt{5}\right)^{15}$ (d) $\left(\sqrt{2}\right)^{20}$

5 If the probability of choosing a boy from a class of 40 students is 0.375 , then the number of girls is ....

(a) 35 (b) 25 (c) 20 (d) 15

6 The solution set of the equation : $(x - 1)^2 = 0$ in $\mathbb{R}$ is ....

(a) $\{-1\}$ (b) $\{1\}$ (c) $\{-1, 1\}$ (d) $\{2\}$

2 **Complete :**

1 The expression : $x^2 - 2x + k$ is perfect square when k = ...

2 If $3^x \times 2^{-x} = 1.5$, then $x =$ ....

3 If $a^2 + b^2 = 7$, $ab = 3$, then $(a - b)^2 =$ ....

4 $x(y - z) + L(y - z) = (y - z)$ (...............)

5 $\left(\frac{\sqrt{3}}{9}\right)^{-1} = \left(\sqrt{3}\right)^{\cdots\cdots}$

3 [a] An integer is added to its multiplicative inverse the result equals 2 Find the number.

[b] Factorize each of the following :

1. $3x^2 - 15x + 12$
2. $\frac{1}{3}x^3 - 9$
3. * $x^4 + 9x^2 + 81$

4 **[a] Simplify :** $\frac{4^{x+1} \times 9^{2-x}}{(6)^{2x}}$, then calculate its value at $x = 1$

[b] If $x = \frac{\sqrt{3}}{2}$, $y = \frac{1}{\sqrt{3}} = z = \frac{\sqrt{2}}{2}$, **find the value :** $x^2 + (xz)^2 \times y^2$

5 [a] A set of cards numbered from 0 to 10 , if a card is drawn randomly

, find the probability of each :

1. Drawing a card carries odd number.
2. Drawing a card carries a number divisible by 5

[b] Factorize each of the following :

1. $ax - 7a + 3x - 21$
2. $9x^2 - 25$

10 El-Gharbia Governorate

Official Languages Schools
The Central Maths Supervision

Answer the following questions :

1 **Choose the correct answer :**

1. If $x^2 - 2x - k = (x + 3)(x - 5)$, then k = ...
   (a) – 2 (b) – 8 (c) 15 (d) 2
2. The expression : $x^2 + 14x + b$ is a perfect square , than b = ...
   (a) 2 (b) 7 (c) 14 (d) 49
3. The solution set of the equation : $x^2 + 9 = 0$ in $\mathbb{R}$ is ....
   (a) $\{3\}$ (b) $\{-3 , 3\}$ (c) $\{-3\}$ (d) $\varnothing$
4. If $3^{2+x} = 4^{x+2}$, then 7^{x+2} = ...
   (a) 7 (b) – 7 (c) – 2 (d) 1
5. In a mixed school there are 320 students. If the probability that the ideal student is a boy equals 0.6 , then the number of girls of the school equals .... girls.
   (a) 256 (b) 192 (c) 128 (d) 196
6. If $\frac{a}{b} = 1$, then 4 a – 4 b = ...
   (a) 8 (b) 4 (c) 0 (d) 1

2 **Complete each of the following :**

1 The solution set of $\frac{x}{4} = \frac{25}{x}$ in $\mathbb{R}$ is ....

2 If $x = (\sqrt{5} - 2)^7$, $y = (\sqrt{5} + 2)^7$, then $x y =$ ....

3 If $y^3 - a = (y - 2)(y^2 + 2y + 4)$, then a = ...

4 If $(x + 1)$ is one of the factors of the expression : $5x^2 - 2x - 7$, then the other factor is ....

5 $1 - \frac{1}{4} =$ .... %

3 **Factorize each of the following expressions completely :**

1 $x^2 + 8x + 15$

2 $x^3 - 27$

3 $2x^2 + 7x + 3$

4 $a x - 7a + 3x - 21$

4 [a] If $\frac{8^x \times 9^x}{18^x} = 64$, **find the value of :** 4^{-x}

[b] Find the solution set of the following equation where $x \in \mathbb{R}$: $x^2 - 8x + 12 = 0$

5 [a] 1 **Simplify :** $\frac{4^n \times 6^{2n}}{2^{4n} \times 3^{2n}}$

2 Find the rational number whose four times its square equals 81

[b] A box contains 3 red balls , 4 yellow balls and 5 green balls.
A ball is drawn randomly from the box. **Find the probability of the drawn ball is :**

1 Yellow 2 Green 3 Not red

11 El-Dakahlia Governorate

Directorate of Education
Maths Supervision

Answer the following questions :

1 **Choose the correct answer :**

1 If $x + y = 8$ and $x^2 - y^2 = 12$, then $x - y =$ ....

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 4 (d) 20

2 The solution set of the equation : $x^2 - 7 = 9$ in $\mathbb{R}$ is ....

(a) $\{4 , -4\}$ (b) $\{4\}$ (c) $\varnothing$ (d) $\{16\}$

3 $\frac{3^x \times 3^x \times 3^x}{3^x + 3^x + 3^x} = 1$, then $x =$ ....

(a) 2 (b) 3 (c) $\frac{1}{2}$ (d) $\frac{-1}{3}$

4 If $(x^2 - 10x + a)$ is a perfect square , then a = ...

(a) -5 (b) 5 (c) 25 (d) ± 25

5 A cube of edge length = 6 cm. , then its volume = ... cm^3

(a) 12 (b) 24 (c) 36 (d) 216

6 If $2^x = 5$, then 8^x = ...

(a) 125 (b) 25 (c) 15 (d) 20

2 Complete :

1 If $7^{x+5} = 3^{x+5}$, then x = ...

2 $1 - \frac{2}{5}$ = ... %

3 $x^3 -$ .... $= (x-2)(x^2$ .... $+ 4)$

4 If $x^{-2} + 1 = x^{-2}$ (.............. + ...)

5 The probability of a certain event = ...

3 Factorize completely each of the following :

1 $2x^2 - 5x + 3$

2 $x^3 + x^2 + 3x + 3$

3 $6x^2 - 24$

4 $8x^3 + 1$

4 [a] Find the solution set in $\mathbb{R}$: $x^2 = 9x - 14$

[b] Find the value of n such that : $\frac{9^{n+1} \times 8^n}{6^{2n+2}} = 16$

5 [a] If $3^x = 81$ and $4^{x+y} = 1$ **, then find the value of :** x , y

[b] A box contains a number of similar balls 8 of them are red balls and the others are white , if the probability of the chosen red ball is $\frac{2}{3}$

Find the number of white balls.

12 Ismailia Governorate

Directorate of Education
Math's Supervision

Answer the following questions :

1 Choose the correct answer :

1 The probability of impossible event = ...

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) $\varnothing$

2 If $3^{x-2} = 1$, then x = ...

(a) 0 (b) 1 (c) 2 (d) 3

3 $\{2, 3, 6\} \cap \{3, 4, 5\} =$ ....

(a) $\{2\}$ (b) $\{4\}$ (c) $\{6\}$ (d) $\{3\}$

4 If $4x^2 + 12x + m$ is a perfect square , then m = ...

(a) 9 (b) 3 (c) 4 (d) 16

5 $3^x + 3^x + 3^x = 3^{\cdots\cdots}$

(a) $3x$ (b) x^3 (c) $x + 1$ (d) x

6 half of $2^6 =$ ....

(a) 2^3 (b) 2^5 (c) 2^{12} (d) 2^4

2 Complete :

1 If $4^x = 7$, then $4^{x+1} =$ ....

2 $12 - 2 \times 4 \div (9 - 5) =$ ....

3 $\left(\frac{3}{4}\right)^x = \frac{27}{64}$, then $x =$ ....

4 The S.S. in $\mathbb{R}$ of : $x^2 + 4 = 0$ is ....

5 $x(a + b) - y(a + b) = (a + b)$ (..............)

3 [a] Factorize each of the following :

1 $x^2 - 11x + 18$

2 $x^3 + 27$

3 ✻ $4x^4 + y^4$

[b] If $2^x = 32$ and $3^{y+1} = 27$, **find the value of :** $x - y$

4 [a] If $\frac{8^x \times 9^x}{(18)^x} = 64$, find the value of x , then find the value of 2^x

[b] Find the S.S. in $\mathbb{R}$ of : $x^2 + 4x - 12 = 0$

5 [a] Factorize :

1 $4x^2 - 9$

2 $5x + ay + 5y + ax$

[b] A box contains 5 red balls , 3 white balls and 3 blue balls. If a ball selected randomly **, find the probability of getting :**

1 A red ball

2 A red or a blue ball

3 Not a red ball

13 El-Beheira Governorate

Edko Directorate
Maths Supervision

Answer the following questions :

1 Choose the correct answer from the given ones :

1) $(x-2)^2 = \dots\dots$

(a) x^2+4 (b) x^2-4 (c) x^2+4x+4 (d) x^2-4x+4

2) $5^x = 25$, then $x = \dots\dots$

(a) 1 (b) 2 (c) 3 (d) 4

3) The S.S. of : $x^2+64=0$ in $\mathbb{R}$ is $\dots\dots$

(a) $\{4\}$ (b) $\varnothing$ (c) $\{-4\}$ (d) $\{4 , -4\}$

4) If $9^{x+1} = 2^{x+1}$, then x is $\dots\dots$

(a) 1 (b) -1 (c) 0 (d) 2

5) $3^x \times 3^x \times 3^x = \dots\dots$

(a) 3^{3x} (b) 3^{x+1} (c) 3^{x+3} (d) 9^{3x}

6) If the expression : $x^2 + ax + 25$ is perfect square , then a = $\dots\dots$

(a) 5 (b) 10 (c) 8 (d) 18

2 Complete each of the following :

1) If $x+y=2$, $x-y=8$, then $x^2-y^2 = \dots\dots$

2) $x^3-27 = (\dots\dots - \dots\dots)(x^2+3x+9)$

3) If $6^{n-2} = 1$, then n = $\dots\dots$

4) Third the number 3^{20} is $\dots\dots$

5) The number $\left(\sqrt{2}\right)^{-2}$ in the simplest form is $\dots\dots$

3 [a] If simplify : $\dfrac{(4)^{x+1} \times (9)^{2-x}}{(6)^{2x}}$, then find the value answer when $x = 1$

[b] Find the S.S. in $\mathbb{Q}$: $x^2 - x = 12$

4 **[a] Factorize :**

1) $3x^2+7x+2$ 2) $ax-7a+3x-21$

[b] If $3^x = 27$, $4^{x+y} = 1$, **find the value of :** x and y

5 [a] The length of rectangle more than its width by 4 cm. and its area 12 cm^2.
, find the dimensions of the rectangle.

[b] A card is chosen randomly from ten cards numbered from 5 to 14 ,

What is the probability that the chosen card is :

1 An even number ? 2 A prime number ?

14 El-Menia Governorate

El-Menia Educational Directorate
Minia Kawmia Language School

Answer the following questions :

1 Complete the following :

1 The solution set of the equation : $x^2 - 1 = 8$, where $x \in \mathbb{Z}^+$ is ....

2 If $3^{x-4} = 1$, then $x =$ ....

3 The S.S. of the equation : $x^2 - 25 = 0$ in $\mathbb{R}$ is ....

4 If $\left(\frac{2}{3}\right)^x = \frac{3}{2}$, then $x =$ ....

5 The volume of a cube of side length 3 cm. equals .... cm^3

2 Choose the correct answer :

1 The S.S. of the equation : $x(x-2) = 0$ in $\mathbb{R}$ is ....

(a) $\{0\}$ (b) $\{2\}$ (c) $\{0, 2\}$ (d) $\{0, -2\}$

2 If $x^3 y^{-3} = 8$, then $\frac{x}{y} =$ ....

(a) $\frac{1}{512}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) 2

3 The expression : $x^2 + kx + 36$ is a perfect square when k equals ....

(a) ± 6 (b) ± 8 (c) ± 12 (d) ± 18

4 $4^3 + 4^3 + 4^3 + 4^3 =$ ....

(a) 4^{12} (b) 16^{12} (c) 16^2 (d) 16^3

5 If the probability of success of a student is 0.75 , then the probability of his failure is ....

(a) 0.20 (b) 0.25 (c) 0.30 (d) 0.35

6 If $(x - 1)$ is one factor of expression : $x - 4x + 3$, then the other factor is ....

(a) $x + 3$ (b) $x + 1$ (c) $x - 3$ (d) $x - y$

3

[a] If $\frac{8^x \times 9^x}{18^x} = 64$ **find :** x

[b] **Find the S.S. of the equation in** $\mathbb{R}$ **:** $x^2 - 1 = 8$

4 **Factorize each of the following expresions :**

[1] $a x - 7 a + 3 x - 21$ [2] $x^3 + 8$ [3] $x^2 - x - 6$

[4] $4 x^2 - 9$ [5] * $x^4 + 324$

5 **[a]** If $(3)^{x-2} = 9$, then find the value of : x

[b] A colored marble is drawn randomly out of a box containing 12 red marbles , 18 white marbles and 20 blue marbles.

Find the probability of selecting :

[1] A white marble. [2] A yellow marble.

[3] A red or blue marble. [4] A non red marble.

15 Aswan Governorate

Aswan Educational Directorate
Amr Farid distinct official Language School

Answer the following questions :

1 **Complete each of the following :**

[1] $\left(\frac{-1}{\sqrt{2}}\right)^6 = \text{..............}$

[2] If $x + y = 5$ and $x - y = 4$, then $x^2 - y^2 = \text{..............}$

[3] A regular die is thrown once and observed the upper face , then the probability of appearance number divisible by 5 is ....

[4] $x^3 - \text{..............} = (x - 2)(\text{..............} + 2x + 4)$

[5] $\sqrt[3]{0.08 \times 0.1} = \text{..............}$

2 **Choose the correct answer from those given :**

[1] The S.S. of the equation : $x^2 - x = 0$ in $\mathbb{R}$ is ....

(a) $\{0\}$ (b) $\varnothing$ (c) $\{0 , 1\}$ (d) $\{1\}$

[2] The probability of the certain events is ....

(a) $\frac{1}{2}$ (b) 0 (c) $\varnothing$ (d) 1

[3] If $5^x = 4$, then 5^{x-1} equals ....

(a) 1.25 (b) 0.8 (c) 0.125 (d) 0.08

[4] If $3^x = 5$ and $3^y = 4$, then $3^{x+y} = \text{..............}$

(a) 15 (b) 20 (c) 9 (d) 1

5 The value of : $2^5 + \left(\sqrt{2}\right)^{10}$ is ....

(a) 2^6 (b) 2^{10} (c) $\left(\sqrt{2}\right)^{15}$ (d) $\left(\sqrt{2}\right)^{20}$

6 $(-1)^9 + (-1)^8 =$ ....

(a) 2 (b) 0 (c) 1 (d) -1

3 [a] If $\left(\frac{2}{5}\right)^{2x-1} = \frac{8}{125}$ **, find :** x

[b] Find the solution set of the following equation where $x \in \mathbb{R}$ **:** $x^2 - 6x = 0$

4 **[a] Find in the simplest form the value of :** $\frac{\left(\sqrt{3}\right)^{-5} \times \left(\sqrt{3}\right)^{-4}}{\left(\sqrt{3}\right)^{-10}}$

[b] A box contains a number of similar balls , 2 of them are green , 4 are blue and the rest are red , at choosing one randomly , and the probability of the drawn ball with green color is $\frac{1}{6}$, then find the number of red balls.

5 **Factorize each of the following expressions :**

1 $x^2 + 8x + 15$

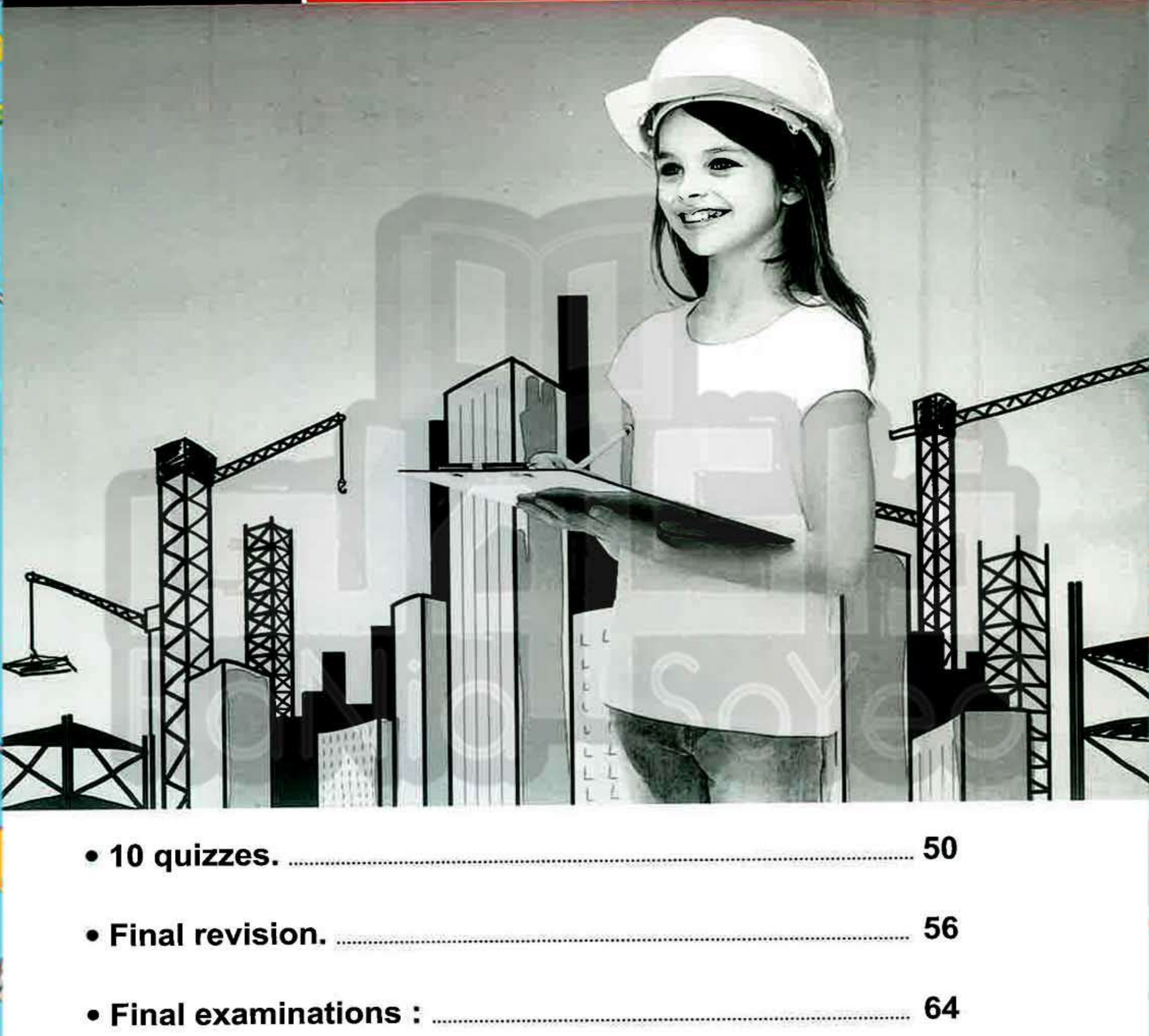
2 $25x^2 - y^2$

3 $x^3 - 8$

4 $18y^2 - 12y + 2$

5 ✻ $81x^4 + 4z^4$

Second Geometry



Quizzes

on Geometry

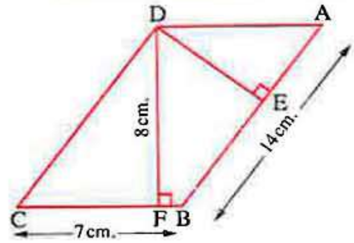
Quiz 1 on lesson 1 – unit 4

time 20 min.

1 Complete the following :

1. Surfaces of two parallelograms with common base and between two parallel straight lines , one is carrying this base are ....
2. A parallelogram of area 60 cm^2 and its height is 5 cm. , then the length of the corresponding base to this height is .... cm.
3. If ABCD is a parallelogram in which : AB = 5 cm. , BC = 10 cm. and the smallest height 4 cm. , then the greatest height equals ....

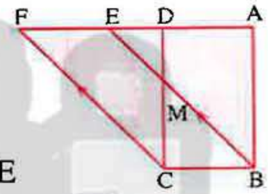
2 [a] In the opposite figure :



ABCD is a parallelogram
, $\overline{DE} \perp \overline{AB}$, $\overline{DF} \perp \overline{CB}$

Find : 1. The area of ▱ ABCD
2. The length of $\overline{DE}$

[b] In the opposite figure :



ABCD is a rectangle , $\overline{BE} \parallel \overline{CF}$
, $E \in \overleftrightarrow{AD}$, $F \in \overleftrightarrow{AD}$

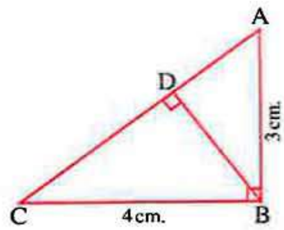
Prove that : Area of the figure ABMD = Area of the figure FCME

Quiz 2 till lesson 2 – unit 4

time 20 min.

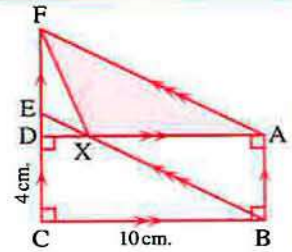
1 Complete the following :

1. If the area of a triangle is 24 cm^2 and its height is 6 cm. , then the length of the corresponding base to this height = ...
2. ABCD is a parallelogram. If the area of Δ ABC equals 25 cm^2 , then the area of the parallelogram ABCD = ... cm^2
3. **In the opposite figure :**



ABC is a right-angled triangle at B
, $\overline{BD} \perp \overline{AC}$, then
BD = ... cm.

2 In the opposite figure :



ABCD is a rectangle
, ABEF is a parallelogram
, DC = 4 cm. , BC = 10 cm.

Find by proof : The area of Δ AXF

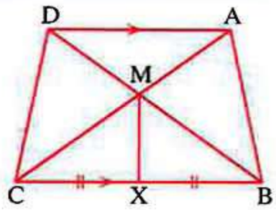
Quiz 3 till lesson 3 – unit 4

time 20 min.

1 Complete the following :

1. Two triangles which have the same base and the vertices opposite to this base are on a straight line parallel to the base have ....
2. The ratio between the area of a triangle and the area of a parallelogram with common base and between two parallel straight lines = ...
3. In Δ ABC , if D is the midpoint of $\overline{BC}$, then the area of Δ ABD = ... the are of Δ ABC

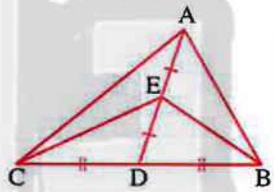
2 [a] In the opposite figure :



ABCD is a quadrilateral in which $\overline{AD}$ // $\overline{BC}$ If the two diagonals intersect at M and X is the midpoint of $\overline{BC}$

Prove that : the area of the figure ABXM = the area of the figure DCXM

[b] In the opposite figure :



The area of Δ ABC = 100 cm^2.

, D is the midpoint of $\overline{BC}$, E is the midpoint of $\overline{AD}$

Find : the area of Δ EBC

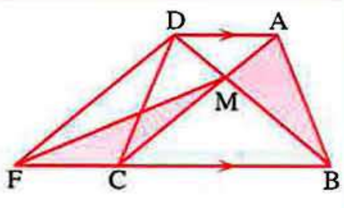
Quiz 4 till lesson 4 – unit 4

time 20 min.

1 Complete the following :

1. The median of a triangle divides its surface into ....
2. The triangle XYZ in which A $\in \overline{YZ}$ where YA = $\frac{1}{2}$ AZ , then the area of Δ XYA = ... the area of Δ XYZ
3. If two triangles are equal in area and drawn on the same base and on one side of it , then their vertices ....

2 In the opposite figure :



$\overline{AD}$ // $\overrightarrow{BC}$, $\overline{AC} \cap \overline{BD}$ = {M}

, the area of Δ ABM = the area of Δ MCF

Prove that : $\overline{MC}$ // $\overline{FD}$

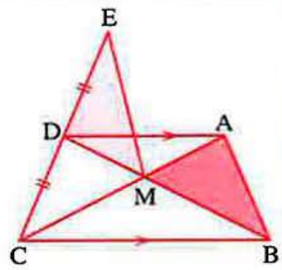
Quiz 5 till lesson 5 – unit 4

time 20 min.

1 Complete the following :

1. The trapezium in which the lengths of the two parallel bases are 5 cm. and 7 cm. and its height is 4 cm. , then its area = ... cm^2.
2. Triangles of bases equal in length and lying between two parallel straight lines are ....
3. A rhombus whose diagonals lengths are 12 cm. and 10 cm. , then its area = ... cm^2.

2 [a] In the opposite figure :



$\overline{AD} \parallel \overline{BC}$, $\overline{AC} \cap \overline{BD} = \{M\}$, $E \in \overrightarrow{CD}$

Such that : CD = DE

Prove that : the area of Δ MDE = the area of Δ AMB

[b] Find the area of the rhombus whose perimeter is 20 cm. and the length of one of its diagonals is 8 cm.

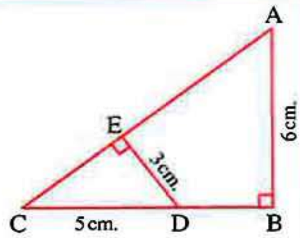
Quiz 6 till lesson 1 – unit 5

time 20 min.

1 Complete the following :

1. If the ratio of magnification between two similar triangles is 3 : 2 and the length of a side from the greatest triangle = 15 cm. , then the length of the corresponding side in the smallest triangle = ... cm.
2. Two triangles are similar if the lengths of their corresponding sides are .... or the measures of their corresponding angles are ....
3. The lengths of the two parallel bases of a trapezium are 14 cm. and 10 cm. and its area is 120 cm^2 , then its height = ... cm.

2 [a] In the opposite figure :



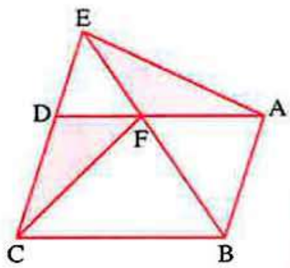
Δ ABC is a right-angled at B

, $\overline{DE} \perp \overline{AC}$, AB = 6 cm. , ED = 3 cm. , CD = 5 cm.

Prove that : Δ CED ~ Δ CBA

and find the length of $\overline{AC}$ and the length of $\overline{BD}$

[b] In the opposite figure :



ABCD is a parallelogram

, $E \in \overrightarrow{CD}$

, $\overline{AD} \cap \overline{BE} = \{F\}$

Prove that : the area of Δ AFE = the area of Δ CDF

Quiz 7 till lesson 2 – unit 5

time 20 min.

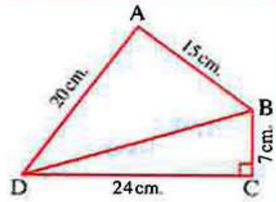
1 Complete the following :

1. In Δ ABC , if $(AC)^2 = (AB)^2 - (BC)^2$, then m (∠ ....) = 90°
2. The area of a square is 98 cm^2 , then the length of its diagonal = ... cm.
3. Two similar triangles , the side lengths of one of them are 9 cm. , 12 cm. , 16 cm. and the perimeter of the other is 111 cm. , then the side lengths of the second triangle are .... , ... , ...

2 [a] In the opposite figure :

m (∠ BCD) = 90° , AB = 15 cm.
, BC = 7 cm. , CD = 24 cm. , AD = 20 cm.

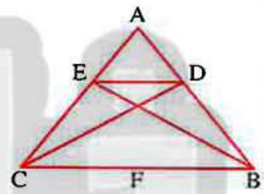
1. **Find :** the length of $\overline{BD}$
2. **Prove that :** m (∠ A) = 90°



[b] In the opposite figure :

The area of Δ ABE = the area of Δ ACD

Prove that : $\overline{DE}$ // $\overline{BC}$



Quiz 8 till lesson 3 – unit 5

time 20 min.

1 Complete the following :

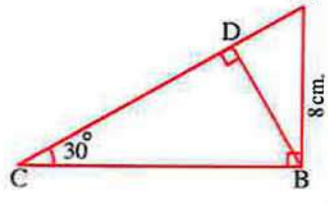
1. If the ratio of magnification between two similar triangles is .... , then the two triangles are congruent.
2. The length of the projection of a given line segment on a given straight line .... the length of the original line segment.
3. The projection of a point on a given straight line is ....

2 [a] ABCD is an isosceles trapezium in which $\overline{AD}$ // $\overline{BC}$, if BC = 2AD = 20 cm. and its area is 180 cm^2 , find the length of each of its legs

[b] In the opposite figure :

ABC is a triangle in which : m (∠ B) = 90°
, m (∠ C) = 30° , AB = 8 cm. , $\overline{BD} \perp \overline{AC}$

Find :
1. The length of the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$
2. The length of the projection of $\overline{BC}$ on $\overleftrightarrow{AC}$



Quiz 9 — till lesson 4 – unit 5

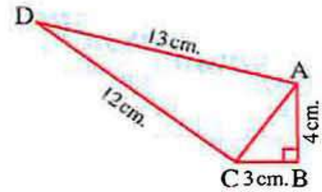
time 20 min.

1 Complete the following :

1. The triangle ABC in which : $(AC)^2 + (AB)^2 - (BC)^2 = 0$, then m (∠ ……………) = 90°
2. The length of the projection of a line segment parallel to a given straight line on this straight line …………… the length of the original line segment.
3. If Δ ABC ~ Δ DEF , AB = $\frac{1}{2}$ DE , then perimeter of Δ ABC = …………… perimeter of Δ DEF

2 [a] In the opposite figure :

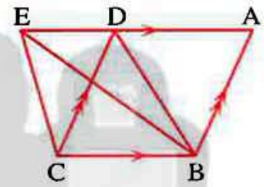
m (∠ ABC) = 90° , AB = 4 cm.
, BC = 3 cm. , AD = 13 cm. , CD = 12 cm.



1. **Prove that :** m (∠ ACD) = 90°
2. **Find :** the length of the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$

[b] In the opposite figure :

ABCD is a parallelogram , E ∈ $\overrightarrow{AD}$



Prove that :

the area of Δ ABD = the area of Δ BCE

Quiz 10 — till lesson 5 – unit 5

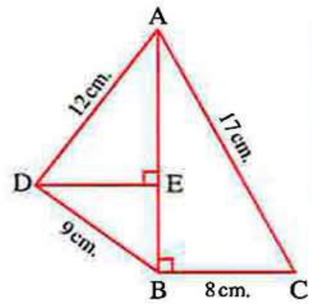
time 20 min.

1 Complete the following :

1. The area of a rhombus = 30 cm^2 , the length of one of its diagonals = 6 cm. , then the length of the other diagonal = …………… cm.
2. In Δ ABC : If $(AC)^2 + (CB)^2 = (AB)^2 - 4$, then the type of the angle C is ……………
3. A triangle whose side lengths are 24 cm. , 7 cm. and 25 cm. , then its area = ……… cm^2

2 [a] In the opposite figure :

m (∠ ABC) = 90° , $\overline{DE} \perp \overline{AB}$
, CB = 8 cm. , AC = 17 cm. , BD = 9 cm.
, AD = 12 cm.

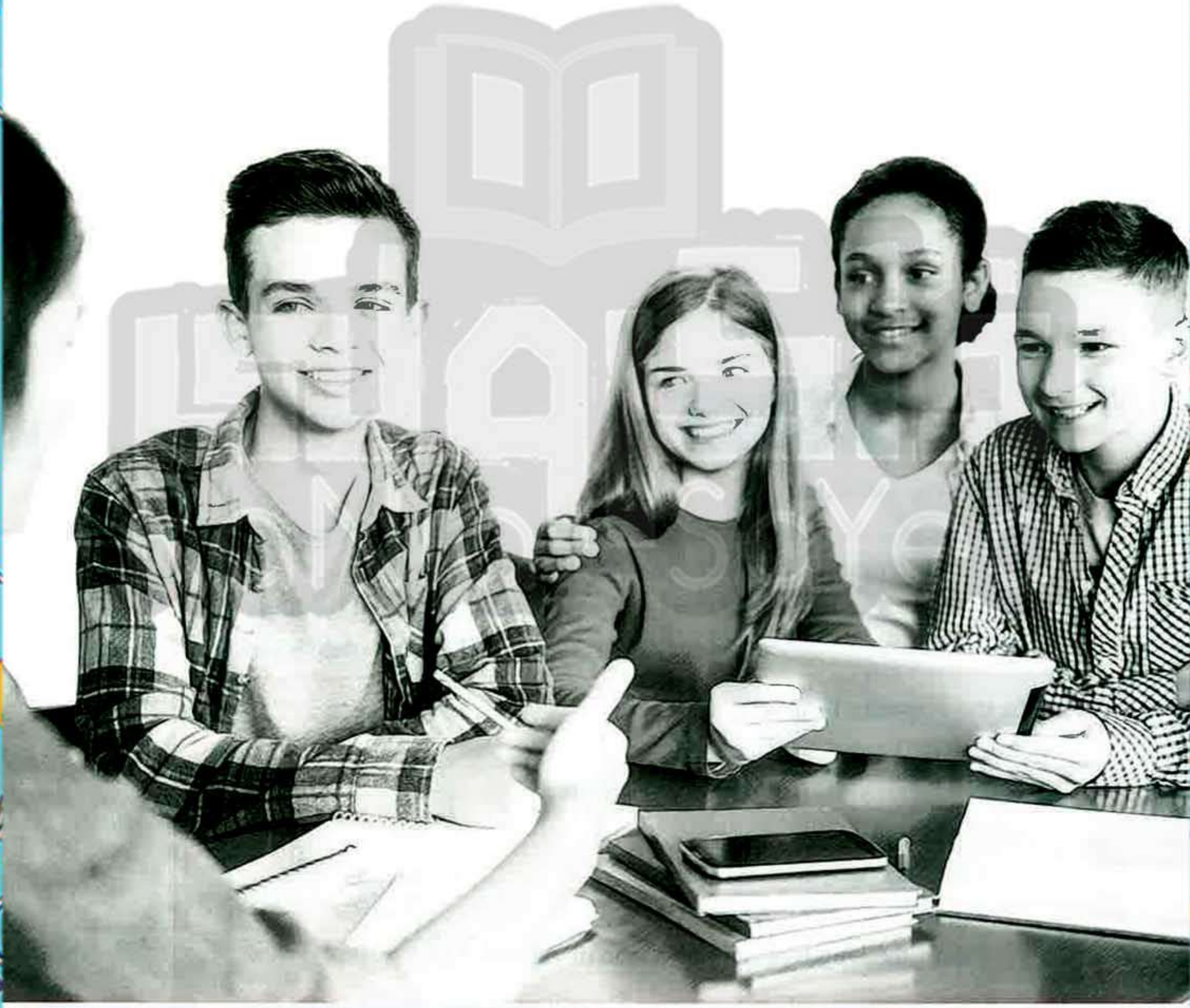


1. **Find :** the length of $\overline{AB}$
2. **Prove that :** m (∠ ADB) = 90°
3. **Find :** the length of the projection of $\overline{AD}$ on $\overleftrightarrow{AB}$

[b] If AB = 6 cm. , BC = 8 cm. , CA = 9 cm.
Determine the type of Δ ABC according to its angles.

Final Revision

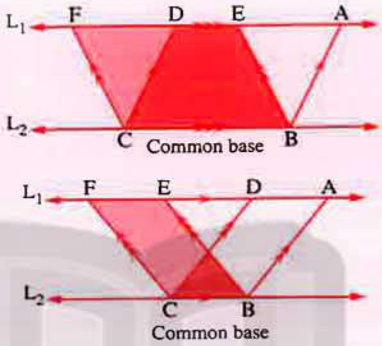
of Geometry



Revision for the important theorems , corollaries and rules of geometry

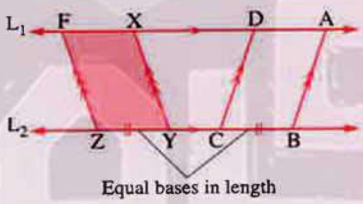
First Remember the relation between the areas of two parallelograms

Surfaces of two parallelograms with common base and between two parallel straight lines , one is carrying this base , are equal in area.



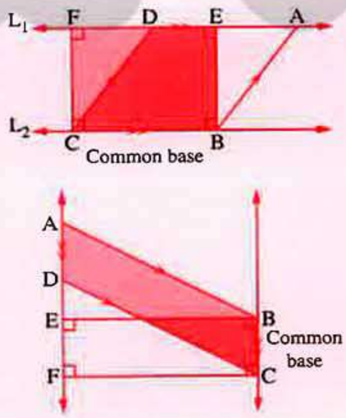
If $\overline{AF}$ // $\overline{BC}$, $\overline{BC}$ is a common base , then :
The area of ▱ ABCD = the area of ▱ EBCF

The parallelograms with bases equal in length and lying on a straight line , while the opposite sides to these bases are on another straight line, are equal in area.



If $\overleftrightarrow{AF}$ // $\overleftrightarrow{BZ}$, BC = YZ , then :
The area of ▱ ABCD = the area of ▱ XYZF

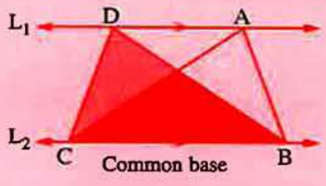
The parallelogram and the rectangle with common base and between two parallel straight lines are equal in area.



If $\overleftrightarrow{AF}$ // $\overleftrightarrow{BC}$, $\overline{BC}$ is a common base , then :
The area of ▱ ABCD = the area of rectangle EBCF

Second Remember the relation between the areas of two triangles

Two triangles which have the same base and the vertices opposite to this base on a straight line parallel to the base have the same area.

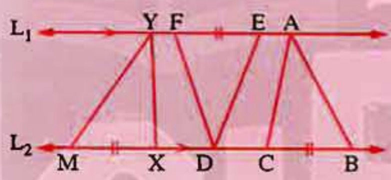


If $\overleftrightarrow{AD} // \overleftrightarrow{BC}$, $\overline{BC}$ is a common base , then :
The area of Δ ABC = the area of Δ DBC

Notice that

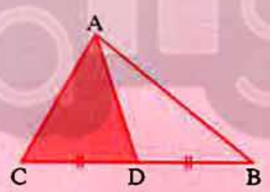
If two triangles are equal in area and drawn on the same base and on one side of it , then their vertices lie on a straight line parallel to this base.

Triangles of bases equal in length and lying between two parallel straight lines are equal in area.



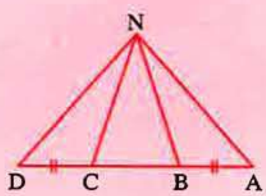
If $L_1 // L_2$, BC = EF = XM , then :
The area of Δ ABC = the area of Δ DEF = the area of Δ YXM

The median of a triangle divides its surface into two triangular surfaces equal in area.



If $\overline{AD}$ is a median in Δ ABC , then :
The area of Δ ABD = the area of Δ ACD $= \frac{1}{2}$ the area of Δ ABC

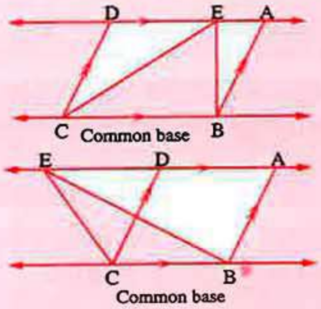
Triangles with congruent bases on one straight line and have a common vertex are equal in areas.



If $\Delta\Delta$ ABN , DCN are common in the vertex N , $\overline{AB}$ and $\overline{CD}$ are on the same straight line , AB = CD , then :
The area of Δ ABN = the area of Δ CDN

Third — Remember the relation between the area of the triangle and the area of the parallelogram

Area of a triangle is equal to half of area of a parallelogram if they have a common base lying on one of two parallel straight lines including them.



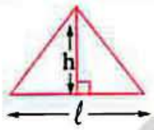
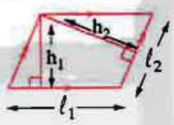
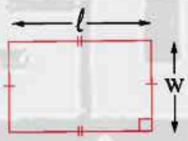
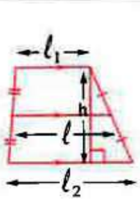
If $E \in \overleftrightarrow{AD}$, $\overleftrightarrow{AD} // \overleftrightarrow{BC}$,

$\overline{BC}$ is a common base ,

then : The area of Δ BEC

$= \frac{1}{2}$ the area of ▱ ABCD

Fourth — Remember areas and perimeters of some geometric figures

The figure		The perimeter	The area
Triangle		The sum of lengths of its three sides	$\frac{1}{2}$ of the base length × its corresponding height $= \frac{1}{2}\ \ell \times h$
Parallelogram		The sum of lengths of two adjacent sides × 2 $= 2(\ell_1 + \ell_2)$	The base length × its corresponding height $= \ell_1 \times h_1 = \ell_2 \times h_2$
Rectangle		2 (Length + Width) $= 2(\ell + w)$	Length × Width $= \ell \times w$
Square		Side length × 4 = 4 ℓ	Square of side length $= \ell^2$ or $\frac{1}{2}$ of the square of its diagonal length $= \frac{1}{2} r^2$
Rhombus		Side length × 4 = 4 ℓ	Side length × height $= \ell \times h$ or $\frac{1}{2}$ the product of the lengths of the two diagonals $= \frac{1}{2} r_1 \times r_2$
Trapezium		The sum of lengths of its sides	$\frac{1}{2}$ the sum of lengths of the two parallel bases × height $= \frac{1}{2}(\ell_1 + \ell_2) \times h$ or the length of the middle base × height $= \ell \times h$

Notice that

In the isosceles trapezium : ❶ The two base angles of it are equal in measure.

❷ The two diagonals of it are equal in length.

Fifth Remember the similarity

1 Similarity of two polygons → It is said that the two polygons are similar if the following two conditions are verified together.

2 Similarity of two triangles → It is said that the two triangles are similar if one of the following two conditions is verified.

→ 1 Their corresponding angles are equal in measure.
2 Their corresponding side lengths are proportional.

Remarks

1. In the two similar polygons P_1 and P_2 , the constant ratio among the lengths of the corresponding sides of P_1 and P_2 is called the ratio of enlargement or the drawing scale.
   If the constant ratio is :
   - Greater than 1 , then the polygon P_1 is an enlargement to the polygon P_2
   - Less than 1 , then the polygon P_1 is a minimizing of the polygon P_2
   - Equal to 1 , then the polygon P_1 is congruent to the polygon P_2
2. If two polygons P_1 and P_2 are similar , then we deduce that :
   - Their corresponding angles are equal in measure.
   - Their corresponding side lengths are proportional.

   i.e. If $\Delta ABC \sim \Delta DEF$, then :
   (1) $m(\angle A) = m(\angle D)$, $m(\angle B) = m(\angle E)$, $m(\angle C) = m(\angle F)$
   (2) $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$
3. The order of corresponding vertices should be kept in givin names of similar polygons.
4. The congruent polygons are similar but it is not necessary that the similar polygons are congruent.
5. If each of two polygons is similar to a third polygon , then they are similar.
6. Any two regular polygons of the same number of sides are similar.
   For example : • Any two equilateral triangles are similar.
   • Any two squares are similar , and so on.
7. The two right-angled triangles are similar if the measure of an acute angle in one of them is equal to the measure of an acute angle in the other.
8. The two isosceles triangles are similar if the measure of an angle in one of them equals the measure of the corresponding angle in the other.
9. The ratio between the perimeters of two similar polygons = the ratio between the lengths of two corresponding sides.

Sixth — Remember the converse of Pythagoras' theorem

In a triangle , if the sum of the areas of two squares on two sides is equal to the area of the square on the third side , then the angle opposite to this side is a right angle.

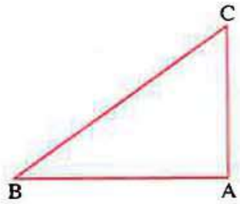
We can state this theorem as follows :

In a triangle , if the square of the length of a side is equal to the sum of the squares of the lengths of the other two sides, then the angle opposite to this side is a right angle.

In Δ ABC :

If $(AB)^2 + (AC)^2 = (BC)^2$

, then : m $(\angle A) = 90°$



Seventh — Remember the projections

1 The projection of a point on a straight line :

- The projection of a point on a straight line is the point of intersection of the perpendicular segment from this point and the straight line.
- If the point lies on the straight line, its projection on it is the same point.

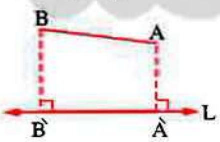
In the opposite triangle :

The projection of the point A on the straight line L is the point À

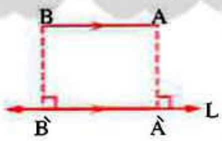
, the projection of the point B on the straight line L is the point B

2 The projection of a line segment on a straight line :

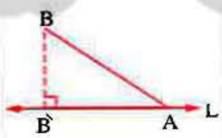
- The projection of a line segment on a given straight line is the line segment whose two endpoints are the projections of the two endpoints of the main line segment on this straight line.



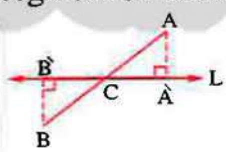
The projection of $\overline{AB}$ on the straight line L is $\overline{\grave{A}\grave{B}}$



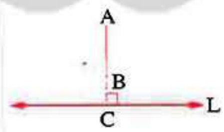
The projection of $\overline{AB}$ on the straight line L is $\overline{\grave{A}\grave{B}}$



The projection of $\overline{AB}$ on the straight line L is $\overline{A\grave{B}}$



The projection of $\overline{AB}$ on the straight line L is $\overline{\grave{A}\grave{B}}$



The projection of $\overline{AB}$ on the straight line L is the point C

Notice that

1. The length of the projection of a line segment on a given straight line $\leq$ the length of the line segment.
2. The projection of a perpendicular line segment to a given straight line is a point , and in this case the length of the projection = zero

3 The projection of a ray on a straight line :

- The projection of a ray on a straight line not perpendicular to it is a ray $\subset$ this straight line.
- The projection of a ray on a straight line perpendicular to it is a point belonging to the straight line.

4 The projection of a straight line on another straight line :

- The projection of a straight line on a straight line not perpendicular to it is a straight line.
- The projection of a straight line on a straight line perpendicular to it is the point of intersection of the two straight lines.

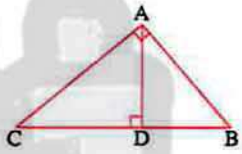
Eighth Remember Euclidean theorem

In the right-angled triangle , the area of the square on a side of the right angle is equal to the area of the rectangle whose dimensions are the length of the projection of this side on the hypotenuse and the length of the hypotenuse.

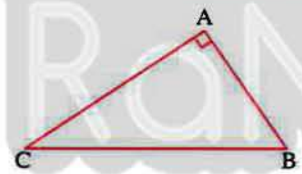
In the opposite figure :

If Δ ABC is right-angled at A , $\overline{AD} \perp \overline{BC}$

, then : $(AB)^2 = BD \times BC$, $(AC)^2 = CD \times CB$



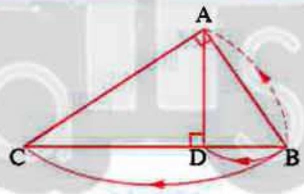
Summary for the important relations of Pythagoras' theorem and Euclidean theorem



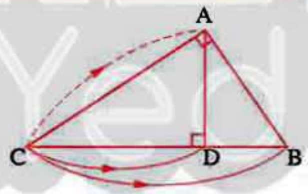
$(BC)^2 = (AB)^2 + (AC)^2$

$(AB)^2 = (BC)^2 - (AC)^2$

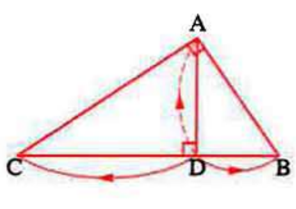
$(AC)^2 = (BC)^2 - (AB)^2$



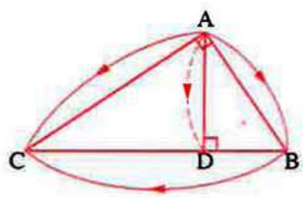
$(BA)^2 = BD \times BC$



$(CA)^2 = CD \times CB$



$(DA)^2 = DB \times DC$



$AD = \frac{AB \times AC}{BC}$

Ninth Remember classifying triangles according to their angles

- In any triangle ABC , if $\overline{AC}$ is the longest side , and :

1. $(AC)^2 = (AB)^2 + (BC)^2$, then Δ ABC is a right-angled at B
2. $(AC)^2 > (AB)^2 + (BC)^2$, then Δ ABC is an obtuse-angled at B
3. $(AC)^2 < (AB)^2 + (BC)^2$, then Δ ABC is an acute-angled.

Generally , to determine the type of the triangle according to its angles , we follow the following :

First : We find the square of the length of each side of its sides.

Second : We compare between the square of the length of the longest side of the triangle and the sum of squares of the lengths of the other two sides.

Third : We determine the type of the triangle according to the previous.

Remarks

* To determine the type of an angle in a triangle , we do the same previous steps with noticing that the comparison between the square length of the side opposite to it and the sum of squares of the other two sides.
* The greatest angle in measure in the triangle is opposite to the longest side.
* In any triangle , there are two acute angles at least.

Final Examinations

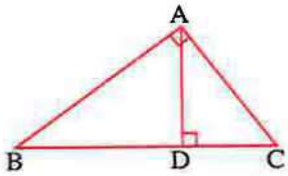
on Geometry

Model Examinations of the School Book on Geometry

Model 1

Answer the following questions :

1 Complete the following :



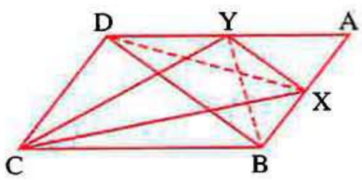
1. **In the opposite figure :**
   AB × .... = BC × AD
2. In Δ ABC , if $(AC)^2 + (BC)^2 = (AB)^2$, then m (∠ ....) = 90°
3. If the point A ∈ the line L , then the projection of the point A on the line L is ....
4. The area of the circle of diameter 14 cm. = ... cm^2 $(\pi = \frac{22}{7})$
5. A trapezium whose bases lengths are 8 cm. , 10 cm. and its height is 5 cm. , then its area equals .... $cm.^2$

2 Choose the correct answer :

1. In Δ ABC , if $(AB)^2 > (BC)^2 + (AC)^2$, then ∠ C is ....
   (a) acute. (b) right. (c) obtuse. (d) straight.
2. A rhombus whose diagonals lengths are 6 cm. , 10 cm. has area .... cm^2
   (a) 60 (b) 30 (c) 15 (d) 10
3. The ratio between the lengths of two corresponding sides of two similar polygons is 3 : 5 , then the ratio between their perimeters is ....
   (a) 2 : 5 (b) 5 : 3 (c) 3 : 5 (d) 1 : 2
4. If the area of a trapezium is 100 cm^2 and its height is 5 cm. , then the length of its middle base = ... cm.
   (a) 20 (b) 30 (c) 40 (d) 50
5. ABCD is a parallelogram in which m (∠ A) = 70° , then m (∠ B) = ...°
   (a) 70 (b) 110 (c) 180 (d) 360
6. Measure of each angle of the regular pentagon is = ...°
   (a) 90 (b) 108 (c) 120 (d) 540

3 [a] The sides lengths of one of two similar triangles are 3 cm. , 4 cm. , 5 cm. and the perimeter of the other triangle is 36 cm. find the side lengths of the other triangle.

[b] **In the opposite figure :**



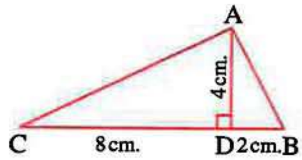
ABCD is a parallelogram , X ∈ $\overline{AB}$
, Y ∈ $\overline{AD}$ such that : The area of Δ CBX = the area of Δ CYD
Prove that : $\overline{XY}$ // $\overline{BD}$

4 [a] **In the opposite figure :**

ABC is a triangle in which : BD = 2 cm.

, CD = 8 cm. , AD = 4 cm. , $\overline{AD} \perp \overline{BC}$

Prove that : m (∠ BAC) = 90°



[b] ABCD is a parallelogram in which : AB = 18 cm. and BC = 12 cm.

We draw $\overline{DE} \perp \overline{BC}$, $\overline{DO} \perp \overline{AB}$, DE = 15 cm.

Calculate the area of parallelogram ABCD and find the length of $\overline{DO}$

5 [a] ABC is a triangle in which m (∠ A) = 50° , m (∠ B) = 60° ,

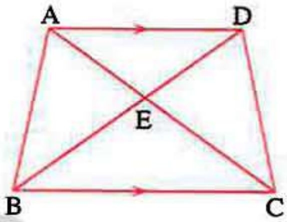
Arrange the lengths of the sides of the triangle in descending order.

[b] **In the opposite figure :**

ABCD is a quadrilateral in which

$\overline{AD} \parallel \overline{BC}$, $\overline{AC} \cap \overline{BD} = \{E\}$

Prove that : the area of Δ ABE = the area of Δ DCE



Model 2

Answer the following questions :

1 **Complete the following :**

1. The two polygons are similar if their corresponding sides are .... and their corresponding angles are ....
2. The area of a rhombus is 24 cm.^{2} , the length of one of its diagonals is 8 cm. , then the length of the other diagonal is ....
3. In Δ ABC , if $(AB)^2 = (AC)^2 - (BC)^2$, then Δ ABC is right-angled at ....
4. A triangle whose side lengths are 6 cm. , 8 cm. and 11 cm. , then its type according to its angles is ....
5. Area of triangle is equal to half of area of a parallelogram if they have a common ....

2 **Choose the correct answer :**

1. A trapezium whose bases lengths are 6 cm. , 8 cm. , then the length of its middle base equals .... cm.

   (a) 48 (b) 24 (c) 14 (d) 7

2. If two polygons are similar and the ratio between the lengths of two corresponding sides is 1 : 3 and the perimeter of the smaller polygon is 15 cm. , then the perimeter of the greater polygon is .... cm.

   (a) 30 (b) 45 (c) 60 (d) 75

3 If the area of the triangle is 24 cm.^{2} and its height = 8 cm. , then the length of the corresponding base .... cm.

(a) 16 (b) 6 (c) 3 (d) 12

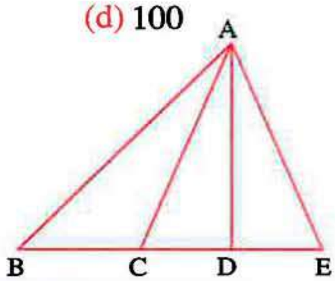
4 ABC is a right-angled triangle at B , $\overline{BD} \perp \overline{AC}$, then the projection of $\overline{BD}$ on $\overleftrightarrow{AC}$ is point ....

(a) A (b) B (c) C (d) D

5 A square of perimeter 20 cm. , then its area equals .... cm^2

(a) 20 (b) 25 (c) 50 (d) 100

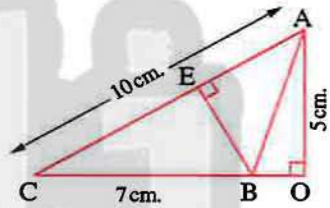
6 The number of the triangles in the opposite figure = ...



(a) 3 (b) 4

(c) 5 (d) 6

3 **In the opposite figure :**



$\overline{AO} \perp \overleftrightarrow{CB}$, $\overline{BE} \perp \overline{AC}$

, AC = 10 cm. , BC = 7 cm. and AO = 5 cm.

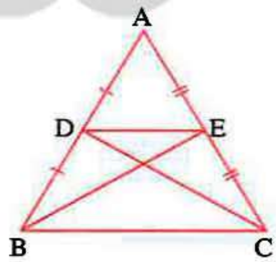
Find : 1 The length of $\overline{BE}$

2 The area of Δ ABC

4 [a] ABCD is a parallelogram in which : AB = 8 cm. , AC = 20 cm. and BD = 12 cm.

Prove that : m ($\angle$ ABD) = 90° , then find the area of this parallelogram.

[b] **In the opposite figure :**



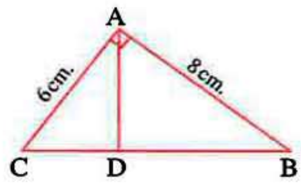
ABC is a triangle in which D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

Prove that :

First : the area of the triangle DBC = the area of triangle EBC

Second : $\overline{DE}$ // $\overline{BC}$

5 [a] **In the opposite figure :**



Δ DBA is a similar to Δ ABC , m ($\angle$ BAC) = 90°

Prove that : $\overline{AD} \perp \overline{BC}$ and if AB = 8 cm. , AC = 6 cm.

Find the length of : $\overline{BD}$

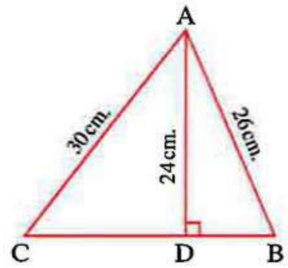
[b] In the opposite figure :

ABC is a triangle , $\overline{AD} \perp \overline{BC}$

If AD = 24 cm. , AB = 26 cm.

and AC = 30 cm.

Find : BC , then calculate area of Δ ABC



Model for the merge students

Answer the following questions :

1 Choose the correct answer from those given :

1. The area of parallelogram whose length of its base 6 cm. and its corresponding height of this base 4 cm. equals .... cm^2.

   (a) 12 (b) 20 (c) 24 (d) 48

2. The triangle whose lengths of its sides 6 cm. , 8 cm. , 10 cm. is ....

   (a) acute-angled triangle. (b) right-angled triangle.
   (c) obtuse-angled triangle. (d) otherwise.

3. The rhombus whose lengths of its diagonals 6 cm. and 10 cm. , then its area = ... cm^2.

   (a) 60 (b) 30 (c) 15 (d) 10

4. Trapezium of length of its middle base 8 cm. and surface area 56 cm^2. , then its height = ... cm.

   (a) 32 (b) 24 (c) 448 (d) 7

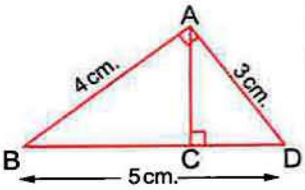
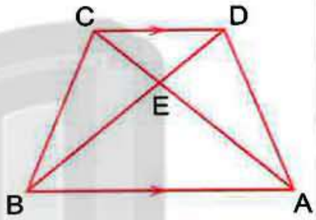
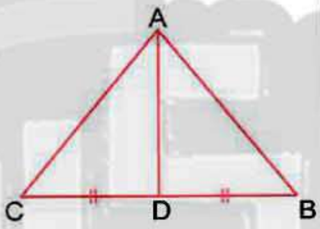
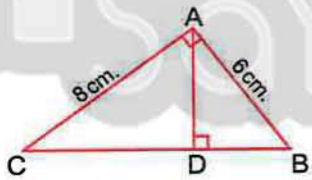
5. All .... are similar.

   (a) squares (b) triangles (c) rectangles (d) parallelograms

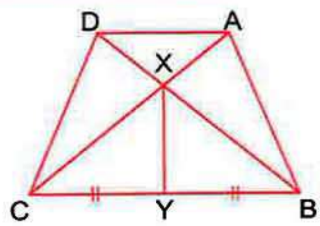
2 Complete each of the following :

1. The projection of point on a straight line is ....
2. If the triangle ABC is obtuse-angled triangle at B , then $(AC)^2$ .... $(AB)^2 + (BC)^2$
3. The square whose length of its diagonal 8 cm. , then its area = ... cm^2.
4. The two triangles have same base and the vertices opposite to this base on straight line parallel to the base ....
5. Area of triangle = $\frac{1}{2}$ × .... × corresponding height.

3 **Join from the column (A) to the suitable one from the column (B) :**

Column (A)	Column (B)
1 **In the opposite figure :** AC = ... cm. 	• BEC
2 **In the opposite figure :** Area of Δ AED = area of Δ .... 	• 2.4
3 **In the opposite figure :** Area of Δ ABD = area of Δ .... 	• Congruent
4 If the ratio of enlargement between two similar triangles = 1, then the two triangles are	• 3.6
5 **In the opposite figure :** The length of the projection of $\overline{AB}$ on $\overline{BC}$ = ... cm. 	• ACD

4 **In the opposite figure :**



Area of the figure ABYX = Area of the figure DCYX

Complete the proof :

To prove that : $\overline{AD}$ // $\overline{BC}$

Given : ....

R.T.P. : ....

Proof : $\overline{XY}$ is median in Δ XBC

∴ Area of Δ .... = area Δ .... (1)

∵ area of the figure ABYX = area of the figure DCYX (2)

By subtracting (1) from (2) :

∴ Area of Δ .... = area of Δ ....

By adding area of Δ ADX to both sides

∴ Area of Δ .... = area of Δ ....

∵ $\overline{AD} // \overline{BC}$

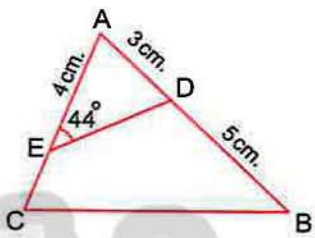
5 **In the opposite figure :**

Δ ABC ~ Δ AED

, m (∠ AED) = 44° , AD = 3 cm. , EA = 4 cm.

, DB = 5 cm. , BC = 8 cm.

Complete to find the length of each of : $\overline{ED}$ and $\overline{EC}$



Solution :

∵ Δ ABC ~ Δ AED

$\therefore \frac{AB}{\cdots\cdots} = \frac{\cdots\cdots}{ED} = \frac{CA}{DA}$

$\therefore \frac{8}{\cdots\cdots} = \frac{\cdots\cdots}{ED} = \frac{CA}{3}$

∴ ED = ... cm. , AC = ... cm. , EC = ... cm.

Schools Examinations of the previous years on Geometry

1 Cairo Governorate

Heliopolis Directorate
Kalousdian-Nubarian Armenian School

Answer the following questions :

1 Choose the correct answer :

1 The area of triangle is equal to .... the area of parallelogram which has a common base and its vertex lies on the straight line parallel to this base.

(a) equal to (b) half (c) twice (d) quarter

2 If $\overline{AB} \parallel \overleftrightarrow{XY}$, then the length of the projection of $\overline{AB}$ on $\overleftrightarrow{XY}$ .... the length of $\overline{AB}$

(a) < (b) > (c) = (d) ≤

3 In Δ ABC : If $(AC)^2 > (AB)^2 + (BC)^2$, then the angle B is ....

(a) acute (b) right (c) obtuse (d) straight

4 The height of the triangle whose area is 24 cm^2 and its corresponding base length is 8 cm. equals .... cm.

(a) 3 (b) 4 (c) 6 (d) 8

5 If the ratio of an enlargement between two triangles equals 1 , then the two triangles are ....

(a) congruent (b) enlargement (c) coincide (d) reduction

6 A rhombus its two diagonals of lengths 8 cm. and 6 cm. , its area equals .... cm^2

(a) 14 (b) 20 (c) 24 (d) 48

2 Complete each of the following :

1 The median of a triangle divides its surface into two triangles ....

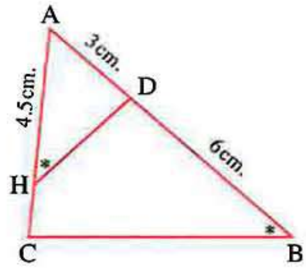
2 If the point A ∈ straight line L , then the projection of the point A on this straight line is ....

3 If two triangles are similar , then their lengths of corresponding sides are ....

4 If the area of a trapezium is 75 cm^2 , and the length of its middle base is 15 cm. , then its height = ... cm.

5 If Δ ABC ~ Δ XYZ , AB = 5 cm. , XY = 10 cm. and YZ = 8 cm. , then BC = ... cm.

3 [a] **In the opposite figure :**

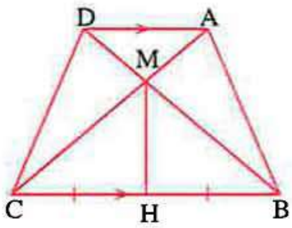


m (∠ AHD) = m (∠ B)

, AD = 3 cm. , AH = 4.5 cm. , and BD = 6 cm.

1. **Prove that :** Δ ADH ~ Δ ACB
2. **Find :** the length of $\overline{HC}$

[b] **In the opposite figure :**

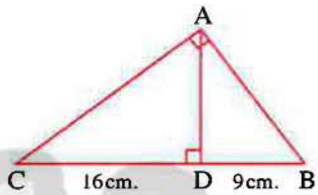


$\overline{AD}$ // $\overline{BC}$, H is the midpoint of $\overline{BC}$

Prove that :

1. The area of Δ AMB = the area of Δ DMC
2. The area of the figure ABHM = the area of the figure DCHM

4 [a] **In the opposite figure :**



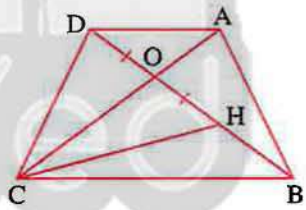
m (∠ BAC) = 90° , $\overline{AD}$ ⊥ $\overline{BC}$

, BD = 9 cm. , DC = 16 cm.

1. **Find :** the length of $\overline{AB}$
2. **Find :** the length of the projection of $\overline{AC}$ on $\overleftrightarrow{AD}$

[b] A trapezium , its area is 6 cm^2. and the length of one of its two parallel bases equals 5 cm. and its height is 7 cm. **Find the length of the other base.**

5 [a] **In the opposite figure :**



ABCD is a quadrilateral its diagonals intersect at the point O

, H ∈ $\overline{BO}$ where OH = OD

, the area of Δ ABO = the area of Δ HOC

Prove that : $\overline{AD}$ // $\overline{BC}$

[b] Identify the type of Δ ABC according to the measures of its angles where AB = 5 cm. , BC = 6 cm. , AC = 7 cm.

2 Cairo Governorate

El-Nozha Directorate of Education
Modern Language Schools

Answer the following questions :

1 **Choose the correct answer :**

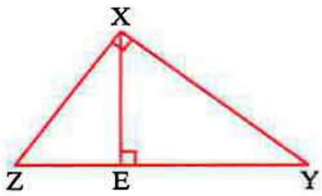
1. The area of the trapezium whose middle base is of length 7 cm. and its height is 6 cm. equals ....

(a) 21 cm^2. (b) 40 cm^2. (c) 42 cm^2. (d) 13 cm^2.

2 In Δ ABC : if m (∠ C) = 90° , AB = 20 cm. and BC = 16 cm.
, then AC = ... cm.

(a) 9 (b) 12 (c) $4\sqrt{41}$ (d) 25

3 **In the opposite figure :**



EY × EZ = ...

(a) $(XE)^2$ (b) $(XZ)^2$

(c) $(YZ)^2$ (d) $(XY)^2$

4 The triangle whose sides lengths are 5 cm. , 8 cm. and 7 cm. is ....-angled triangle.

(a) right (b) acute (c) obtuse (d) straight

5 If the triangle base length is 6 cm. and its area is 24 cm^2
, then its corresponding height is ....

(a) 18 cm. (b) 8 cm. (c) 4 cm. (d) 10 cm.

6 If $\overline{AB}$ // $\overline{XY}$, then the length of the projection of $\overline{AB}$ on $\overleftrightarrow{XY}$ .... the length of $\overline{AB}$

(a) < (b) > (c) = (d) ≥

2 Complete :

1 The two polygons are similar if their corresponding side lengths are .... and their corresponding angles are ....

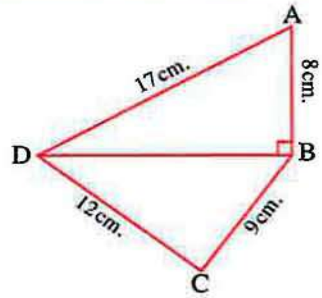
2 If Δ ABC ~ Δ XYZ , m (∠ A) + m (∠ B) = 60° , then m (∠ Z) = ...

3 If Δ ABC is an obtuse-angled triangle at B , then $(AC)^2$ .... $(AB)^2 + (BC)^2$

4 If the length of the diagonal of a square is 10 cm. , then its area = ... cm^2

5 If the ratio between the length of two corresponding sides of two similar polygons is 2 : 5 and the perimeter of the smaller one is 12 cm. , then the perimeter of the other one is ....

3 [a] In the opposite figure :



ABCD is a quadrilateral in which :

AB = 8 cm. , BC = 9 cm.

, CD = 12 cm. , AD = 17 cm.

and $\overline{DB} \perp \overline{AB}$

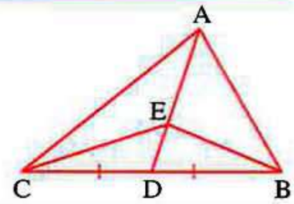
1 **Find :** the length of $\overline{BD}$

2 **Prove that :** m (∠ C) = 90°

[b] In the opposite figure :

Δ ABC with a median $\overline{AD}$, E $\in \overline{AD}$

Prove that : the area of Δ ABE = the area of Δ ACE

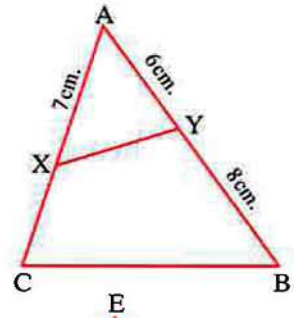


4 [a] In the opposite figure :

Δ AXY ~ Δ ABC , AX = 7 cm.

, AY = 6 cm. , YB = 8 cm.

Find : the length of $\overline{XC}$

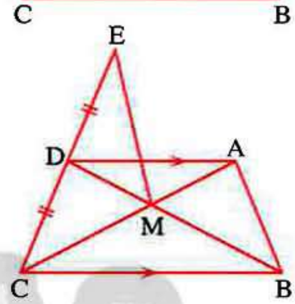


[b] In the opposite figure :

$\overline{AD}$ // $\overline{BC}$, $\overline{AC} \cap \overline{BD} = \{M\}$

, D is the midpoint of $\overline{EC}$

Prove that : the area of Δ MDE = the area of Δ AMB



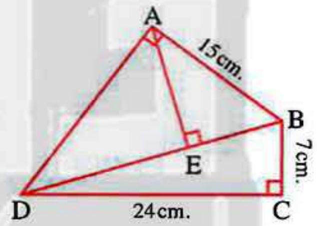
5 In the opposite figure :

ABCD is a quadrilateral , m ($\angle$ BCD) = m ($\angle$ BAD) = 90°

, $\overline{AE} \perp \overleftrightarrow{BE}$

1 **Find :** the length of $\overline{BD}$ and $\overline{AD}$

2 **Find :** the length of the projection of $\overline{AB}$ on $\overleftrightarrow{BD}$



3 Cairo Governorate

El Zawia Educational Zone
Geel Almostakbal E.L.S.

Answer the following questions :

1 Complete each of the following :

1 The length diagonal of a square whose area 50 cm^2 equals ....

2 The lengths of two adjacent sides in a parallelogram are 9 cm. , 6 cm. and the smallest height is 4 cm. , then the length of the other height is ....

3 The two polygons are similar to a third are ....

4 The median of a triangle divide it into two triangle .... in area.

5 The area of trapezium whose parallel bases are 6 cm. , 10 cm. and height 5 cm. equals ....

2 Choose the correct answer :

1. If the area of a rhombus is 24 cm^2. and the length of one of its diagonal is 6 cm. , then the length of the other diagonal is ....

(a) 4 cm. (b) 8 cm. (c) 10 cm. (d) 12 cm.

2. The length of the projection of a given line segment .... the length of the original line segment.

(a) ≥ (b) > (c) ≤ (d) =

3. ABC is an obtuse angle triangle at A in which AB = 5 cm. , BC = 8 cm. , then AC = ... cm.

(a) 5 (b) 7 (c) 8 (d) 13

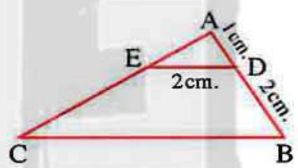
4. Δ ABC in which $(AB)^2 = (AC)^2 + (BC)^2$, m (∠ B) = 40° , then m (∠ A) = ...

(a) 40° (b) 50° (c) 90° (d) 130°

5. The diagonals of an isosceles trapezium are ....

(a) congruent (b) perpendicular (c) bisect each other (d) parallel

6. **In the opposite figure :**



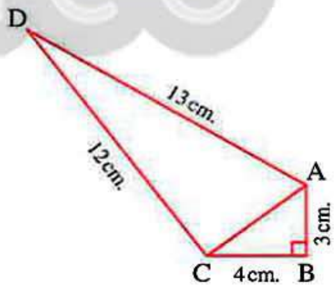
If Δ ADE ~ Δ ABC

, then the length of

$\overline{BC}$ in cm. equals .... cm.

(a) 3 (b) 4 (c) 6 (d) 8

3 [a] In the opposite figure :



AB = 3 cm. , BC = 4 cm.

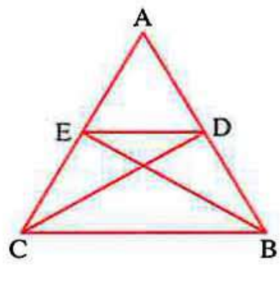
, AD = 13 cm. , CD = 12 cm.

, m (∠ B) = 90°

Prove that :

m (∠ ACD) = 90°

[b] In the opposite figure :

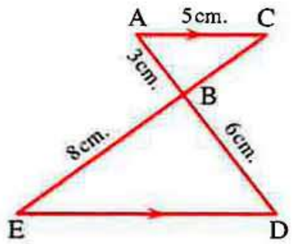


If the area of Δ ADC = the area of Δ AEB

Prove that :

$\overline{DE}$ // $\overline{BC}$

4 **[a] In the opposite figure :**



$\overline{AC} \parallel \overline{DE}$

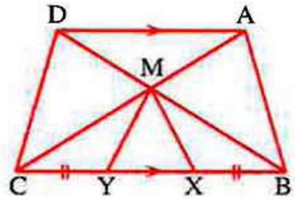
, $\overline{AD} \cap \overline{CE} = \{B\}$

, AB = 3 cm. , AC = 5 cm. , BD = 6 cm. , EB = 8 cm.

1 **Prove that :** $\Delta ABC \sim \Delta DBE$

2 **Find the length of :** $\overline{BC}$, $\overline{DE}$

[b] In the opposite figure :



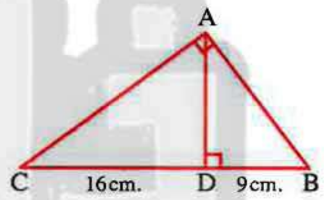
$\overline{AD} \parallel \overline{BC}$, $\overline{AC} \cap \overline{BD} = \{M\}$

, $X \in \overline{BC}$, $Y \in \overline{BC}$ such that BX = CY

Prove that :

The area of the shape ABXM = the area of the shape DCYM

5 **In the opposite figure :**



ABC is a right-angled triangle at A

, $\overline{AD} \perp \overline{BC}$, BD = 9 cm. , CD = 16 cm. **Find :**

1 The length of $\overline{AB}$

2 The length of the projection of $\overline{AC}$ on $\overleftrightarrow{AD}$

3 The area of ΔABC

4 Giza Governorate

Math inspection

Answer the following questions :

1 **Complete each of the following :**

1 A square its diagonal length = 12 cm. its area = ... cm^2.

2 If $\overline{AB} \perp \overleftrightarrow{CD}$, then the length of projection of $\overline{AB}$ on $\overleftrightarrow{CD}$ = ...

3 The two polygons are similar if their corresponding angles are .... and their corresponding sides are ....

4 Area of a triangle is equal to half of area of parallelogram if they have a common base and ....

5 ABC is a triangle , AB = 8 cm. , BC = 9 cm. and AC = 6 cm. , then its type according to its angles is ....

2 Choose the correct answer :

1 Area of a parallelogram = 24 cm^2 and its base length is 6 cm. , then its corresponding height = ... cm.

(a) 8 (b) 4 (c) 48 (d) 12

2 The median of a triangle divides its surface into two triangles are ....

(a) similar (b) congruent (c) equal in area (d) equal in perimeter

3 In Δ ABC if $(AC)^2 < (AB)^2 + (BC)^2$, then the type of ∠ B is ....

(a) obtuse (b) right (c) acute (d) straight

4 The trapezium whose middle base length = 9 cm. and its height = 6 cm. its area = ... cm^2

(a) 27 (b) 54 (c) 15 (d) 108

5 $\overline{AB} \parallel \overleftrightarrow{CD}$, then the length of projection of $\overline{AB}$ on $\overleftrightarrow{CD}$ .... the length of $\overline{AB}$

(a) > (b) < (c) = (d) ≤

6 A rhombus whose diagonal lengths are 12 cm. , 8 cm. its area = ... cm^2

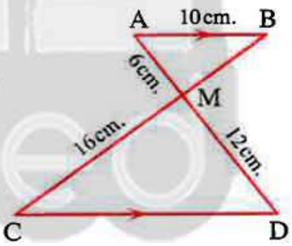
(a) 48 (b) 96 (c) 20 (d) 144

3 [a] In the opposite figure :

$\overline{AB} \parallel \overline{DC}$, $\overline{AD} \cap \overline{BC} = \{M\}$

, AB = 10 cm. , AM = 6 cm.

, MD = 12 cm. , MC = 16 cm.



1 **Prove that :** Δ AMB ~ Δ DMC

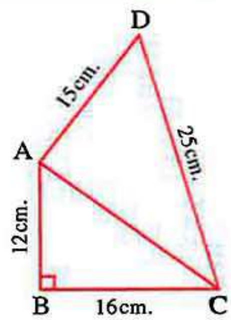
2 **Find the length of :** $\overline{CD}$

[b] Find height of trapezium its area = 450 cm^2 and the two base lengths are 24 cm. , 12 cm.

4 [a] In the opposite figure :

ABCD is a quadrilateral , m (∠ B) = 90° , AB = 12 cm.

, BC = 16 cm. , CD = 25 cm. and DA = 15 cm.



1 **Find :** AC

2 **Prove that :** m (∠ DAC) = 90°

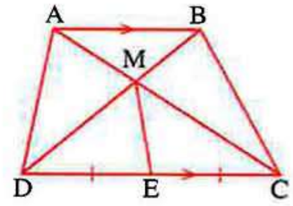
[b] Determine the type of triangle ABC according to its angles If AB = 12 cm. , BC = 5 cm. and AC = 13 cm. , then find its area.

5 [a] **In the opposite figure :**

$\overline{AB} \parallel \overline{DC}$, $\overline{AC} \cap \overline{BD} = \{M\}$

, E is the midpoint of $\overline{CD}$

Prove that : the area of the figure ADEM = the area of the figure BCEM



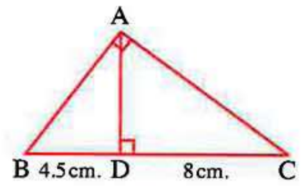
[b] **In the opposite figure :**

ABC is a triangle , m ($\angle$ BAC) = 90°

, $\overline{AD} \perp \overline{BC}$

, BD = 4.5 cm. , DC = 8 cm.

Find : AD , AB , AC



5 Giza Governorate

Omrania Directorate
El-Sadat Governmental Language School

Answer the following questions :

1 Choose the correct answer :

1. In Δ ABC , if $(AB)^2 > (BC)^2 + (AC)^2$, then $\angle$ C is ....
   (a) acute. (b) right. (c) obtuse. (d) straight.
2. A rhombus whose diagonal lengths are 6 cm. , 10 cm. its area = ... cm^2
   (a) 60 (b) 30 (c) 15 (d) 10
3. If the length of the middle base of a trapezium is 8 cm. and its surface area is 56 cm^2 , then its height = ... cm.
   (a) 32 (b) 24 (c) 448 (d) 7
4. If the ratio of enlargement between two triangles equals 1 , then the two triangles are ....
   (a) congruent. (b) enlargement. (c) coincide. (d) reduction.
5. Any triangle has at least two .... angles.
   (a) right (b) obtuse (c) acute (d) straight
6. The isosceles triangle has .... axis of symmetry.
   (a) zero (b) one (c) two (d) three

2 Complete each of the following :

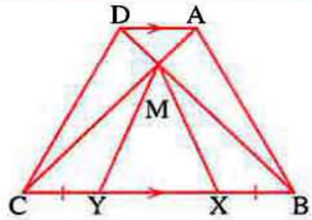
1. The median of a triangle divides it into two triangles .... in area.
2. Two triangles are similar if their corresponding side lengths are ....

3 ABCD is a rectangle , the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is ....

4 Surfaces of two parallelograms with common base and between two parallel straight lines one is carrying this base are ....

5 ABCD is a parallelogram its area = 36 cm^2, E ∈ $\overline{AD}$, then the area of Δ EBC = ... cm^2

3 **[a] In the opposite figure :**

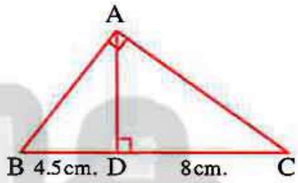


$\overline{AD}$ // $\overline{BC}$, $\overline{AC} \cap \overline{BD} = \{M\}$

, X ∈ $\overline{BC}$, Y ∈ $\overline{BC}$ such that BX = CY

Prove that : the area of the shape ABXM = the area of the shape DCYM

[b] In the opposite figure :

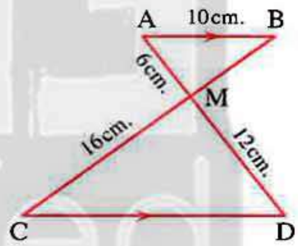


ABC is a triangle in which : m (∠ BAC) = 90° , $\overline{AD} \perp \overline{BC}$

, BD = 4.5 cm. , DC = 8 cm.

Find : the length of each of $\overline{AC}$, $\overline{AB}$, $\overline{AD}$

4 **[a] In the opposite figure :**



$\overline{AB}$ // $\overline{DC}$, $\overline{AD} \cap \overline{BC} = \{M\}$

, AB = 10 cm. , AM = 6 cm.

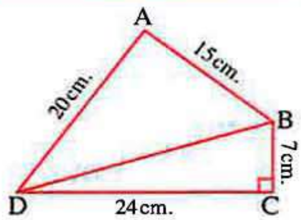
, MD = 12 cm. , MC = 16 cm.

1 **Prove that :** Δ AMB ~ Δ DMC

2 **Find :** the length of $\overline{CD}$, $\overline{MB}$

[b] Identify the type of Δ ABC according to the measures of its angles where AB = 5 cm. , BC = 6 cm. , AC = 7 cm.

5 **[a] In the opposite figure :**

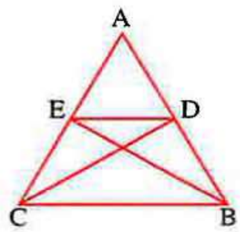


m (∠ BCD) = 90° , AB = 15 cm.

, BC = 7 cm. , CD = 24 cm. , AD = 20 cm.

Prove that : m (∠ BAD) = 90°

[b] In the opposite figure :



If the area of Δ ADC = the area of Δ AEB

Prove that : $\overline{DE}$ // $\overline{BC}$

6 Giza Governorate

6th October Directorate
Om Al Moamneen Language School

Answer the following questions :

1 Choose the correct answer :

1. If the area of parallelogram is 40 cm^2 and its base length is 5 cm. , then the corresponding height to this base = ... cm.
   (a) 16 (b) 8 (c) 20 (d) 5
2. If a triangle ABC in which $(AC)^2 - (AB)^2 = (BC)^2$, then ∠ B is ....
   (a) acute. (b) right. (c) reflex. (d) obtuse.
3. If the area of square is 72 cm^2 , then its diagonal length = ... cm.
   (a) 6 (b) 8 (c) 36 (d) 12
4. ABC is a right-angled triangle at B , $\overline{BD} \perp \overline{AC}$, D $\in \overline{AC}$, then the projection of $\overline{BD}$ on $\overline{AC}$ is ....
   (a) A (b) B (c) C (d) D
5. ABC is a triangle in which $(AB)^2 = (BC)^2 + (AC)^2$ and m (∠ B) = 40° , then m (∠ A) = ...
   (a) 40° (b) 50° (c) 90° (d) 130°
6. If $\overline{XL}$ is a median in Δ XYZ , then the area of Δ XYZ = ... the area of Δ XYL
   (a) 2 (b) 4 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

2 Complete each of the following :

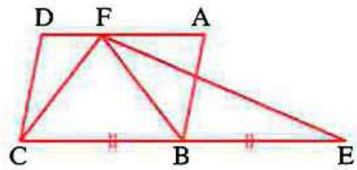
1. If the ratio of enlargement between two triangles equals 1 , then the two triangles are ....
2. A trapezium whose base lengths are 8 cm. , 10 cm. , and its height is 5 cm. , then its area equals .... cm^2.
3. ABC is a right-angled triangle at A , $\overline{AD} \perp \overline{BC}$, D $\in \overline{BC}$, DC = 9 cm. , BD = 16 cm. , then the length of $\overline{AC}$ = ... cm.
4. A triangle whose side lengths 6 cm. , 8 cm. , 11 cm. , then its type according to its angles is ....
5. If ABC is a triangle m (∠ A) = 3 x° , m (∠ B) = 5 x° , m (∠ C) = 4 x° , then m (∠ B) = ...

3 [a] In the opposite figure :

ABCD is a parallelogram.

, E $\in \overrightarrow{CB}$, where BC = BE

Prove that : The area of Δ EFC = The area of ▱ ABCD



[b] In the opposite figure :

ABCD is a quadrilateral ,

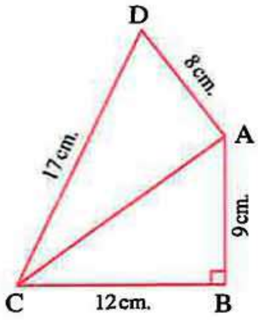
m (∠ B) = 90°

, AB = 9 cm.

, BC = 12 cm. , DA = 8 cm. , CD = 17 cm.

Prove that :

m (∠ DAC) = 90°



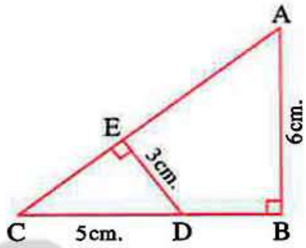
4 **[a] In the opposite figure :**

ABC is a right-angled triangle at B

, $\overline{ED} \perp \overline{AC}$, CD = 5 cm. , AB = 6 cm. , ED = 3 cm.

1 **Prove that :** Δ CED ~ Δ CBA

2 **Find :** The length of $\overline{AC}$



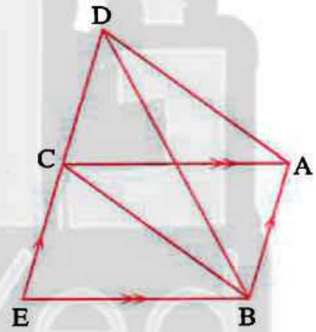
[b] In the opposite figure :

ABEC is a parallelogram.

, D ∈ $\overrightarrow{EC}$

Such that : The area of Δ DBC = the area of Δ EBC

Prove that : $\overline{AD}$ // $\overline{BC}$



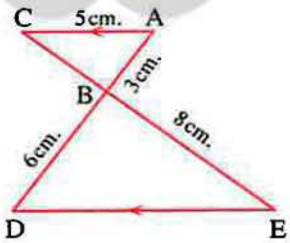
5 **[a] In the opposite figure :**

$\overline{AC}$ // $\overline{ED}$, $\overline{AD} \cap \overline{CE} = \{B\}$, AC = 5 cm.

, AB = 3 cm. , BD = 6 cm. , BE = 8 cm.

1 **Prove that :** Δ ABC ~ Δ DBE

2 **Find :** The perimeter of triangle BED



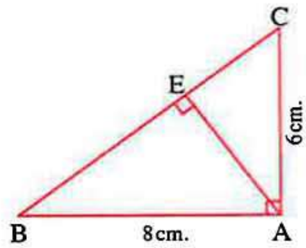
[b] In the opposite figure :

ABC is a triangle in which : $\overline{AE} \perp \overline{BC}$

Find :

1 The length of the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$

2 The length of $\overline{EC}$



7 Alexandria Governorate

West Educational Zone
Inspectorate of Mathematics

Answer the following questions :

1 Complete each of the following :

1. The area of the rhombus whose diagonal lengths are12 cm. , 8 cm. equals .... cm^2.
2. If $\overline{AD} \perp \overline{BC}$, then the length of projection of $\overline{AD}$ on $\overleftrightarrow{BC}$ equals .... cm.
3. ABC is a right-angled triangle at B in which AB = 5 cm. , BC = 12 cm. , then AC = ... cm.
4. In Δ ABC , AB = 8 cm. , BC = 9 cm. and AC = 6 cm. , then the type of this triangle according to its measures of angles is ....
5. The number of axes of symmetry of an isosceles triangle equals ....

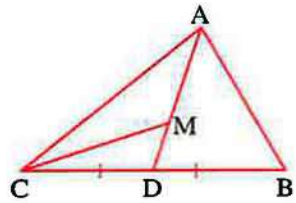
2 Choose the correct answer :

1. The diagonals of an isosceles trapezium are ....
   (a) congruent. (b) perpendicular.
   (c) bisect each other. (d) parallel.
2. If the ratio between two corresponding sides of two similar triangles is 1 : 2 and the measure of an angle of the first triangle equals 60° , then the measure of its corresponding angle in the other triangle equals ....
   (a) 30° (b) 120° (c) 60° (d) 62°
3. The image of the point (2 , 0) is itself by reflection on ....
   (a) X-axis (b) y-aixs
   (c) origin point. (d) X-axis followed by y-axis
4. The perpendicular segment drawn from the right angle of a triangle to the hypotenuse divides it into two .... triangles.
   (a) obtuse-angled (b) acute-angled
   (c) equilateral (d) similar
5. The measure of the complementary angle of an angle whose measure $X°$ equals ....
   (a) 90° (b) $90° - X°$ (c) $X° - 90°$ (d) $90\,X°$
6. ABCD is a parallelogram , E $\in \overline{BC}$, then the area of ▱ ABCD = ... area of Δ EAD
   (a) the same (b) half (c) twice (d) third

3 **[a] In the opposite figure :**

ABC is a triangle , M is the point of concurrence

Prove that : the area of Δ AMC = $\frac{1}{3}$ the area of Δ ABC

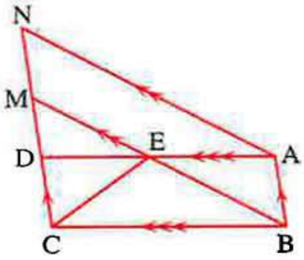


[b] In the opposite figure :

ABCD and ABMN are two parallelograms

Prove that :

The area of Δ EBC = $\frac{1}{2}$ the area of ▱ ABMN



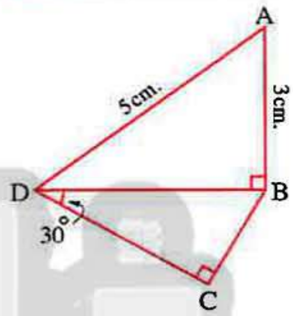
4 **[a] In the opposite figure :**

ABCD is a quadrilateral in which :

m ($\angle$ ABD) = 90° , m ($\angle$ BCD) = 90°

, m ($\angle$ BDC) = 30° , AB = 3 cm. , AD = 5 cm.

Find : the lengths of $\overline{BD}$ and $\overline{BC}$



[b] In the opposite figure :

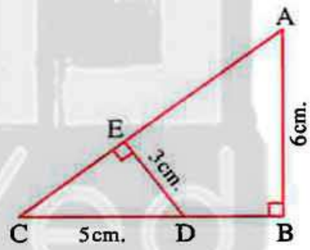
ABC is a right-angled triangle at B

, $\overline{DE} \perp \overline{AC}$, AB = 6 cm.

, ED = 3 cm. , CD = 5 cm.

1 **Prove that :** Δ ABC ~ Δ DEC

2 **Find :** the length of $\overline{AC}$



5 **[a]** The area of a trapezium is 88 cm^2 , its height is 8 cm. and the length of one of the two parallel bases is 10 cm. find the length of the other base.

[b] In the opposite figure :

m ($\angle$ ABD) = 90° , $\overline{CE} \perp \overline{AB}$

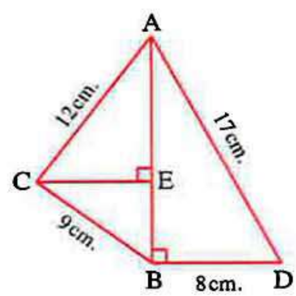
, CB = 9 cm. , AD = 17 cm.

, BD = 8 cm. , AC = 12 cm.

1 **Find :** the length of $\overline{AB}$

2 **Prove that :** m ($\angle$ ACB) = 90°

3 **Find :** the length of the projection of $\overline{AC}$ on $\overleftrightarrow{AB}$



8 Alexandria Governorate

El-Montaza Educational Zone
Math's Supervision

Answer the following questions :

1 Choose the correct answer :

1. Any triangle has at least two .... interior angles.
   (a) right (b) obtuse (c) acute (d) straight
2. If the area of a trapezium is 100 cm^2. and its height is 5 cm. , then the length of its middle base = ... cm.
   (a) 20 (b) 30 (c) 50 (d) 40
3. ABCD is a parallelogram , $E \in \overline{BC}$, the area of ▱ ABCD = ... area of Δ EAD
   (a) the same (b) half (c) twice (d) third
4. If Δ XYZ is an obtuse-angled triangle at Y , then $(XZ)^2$ .... $(XY)^2 + (YZ)^2$
   (a) < (b) > (c) = (d) ≥
5. ABCD is a square , then the projection of $\overline{AC}$ on $\overleftrightarrow{BC}$ is ....
   (a) $\overline{AB}$ (b) $\overline{BC}$ (c) $\overline{CD}$ (d) $\overline{AD}$
6. A rhombus whose diagonal lengths are 6 cm. , 10 cm. has area .... cm^2.
   (a) 60 (b) 30 (c) 15 (d) 10

2 Complete each of the following :

1. If two straight lines intersect , then each two vertically opposite angles are ....
2. In the triangle ABC : if $(AB)^2 = (BC)^2 + (AC)^2$, then m (∠ ....) = 90°
3. Δ ABC ~ Δ XYZ , and m (∠ B) + m (∠ C) = 70° , then m (∠ X) = ...°
4. The number of axes of symmetry of the rectangle is ....
5. The ratio between the perimeters of two similar polygons is 3 : 5 , then the ratio between their lengths of two corresponding sides is ....

3 [a] In the opposite figure :

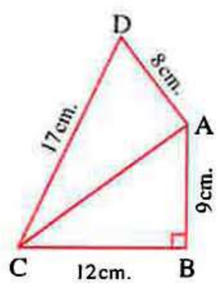
ABCD is a quadrilateral

, m (∠ B) = 90°

, AB = 9 cm. , BC = 12 cm.

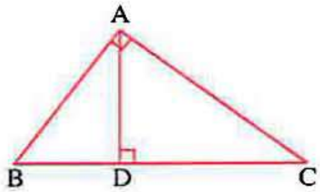
, CD = 17 cm. and DA = 8 cm.

Prove that : m (∠ DAC) = 90°



[b] In the opposite figure :

ABC is a triangle , m (∠ BAC) = 90° , $\overline{AD} \perp \overline{BC}$



Then complete :

1 The projection of $\overline{AC}$ on $\overleftrightarrow{BC}$ is ....

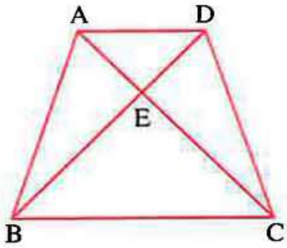
2 The projection of $\overline{AD}$ on $\overleftrightarrow{BC}$ is ....

4 **[a] In the opposite figure :**

The area of Δ AEB

= the area of Δ DEC

Prove that : $\overline{AD}$ // $\overline{BC}$



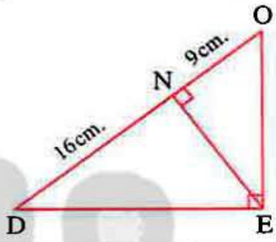
[b] In the opposite figure :

OED is a right-angled triangle at E

, $\overline{EN} \perp \overline{DO}$, DN = 16 cm.

and ON = 9 cm.

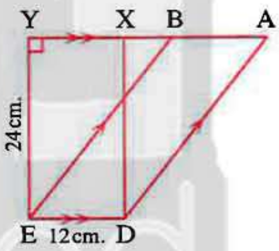
Find the length of : $\overline{EN}$, $\overline{EO}$, $\overline{ED}$



5 **In the opposite figure :**

$\overleftrightarrow{AB}$ // $\overleftrightarrow{DE}$, X ∈ $\overleftrightarrow{AB}$ and Y ∈ $\overleftrightarrow{AB}$

, XDEY is a rectangle and $\overline{AD}$ // $\overline{BE}$



1 **Prove that :** the area of figure ABED = the area of rectangle XYED

2 **Find :** the area of the figure ABED

3 If AD = 30 cm. Find the length of the perpendicular from B to $\overline{AD}$

9 El-Kalyoubia Governorate

Math's Inspection

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The sum of measures of interior angles of a triangle equals .... °

(a) 90 (b) 180 (c) 80 (d) 360

2 The area of the rhombus whose diagonal lengths are 6 cm. and 8 cm. = ... cm^2.

(a) 7 (b) 24 (c) 48 (d) 14

3 ABCD is a parallelogram in which m (∠ A) = 120° , then m (∠ B) = ... °

(a) 120 (b) 60 (c) 90 (d) 180

4 If $\Delta ABC \equiv \Delta XYZ$ and $m(\angle X) = 70^\circ$, then $m(\angle A) = \cdots\cdots\cdots\cdots ^\circ$

(a) 70 (b) 55 (c) 50 (d) 80

5 If $\Delta ABC \sim \Delta XYZ$, then $m(\angle B) = m(\angle \cdots\cdots\cdots\cdots)$

(a) C (b) Z (c) X (d) Y

6 ΔABC in which $(AB)^2 > (BC)^2 + (AC)^2$, then angle C is $\cdots\cdots\cdots\cdots$

(a) acute (b) right (c) obtuse (d) straight

2 Complete each of the following :

1 Area of the parallelogram = length of the base × $\cdots\cdots\cdots\cdots$

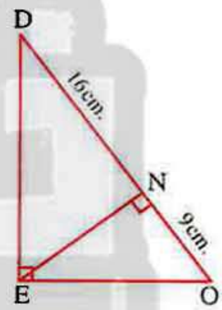
2 Two polygons are similar if $\cdots\cdots\cdots\cdots$

3 If the point $A \in$ the line L , then the projection of the point A on the line L is $\cdots\cdots\cdots\cdots$

4 In ΔABC if $(AB)^2 = (BC)^2 + (AC)^2$, then $m(\angle \cdots\cdots\cdots\cdots) = 90^\circ$

5 Surfaces of two parallelograms with common base and between two parallel straight lines , one is carrying this base are $\cdots\cdots\cdots\cdots$

3 [a] In the opposite figure :



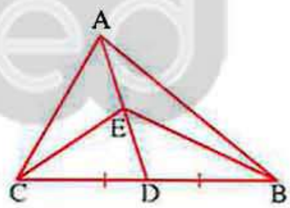
DEO is a right-angled triangle at E

, $\overline{EN} \perp \overline{DO}$, DN = 16 cm.

and ON = 9 cm.

Find the length of : $\overline{EN}$, $\overline{DE}$, $\overline{EO}$

[b] In the opposite figure :

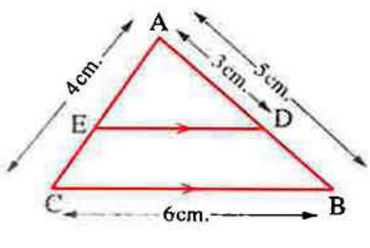


$\overline{AD}$ is a median in ΔABC , $E \in \overline{AD}$

and $\overline{BE}$, $\overline{CE}$ are drawn.

Prove that : the area of ΔABE = the area of ΔACE

4 [a] In the opposite figure :



ABC is a triangle in which AB = 5 cm.

, BC = 6 cm. , AC = 4 cm. and $D \in \overline{AB}$

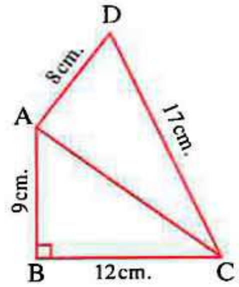
where AD = 3 cm. , $\overline{DE} \parallel \overline{BC}$, $\overline{DE} \cap \overline{AC} = \{E\}$

1 **Prove that :** $\Delta ADE \sim \Delta ABC$

2 Find the length of each of : $\overline{DE}$ and $\overline{AE}$

[b] The area of a trapezium is 108 cm^2. and the length of one of its parallel bases is 15 cm. , find the length of the other base , if the height of the trapezium is 8 cm.

5 **[a] In the opposite figure :**



ABCD is a quadrilateral

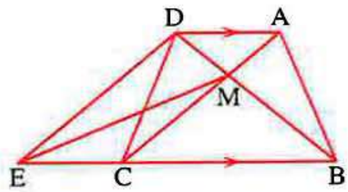
, m (∠ B) = 90° , AB = 9 cm.

, BC = 12 cm. , CD = 17 cm.

and DA = 8 cm.

Prove that : m (∠ DAC) = 90°

[b] In the opposite figure :



ABCD is a quadrilateral , $\overline{AD}$ // $\overline{BC}$, E ∈ $\overrightarrow{BC}$

, $\overline{AC} \cap \overline{BD} = \{M\}$

, the area of Δ ABM = the area of Δ ECM

Prove that : $\overline{DE}$ // $\overleftrightarrow{AC}$

10 El-Sharkia Governorate

Directorate of Education
Dep. of Governmental L. Schools

Answer the following questions :

1 **Choose the correct answer from those given :**

1. ABCD is a parallelogram in which AB = 5 cm. , BC = 10 cm. and its smaller height is 4 cm. , then its area = ... cm^2.

(a) 40 (b) 20 (c) 10 (d) 50

2. Number of axis of symmetry of square is ....

(a) 1 (b) 2 (c) 3 (d) 4

3. In Δ ABC , if $(AB)^2 > (BC)^2 + (AC)^2$, then angle C is .... angle.

(a) acute (b) obtuse (c) right (d) straight

4. If the projection of a line segment on a straight line is a point , then the line segment .... straight line.

(a) // (b) ⊥ (c) ≡ (d) ⊂

5. If M is the midpoint of $\overline{AB}$, then $(AB)^2$ = ... $(AM)^2$

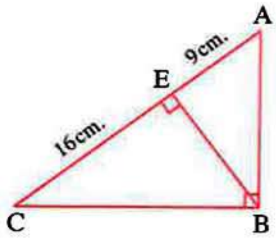
(a) 2 (b) $\frac{1}{2}$ (c) 4 (d) $\frac{1}{4}$

6. The area of the rhombus whose diagonal lengths are 16 cm. , 18 cm. = ... cm^2.

(a) 144 (b) 148 (c) 128 (d) 288

2 Complete each of the following :

1 **In the opposite figure :**



m (∠ ABC) = 90° , $\overline{BE} \perp \overline{AC}$

, EA = 9 cm. , EC = 16 cm. , then AB = ... cm.

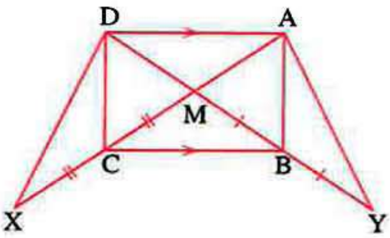
2 In Δ ABC , if $(AB)^2 = (AC)^2 + (BC)^2$, then m (...............) = 90°

3 The two triangles are similar if their corresponding side lengths are ....

4 The median of a triangle divides its surface into ....

5 The area of rectangle = ...

3 [a] **In the opposite figure :**



$\overline{AD}$ // $\overline{BC}$, MB = BY

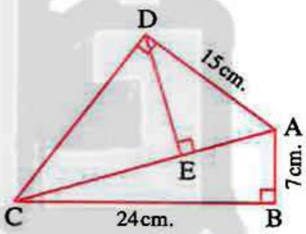
, MC = CX , $\overline{AX} \cap \overline{DY} = \{M\}$

Prove that : 1 the area of Δ MAB = the area of Δ MDC

2 the area of Δ ABY = the area of Δ DCX

[b] Determine the type of triangle ABC according to its angles where AB = 7 cm. , BC = 6 cm. , AC = 9 cm.

4 [a] **In the opposite figure :**



m (∠ ADC) = m (∠ ABC) = 90° , $\overline{DE} \perp \overline{AC}$

, AB = 7 cm. , BC = 24 cm.

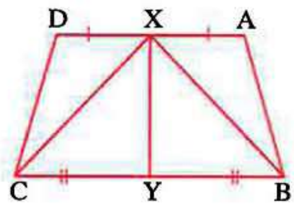
, AD = 15 cm.

Find : 1 The length of $\overline{AC}$ and $\overline{DC}$

2 The length of the projection of $\overline{AD}$ on $\overleftrightarrow{AC}$

[b] Find the height of a trapezium with area of 450 cm^2. and two bases lengths are 24 cm. , 12 cm.

5 [a] **In the opposite figure :**



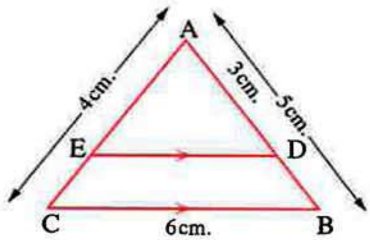
ABCD is a quadrilateral , YC = YB

, X is the midpoint of $\overline{AD}$, Y is the midpoint of $\overline{BC}$

, the area of the figure ABYX = the area of the figure DCYX

Prove that : $\overline{AD}$ // $\overline{BC}$

[b] **In the opposite figure :**



ABC is a triangle , AB = 5 cm.

, BC = 6 cm. , AC = 4 cm.

, AD = 3 cm. , $\overline{DE}$ // $\overline{BC}$

1 **Prove that :** Δ ADE ~ Δ ABC

2 **Find the length of :** $\overline{DE}$ and $\overline{EC}$

11 El-Dakahlia Governorate

Directorate of Education
Maths Supervision

Answer the following questions :

1 Choose the correct answer :

1. The area of a rhombus whose diagonal lengths are 6 cm. and 10 cm. is .... cm^2.
   (a) 60 (b) 120 (c) 30 (d) 15
2. In Δ XYZ , if $(XZ)^2 = (XY)^2 - (ZY)^2$, then $\angle$ Z is .... angle.
   (a) an acute (b) a straight (c) an obtuse (d) a right
3. If the perimeter of a square is 20 cm. , then its area = ... cm^2.
   (a) 400 (b) 10 (c) 25 (d) 12.5
4. If Δ ABC ~ Δ XYZ , then m ($\angle$ ACB) = m ($\angle$ ....)
   (a) XYZ (b) YXZ (c) ZYX (d) XZY
5. If the diameter length of a circle = 14 cm. , then its area = ... cm^2.
   (a) 154 (b) 44 (c) 616 (d) 22
6. If ABC is a triangle , m ($\angle$ B) = 90° , $\overline{BD} \perp \overline{AC}$, then $(AB)^2$ = AD × ....
   (a) AC (b) CD (c) BC (d) AD

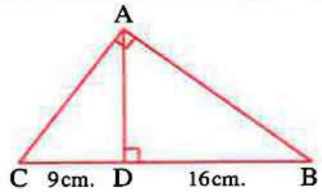
2 Complete each of the following :

1. If the area of a square is 50 cm^2. , then its diagonal length = ...
2. Two polygons are similar if the corresponding side lengths are ....
3. If A ∈ straight line L , then its projection on straight line L is ....
4. ABCD is a parallelogram , if m ($\angle$ A) = 80° , then m ($\angle$ B) = ...
5. If the perimeter of an equilateral triangle is 30 cm. and its height is 5 cm. , then its area = ... cm^2.

3 **[a] In the opposite figure :**

m ($\angle$ BAC) = 90° , $\overline{AD} \perp \overline{BC}$, CD = 9 cm. and DB = 16 cm.

Find : AC and AD

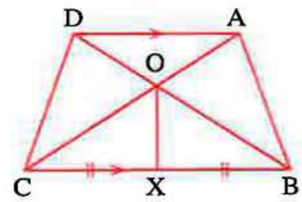


[b] In the opposite figure :

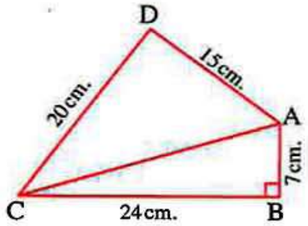
$\overline{DA} // \overline{CB}$, XB = XC

Prove that :

the area of the figure ABXO = the area of the figure DCXO



4 **[a] In the opposite figure :**



m (∠ B) = 90° , AB = 7 cm.

, BC = 24 cm. , AD = 15 cm.

and CD = 20 cm.

Prove that : m (∠ D) = 90°

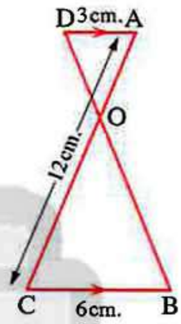
[b] In Δ ABC , AB = 5 cm. , BC = 8 cm. and AC = 10 cm.

What is the type of the triangle according to its angles ? (write steps)

---

5 **[a]** A trapezium , the lengths of the two parallel bases are 4 cm. and 10 cm. , if its height is 5 cm. Calculate its middle base and its area.

[b] In the opposite figure :



$\overline{DA}$ // $\overline{CB}$, AD = 3 cm. , BC = 6 cm.

and AC = 12 cm.

1 **Prove that :** Δ AOD ~ Δ COB

2 **Find :** the length of $\overline{AO}$

12 Suez Governorate

Directorate of Education
Inspection of Mathematics

Answer the following questions :

1 **Choose the correct answer :**

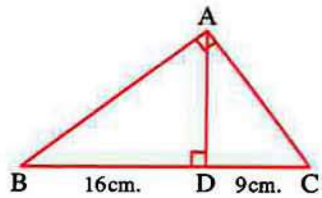
1 ABCD is a parallelogram in which m (∠ A) = 70° , then m (∠ B) = ...

(a) 70° (b) 110° (c) 180° (d) 540°

2 If ABC is a triangle , $(AB)^2 > (BC)^2 + (AC)^2$, then angle C is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

3 **In the opposite figure :**



m (∠ BAC) = 90° , $\overline{AD} \perp \overline{BC}$

, DC = 9 cm. , DB = 16 cm.

, then AD = ... cm.

(a) 144 (b) 25 (c) 50 (d) 12

4 The area of the square whose diagonal length is 10 cm. is ....

(a) 100 cm^2. (b) 50 cm^2. (c) 40 cm^2. (d) 20 cm^2.

5 A trapezium whose lengths of two parallel bases are 6 cm. and 8 cm. , then the length of its middle base equals .... cm.

(a) 48 (b) 24 (c) 14 (d) 7

6 The ratio between the lengths of two corresponding sides of two similar polygons is 3 : 5 , then the ratio between their perimeters is ....

(a) 2 : 5 (b) 2 : 3 (c) 3 : 5 (d) 1 : 2

2 Complete each of the following :

1 The median of a triangle divides its surface into two triangular surfaces equals ....

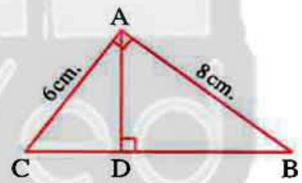
2 If ABC is a triangle , $(AC)^2 + (BC)^2 = (AB)^2$, then m (∠ ....) = 90°

3 All the regular polygons that have the same number of sides are ....

4 The area of rhombus is 24 cm^2. , if the length of one of its diagonals 8 cm. , then the length of the other diagonal is ....

5 If the point A ∈ the striaght line L , then the projection of the point A on the line L is ....

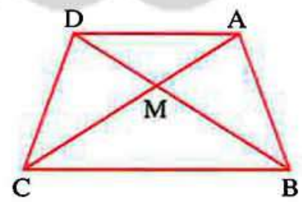
3 [a] In the opposite figure :



m (∠ CAB) = 90° , $\overline{AD} \perp \overline{BC}$, AC = 6 cm. , AB = 8 cm.

Find : the length of the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$

[b] In the opposite figure :

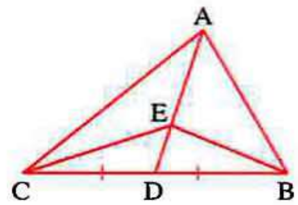


ABCD is a quadrilateral , the area of Δ AMB = the area of Δ DMC

Prove that : $\overline{AD}$ // $\overline{BC}$

4 [a] Determine the type of the greatest angle in Δ ABC where AB = 9 cm. , BC = 10 cm. , AC = 12 cm.

[b] In the opposite figure :



$\overline{AD}$ is the median of Δ ABC , E ∈ $\overline{AD}$

Prove that :

The area of Δ ABE = the area of Δ ACE

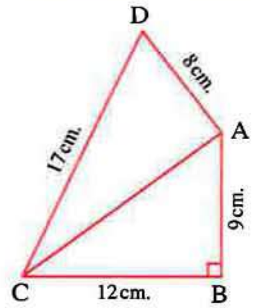
5 **[a] In the opposite figure :**

m (∠ B) = 90° , AB = 9 cm.

, BC = 12 cm. , AD = 8 cm.

, DC = 17 cm.

Prove that : m (∠ DAC) = 90°

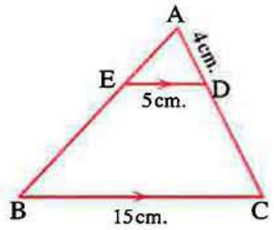


[b] In the opposite figure :

$\overline{ED}$ // $\overline{BC}$, AD = 4 cm.

, ED = 5 cm. , BC = 15 cm.

Find with proof : The length of $\overline{DC}$



13 Port Said Governorate

East Educational Directorate
mathematics Inspection

Answer the following questions :

1 **Choose the correct answer :**

1. In Δ ABC , if $(AB)^2 > (BC)^2 + (AC)^2$, then the angle C is ……………
   (a) acute. (b) right. (c) obtuse. (d) straight.
2. If the lengths of the diagonals of a rhombus are 6 cm. and 8 cm. , then its perimeter = …………… cm.
   (a) 24 (b) 28 (c) 14 (d) 20
3. If ABCD is a parallelogram of area 20 cm^2 and E ∈ $\overline{AD}$, then the area of Δ EBC = …………… cm^2
   (a) 10 (b) 5 (c) 20 (d) 40
4. If the projection of a line segment on a straight line is a point , then the line segment …………… the straight line.
   (a) // (b) ⊥ (c) ≡ (d) ⊂
5. The two triangles drawn on a common base and their vertices located on straight line parallel to the base are ……………
   (a) congruent. (b) similar.
   (c) equal in perimeter. (d) equal in area.
6. ABCD is a parallelogram in which : AB = 5 cm. , BC = 10 cm. and its smaller height is 4 cm. , then its greater height = …………… cm.
   (a) 2 (b) 4 (c) 8 (d) 10

2 **Complete each of the following :**

1. If ∠ A complements ∠ B and ∠ B supplements ∠ C , if m (∠ A) = 30°
, then m (∠ C) = ...°

2. In Δ ABC , if $(AB)^2 = (BC)^2 + (AC)^2$, then m (∠ ....) = 90°

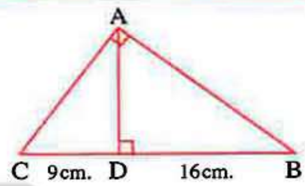
3. If two polygons are similar and the ratio between the lengths of two corresponding sides is 2 : 5 , then the ratio between their perimeters is ....

4. A trapezium whose lengths of the two parallel bases are 4 cm. and 6 cm. , then the length of its middle base = ... cm.

5. The rectangle is a parallelogram in which one of its angles is ....

3 **[a] In the opposite figure :**

Find : the length of $\overline{AB}$, $\overline{AC}$ and $\overline{AD}$



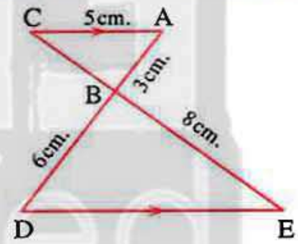
[b] ABCD is a trapezium in which $\overline{AD}$ // $\overline{BC}$, if BC = 2 AD = 20 cm. and its area = 180 cm^2.
Find its height.

4 **[a] In the opposite figure :**

$\overline{AC}$ // $\overline{ED}$, AB = 3 cm. , BD = 6 cm.
, AC = 5 cm. , BE = 8 cm.

Prove that : Δ ABC ~ Δ DBE

, then find : the length of $\overline{ED}$, $\overline{BC}$

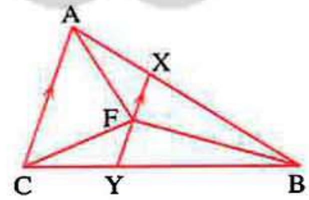


[b] In the opposite figure :

$\overline{AC}$ // $\overline{XY}$ and F is the midpoint of $\overline{XY}$

Prove that :

The area of Δ ABF = the area of Δ CBF



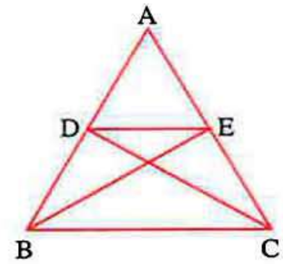
5 **[a] In the opposite figure :**

ABC is a triangle in which :

D ∈ $\overline{AB}$ and E ∈ $\overline{AC}$

, such that the area of Δ ABE = the area of Δ ACD

Prove that : $\overline{DE}$ // $\overline{BC}$



[b] In the opposite figure :

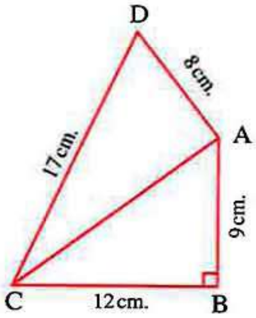
ABCD is a quadrilateral

, m (∠ B) = 90° , AB = 9 cm.

, BC = 12 cm. , CD = 17 cm.

and DA = 8 cm.

Prove that : m (∠ DAC) = 90°



14 El-Beheira Governorate

Bandar Damanhour Educational Zone
Amr Ibn El-Aass G.L.S.

Answer the following questions :

1 Choose the correct answer :

1. In Δ ABC if $(AB)^2 = (AC)^2 - (BC)^2$, then ∠ C is .... angle.
   (a) an acute. (b) an obtuse. (c) a right. (d) a straight.
2. If the lengths of two adjacent sides in a parallelogram are 9 cm. and 6 cm. and the length of the smaller height 4 cm. , then its area is .... cm^2.
   (a) 30 (b) 36 (c) 24 (d) 15
3. The diagonal length of a square whose area 50 cm^2 equals .... cm.
   (a) 10 (b) 20 (c) 30 (d) 40
4. The median of a triangle divides its surface into two triangles ....
   (a) congruent. (b) equal in area. (c) similar. (d) coincide.
5. If $\overline{AB}$ // $\overleftrightarrow{XY}$, then the length of the projection of $\overline{AB}$ on $\overleftrightarrow{XY}$ .... length of $\overline{AB}$
   (a) > (b) < (c) = (d) ≡
6. The number of axes of symmetry of the rhombus = ...
   (a) zero (b) 1 (c) 2 (d) 4

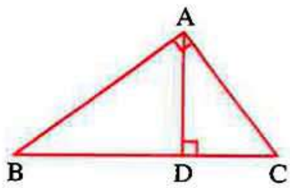
2 Complete each of the following :

1. In Δ ABC if $(AB)^2 = (BC)^2 + (AC)^2$, m (∠ B) = 50° , then m (∠ A) = ... °
2. In the two similar polygons there corresponding angles are ....
3. The area of rhombus is 24 cm^2 , the length of one of its diagonals 8 cm. , then the length of the other diagonal is .... cm.
4. The sum of the measures of the exterior angles of a triangle = ...°
5. **In the opposite figure :**

   Δ ABC is a right-angled triangle at A

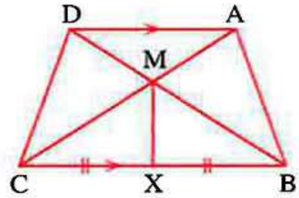
   , $\overline{AD} \perp \overline{BC}$

   $(AD)^2$ = ... × ....



3 **[a]** The area of a trapezium is 180 cm^2, its height is 9 cm. find the lengths of its parallel bases if the ratio between their lengths is 3 : 5

[b] In the opposite figure :

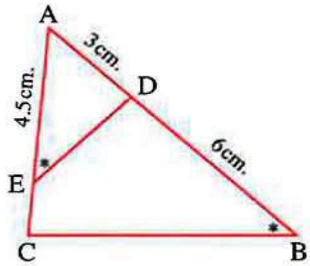


$\overline{AD} \parallel \overline{BC}$, $\overline{AC} \cap \overline{BD} = \{M\}$

, X is the midpoint of $\overline{BC}$

Prove that : the area of the shape ABXM
= the area of the shape DCXM

4 **[a] In the opposite figure :**



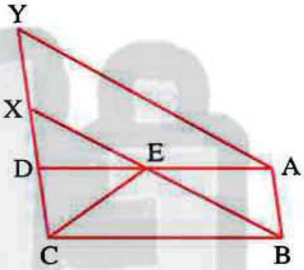
m (∠ AED) = m (∠ B)

, AD = 3 cm. , AE = 4.5 cm. , DB = 6 cm.

1 **Prove that :** Δ AED ~ Δ ABC

2 **Find :** the length of $\overline{CE}$

[b] In the opposite figure :

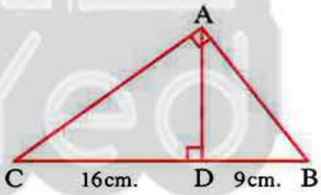


ABCD and ABXY are two parallelograms.

Prove that :

the area of Δ EBC
$= \frac{1}{2}$ the area of ▱ ABXY

5 **[a] In the opposite figure :**



ABC is a right-angled triangle at A ,

D ∈ $\overline{CB}$, $\overline{AD} \perp \overline{CB}$, CD = 16 cm. , DB = 9 cm.

Find : the length of $\overline{AC}$, $\overline{AB}$, $\overline{AD}$

[b] ABC is a triangle in which , AB = 7 cm. , BC = 10 cm. , AC = 8 cm.
Determine the type of Δ ABC according to its angles.

15 Beni Suef Governorate

Directorate of official Language School
Education administration

Answer the following questions :

1 **Choose the correct answer :**

1 If the area of a parallelogram is 35 cm^2 and its height is 5 cm. , then the length of the corresponding base to this height is .... cm.

(a) 5 (b) 7 (c) 9 (d) 30

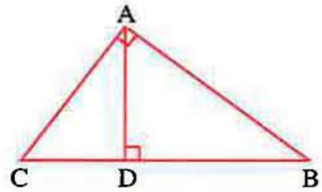
2 If the area of a trapezium is 32 cm^2 and its height is 4 cm. , then the length of its middle base equals .... cm.

(a) 4 (b) 8 (c) 14 (d) 16

3 If two polygons are similar and the ratio between the lengths of two corresponding sides is 2 : 3 , then the ratio between their perimeters is ....

(a) 1 : 2 (b) 4 : 9 (c) 2 : 3 (d) 9 : 4

4 **In the opposite figure :**



Δ ABC is right-angled at A

, $\overline{AD} \perp \overline{BC}$, then $(AB)^2$ = ...

(a) BD × BC (b) DC × BC

(c) BD × DC (d) AD × BC

5 If $\overline{AB}$ // $\overleftrightarrow{XY}$, then the length of the projection of $\overline{AB}$ on $\overleftrightarrow{XY}$ .... the length of $\overline{AB}$

(a) = (b) > (c) < (d) ≠

6 The smallest number of the acute angle in any triangle is ....

(a) zero (b) 1 (c) 2 (d) 3

2 Complete each of the following :

1 Triangles with congruent bases on one straight line and have a common vertex are ....

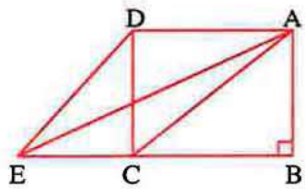
2 A triangle whose side lengths are 7 cm. , 14 cm. and 16 cm. , then its type according to its angles is ....

3 In Δ ABC , if $(AC)^2 < (AB)^2 + (BC)^2$, then the type of the angle B is ....

4 The sum of the measures of the interior angles of a triangle equals .... °

5 Each two opposite angles in a parallelogram are ....

3 [a] **In the opposite figure :**

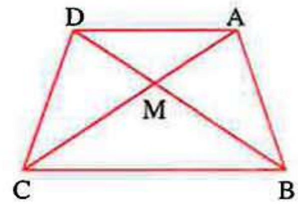


ABCD is a rectangle and E $\in \overrightarrow{BC}$

Prove that :

The area of Δ DAE = the area of Δ ABC

[b] **In the opposite figure :**



ABCD is a quadrilateral

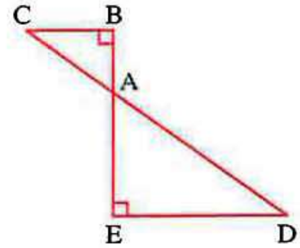
, its diagonals intersect at M

and the area of Δ ABM = the area of Δ DCM

Prove that : $\overline{AD}$ // $\overline{BC}$

4 [a] Find the area of a rhombus whose diagonals lengths are 10 cm. and 8 cm.

[b] **In the opposite figure :**

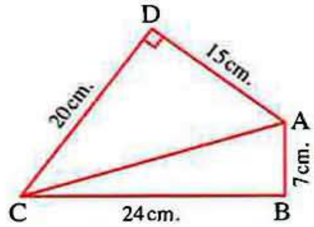


$\overline{BE} \cap \overline{DC} = \{A\}$ and

m (∠ B) = m (∠ E) = 90°

Prove that : Δ ABC ~ Δ AED

5 [a] **In the opposite figure :**

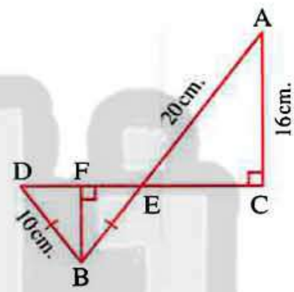


ABCD is a quadrilateral in which m (∠ ADC) = 90°

, AB = 7 cm. , BC = 24 cm. , CD = 20 cm. , DA = 15 cm.

Prove that : m (∠ ABC) = 90°

[b] **In the opposite figure :**



$\overline{AB} \cap \overline{CD} = \{E\}$

, E is the midpoint of $\overline{CD}$, $\overline{BF} \perp \overline{CD}$

, AC = 16 cm. , AE = 20 cm. and BD = BE = 10 cm.

Find the length of the projection of $\overline{BD}$ on $\overleftrightarrow{CD}$

Final Examinations

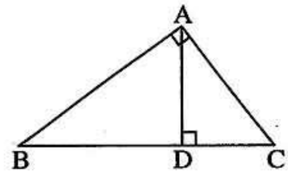
of Geometry

Model Examinations of the School Book on Geometry

Model 1

Answer the following questions :

1 Complete the following :



1. **In the opposite figure :**

   AB × ………… = BC × AD

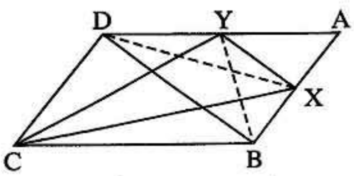
2. In Δ ABC , if $(AC)^2 + (BC)^2 = (AB)^2$, then m (∠ …………) = 90°
3. If the point A ∈ the line L , then the projection of the point A on the line L is …………
4. The area of the circle of diameter 14 cm. = ………… cm^2. ($\pi = \frac{22}{7}$)
5. A trapezium whose bases lengths are 8 cm. , 10 cm. and its height is 5 cm. , then its area equals ………… cm^2

2 Choose the correct answer :

1. In Δ ABC , if $(AB)^2 > (BC)^2 + (AC)^2$, then ∠ C is …………
   (a) acute. (b) right. (c) obtuse. (d) straight.
2. A rhombus whose diagonals lengths are 6 cm. , 10 cm. has area ………… cm^2.
   (a) 60 (b) 30 (c) 15 (d) 10
3. The ratio between the lengths of two corresponding sides of two similar polygons is 3 : 5 , then the ratio between their perimeters is …………
   (a) 2 : 5 (b) 5 : 3 (c) 3 : 5 (d) 1 : 2
4. If the area of a trapezium is 100 cm^2. and its height is 5 cm. , then the length of its middle base = ………… cm.
   (a) 20 (b) 30 (c) 40 (d) 50
5. ABCD is a parallelogram in which m (∠ A) = 70° , then m (∠ B) = …………°
   (a) 70 (b) 110 (c) 180 (d) 360
6. Measure of each angle of the regular pentagon is = …………°
   (a) 90 (b) 108 (c) 120 (d) 540

3 **[a]** The sides lengths of one of two similar triangles are 3 cm. , 4 cm. , 5 cm. and the perimeter of the other triangle is 36 cm. find the side lengths of the other triangle.

[b] In the opposite figure :



ABCD is a parallelogram , X ∈ $\overline{AB}$

, Y ∈ $\overline{AD}$ such that : The area of Δ CBX = the area of Δ CYD

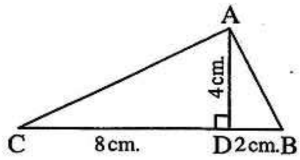
Prove that : $\overline{XY}$ // $\overline{BD}$

4 **[a] In the opposite figure :**

ABC is a triangle in which : BD = 2 cm.

, CD = 8 cm. , AD = 4 cm. , $\overline{AD} \perp \overline{BC}$

Prove that : m (∠ BAC) = 90°



[b] ABCD is a parallelogram in which : AB = 18 cm. and BC = 12 cm.

We draw $\overline{DE} \perp \overline{BC}$, $\overline{DO} \perp \overline{AB}$, DE = 15 cm.

Calculate the area of parallelogram ABCD and find the length of $\overline{DO}$

5 **[a]** ABC is a triangle in which m (∠ A) = 50° , m (∠ B) = 60° ,

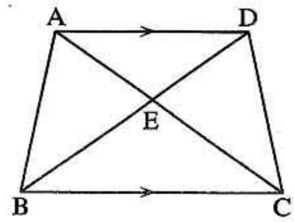
Arrange the lengths of the sides of the triangle in descending order.

[b] In the opposite figure :

ABCD is a quadrilateral in which

$\overline{AD} // \overline{BC}$, $\overline{AC} \cap \overline{BD} = \{E\}$

Prove that : the area of Δ ABE = the area of Δ DCE



Model 2

Answer the following questions :

1 **Complete the following :**

1 The two polygons are similar if their corresponding sides are …………… and their corresponding angles are ……………

2 The area of a rhombus is 24 cm.^{2} , the length of one of its diagonals is 8 cm. , then the length of the other diagonal is ……………

3 In Δ ABC , if $(AB)^2 = (AC)^2 - (BC)^2$, then Δ ABC is right-angled at ……………

4 A triangle whose side lengths are 6 cm. , 8 cm. and 11 cm. , then its type according to its angles is ……………

5 Area of triangle is equal to half of area of a parallelogram if they have a common ……………

2 **Choose the correct answer :**

1 A trapezium whose bases lengths are 6 cm. , 8 cm. , then the length of its middle base equals …………… cm.

(a) 48 (b) 24 (c) 14 (d) 7

2 If two polygons are similar and the ratio between the lengths of two corresponding sides is 1 : 3 and the perimeter of the smaller polygon is 15 cm. , then the perimeter of the greater polygon is …………… cm.

(a) 30 (b) 45 (c) 60 (d) 75

3 If the area of the triangle is 24 cm.^{2} and its height = 8 cm. , then the length of the corresponding base .... cm.

(a) 16 (b) 6 (c) 3 (d) 12

4 ABC is a right-angled triangle at B , $\overline{BD} \perp \overline{AC}$, then the projection of $\overline{BD}$ on $\overleftrightarrow{AC}$ is point ....

(a) A (b) B (c) C (d) D

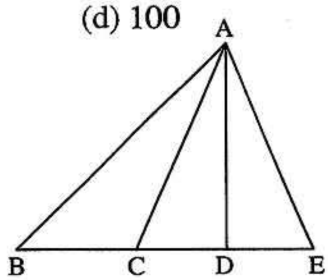
5 A square of perimeter 20 cm. , then its area equals .... cm.^{2}

(a) 20 (b) 25 (c) 50 (d) 100

6 The number of the triangles in the opposite figure = ...

(a) 3 (b) 4

(c) 5 (d) 6



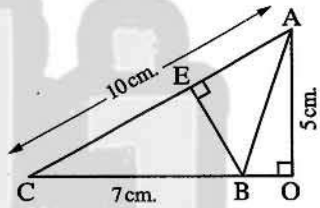
3 In the opposite figure :

$\overline{AO} \perp \overleftrightarrow{CB}$, $\overline{BE} \perp \overline{AC}$

, AC = 10 cm. , BC = 7 cm. and AO = 5 cm.

Find : 1 The length of $\overline{BE}$

2 The area of Δ ABC



4 [a] ABCD is a parallelogram in which : AB = 8 cm. , AC = 20 cm. and BD = 12 cm.

Prove that : m ($\angle$ ABD) = 90° , then find the area of this parallelogram.

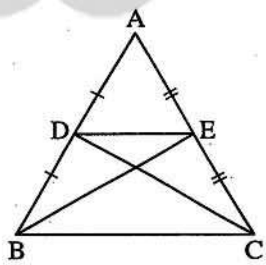
[b] In the opposite figure :

ABC is a triangle in which D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

Prove that :

First : the area of the triangle DBC = the area of triangle EBC

Second : $\overline{DE}$ // $\overline{BC}$

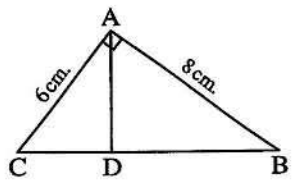


5 **[a] In the opposite figure :**

Δ DBA is a similar to Δ ABC , m ($\angle$ BAC) = 90°

Prove that : $\overline{AD} \perp \overline{BC}$ and if AB = 8 cm. , AC = 6 cm.

Find the length of : $\overline{BD}$



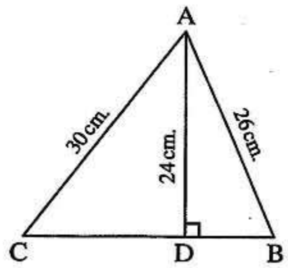
[b] In the opposite figure :

ABC is a triangle , $\overline{AD} \perp \overline{BC}$

If AD = 24 cm. , AB = 26 cm.

and AC = 30 cm.

Find : BC , then calculate area of Δ ABC



Model for the merge students

Answer the following questions :

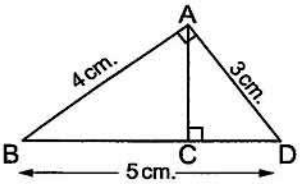
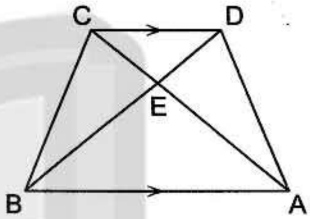
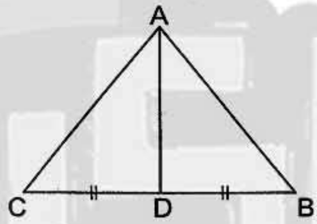
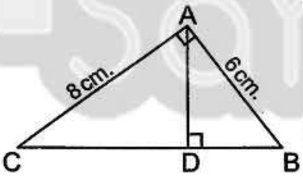
1 Choose the correct answer from those given :

1. The area of parallelogram whose length of its base 6 cm. and its corresponding height of this base 4 cm. equals .... cm^2.
   (a) 12 (b) 20 (c) 24 (d) 48
2. The triangle whose lengths of its sides 6 cm. , 8 cm. , 10 cm. is ....
   (a) acute-angled triangle. (b) right-angled triangle.
   (c) obtuse-angled triangle. (d) otherwise.
3. The rhombus whose lengths of its diagonals 6 cm. and 10 cm. , then its area = ... cm^2.
   (a) 60 (b) 30 (c) 15 (d) 10
4. Trapezium of length of its middle base 8 cm. and surface area 56 cm^2. , then its height = ... cm.
   (a) 32 (b) 24 (c) 448 (d) 7
5. All .... are similar.
   (a) squares (b) triangles (c) rectangles (d) parallelograms

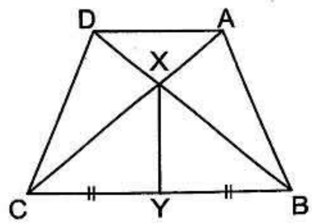
2 Complete each of the following :

1. The projection of point on a straight line is ....
2. If the triangle ABC is obtuse-angled triangle at B , then $(AC)^2$ .... $(AB)^2 + (BC)^2$
3. The square whose length of its diagonal 8 cm. , then its area = ... cm^2.
4. The two triangles have same base and the vertices opposite to this base on straight line parallel to the base ....
5. Area of triangle = $\frac{1}{2}$ × .... × corresponding height.

3 **Join from the column (A) to the suitable one from the column (B) :**

Column (A)	Column (B)
1 **In the opposite figure :** AC = ... cm. 	• BEC
2 **In the opposite figure :** Area of Δ AED = area of Δ .... 	• 2.4
3 **In the opposite figure :** Area of Δ ABD = area of Δ .... 	• Congruent
4 If the ratio of enlargement between two similar triangles = 1 , then the two triangles are	• 3.6
5 **In the opposite figure :** The length of the projection of $\overline{AB}$ on $\overline{BC}$ = ... cm. 	• ACD

4 **In the opposite figure :**



Area of the figure ABYX = Area of the figure DCYX

Complete the proof :

To prove that : $\overline{AD}$ // $\overline{BC}$

Given : ....

R.T.P. : ....

Proof : $\overline{XY}$ is median in Δ XBC

∴ Area of Δ .... = area Δ .... (1)

∵ area of the figure ABYX = area of the figure DCYX (2)

By subtracting (1) from (2) :

∴ Area of Δ .... = area of Δ ....

By adding area of Δ ADX to both sides

∴ Area of Δ .... = area of Δ ....

∵ $\overline{AD}$ // $\overline{BC}$

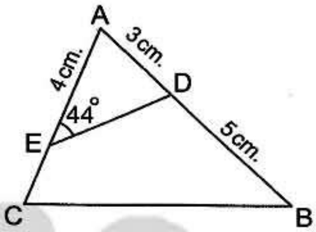
5 In the opposite figure :

Δ ABC ~ Δ AED

, m (∠ AED) = 44° , AD = 3 cm. , EA = 4 cm.

, DB = 5 cm. , BC = 8 cm.

Complete to find the length of each of : $\overline{ED}$ and $\overline{EC}$



Solution :

∵ Δ ABC ~ Δ AED

$\therefore \frac{AB}{..........} = \frac{..........}{ED} = \frac{CA}{DA}$

$\therefore \frac{8}{..........} = \frac{..........}{ED} = \frac{CA}{3}$

∴ ED = ... cm. , AC = ... cm. , EC = ... cm.

Schools Examinations on Geometry

1 Cairo Governorate

El-Nozha Directorate of Education
Modern Language Schools

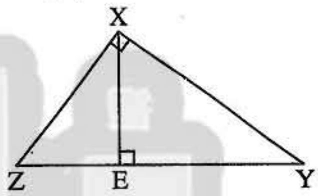
Answer the following questions :

1 Choose the correct answer :

1. The area of the trapezium whose middle base is of length 7 cm. and its height is 6 cm. equals ....
   (a) 21 cm^2 (b) 40 cm^2 (c) 42 cm^2 (d) 13 cm^2

2. In Δ ABC : if m (∠ C) = 90° , AB = 20 cm. and BC = 16 cm. , then AC = ... cm.
   (a) 9 (b) 12 (c) $4\sqrt{41}$ (d) 25

3. **In the opposite figure :**
   EY × EZ = ...
   (a) $(XE)^2$ (b) $(XZ)^2$
   (c) $(YZ)^2$ (d) $(XY)^2$



4. The triangle whose sides lengths are 5 cm. , 8 cm. and 7 cm. is ....-angled triangle.
   (a) right (b) acute (c) obtuse (d) straight

5. If the triangle base length is 6 cm. and its area is 24 cm^2 , then its corresponding height is ....
   (a) 18 cm. (b) 8 cm. (c) 4 cm. (d) 10 cm.

6. If $\overline{AB}$ // $\overline{XY}$, then the length of the projection of $\overline{AB}$ on $\overleftrightarrow{XY}$ .... the length of $\overline{AB}$
   (a) < (b) > (c) = (d) ≥

2 Complete :

1. The two polygons are similar if their corresponding side lengths are .... and their corresponding angle are ....
2. If Δ ABC ~ Δ XYZ , m (∠ A) + m (∠ B) = 60° , then m (∠ Z) = ...
3. If Δ ABC is an obtuse-angled triangle at B , then $(AC)^2$ .... $(AB)^2 + (BC)^2$
4. If the length of the diagonal of a square is 10 cm. , then its area = ... cm^2
5. If the ratio between the length of two corresponding sides of two similar polygons is 2 : 5 and the perimeter of the smaller one is 12 cm. , then the perimeter of the other one is ....

3 **[a] In the opposite figure :**

ABCD is a quadrilateral in which :

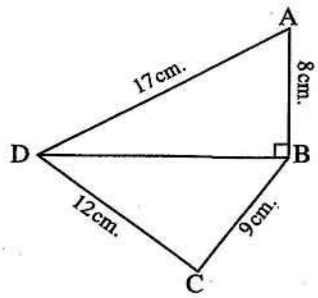
AB = 8 cm. , BC = 9 cm.

, CD = 12 cm. , AD = 17 cm.

and $\overline{DB} \perp \overline{AB}$

[1] **Find :** the length of $\overline{BD}$

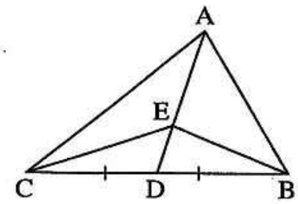
[2] **Prove that :** m (∠ C) = 90°



[b] In the opposite figure :

Δ ABC with a median $\overline{AD}$, E ∈ $\overline{AD}$

Prove that : the area of Δ ABE = the area of Δ ACE

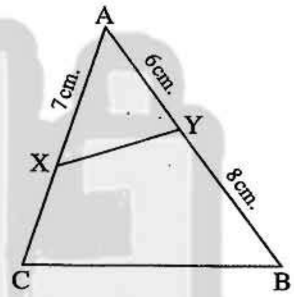


4 **[a] In the opposite figure :**

Δ AXY ~ Δ ABC , AX = 7 cm.

, AY = 6 cm. , YB = 8 cm.

Find : the length of $\overline{XC}$

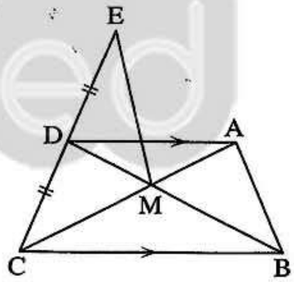


[b] In the opposite figure :

$\overline{AD}$ // $\overline{BC}$, $\overline{AC}$ ∩ $\overline{BD}$ = {M}

, D is the midpoint of $\overline{EC}$

Prove that : the area of Δ MDE = the area of Δ AMB



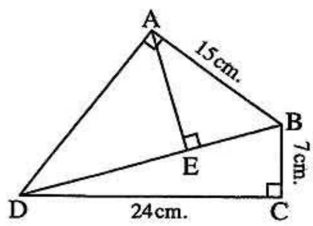
5 **In the opposite figure :**

ABCD is a quadrilateral , m (∠ BCD) = m (∠ BAD) = 90°

, $\overline{AE} \perp \overleftrightarrow{BE}$

[1] **Find :** the length of $\overline{BD}$ and $\overline{AD}$

[2] **Find :** the length of the projection of $\overline{AB}$ on $\overleftrightarrow{BD}$



2 Cairo Governorate

Shoubra Educational Zone

Answer the following questions :

1 Choose the correct answer :

1. In Δ ABC , if $(AB)^2 > (BC)^2 + (AC)^2$, then m ($\angle$ C) is ....
   (a) acute. (b) right. (c) obtuse. (d) straight.
2. ABCD is a parallelogram in which m ($\angle$ A) = 70° , then m ($\angle$ B) = ...°
   (a) 70 (b) 110 (c) 180 (d) 360
3. The diagonals of isosceles trapezium are ....
   (a) parallel (b) perpendicular (c) congruent (d) not equal
4. A rhombus its two diagonals of lengths 8 cm. and 6 cm. , its area equals .... cm^2
   (a) 14 (b) 20 (c) 24 (d) 48
5. $\overline{AB} \parallel \overleftrightarrow{CD}$, then the length of the projection of $\overline{AB}$ on $\overleftrightarrow{CD}$ .... the length of $\overline{AB}$
   (a) > (b) < (c) = (d) ≤
6. Any triangle has at least two .... angles.
   (a) right. (b) obtuse. (c) acute. (d) straight.

2 Complete each of the following :

1. The square of diagonal length 12 cm. , then its area = ... cm^2
2. If each of two triangles is similar to a third triangle , then they are ....
3. The median of a triangle divides it into two triangles ....
4. If Δ ADE ~ Δ ABC , AE : AC = 1 : 2 and DE = 5 cm. , then BC = ... cm.
5. Each two opposite angles in a parallelogram are ....

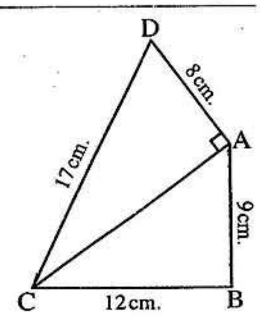
3 [a] In the opposite figure :

m ($\angle$ DAC) = 90° , AD = 8 cm.

, DC = 17 cm. , AB = 9 cm.

, BC = 12 cm.

Prove that : m ($\angle$ B) = 90°



[b] Find the area of a trapezium :

Whose two parallel bases lengths are 12 cm. , 7 cm. and its height is 5 cm.

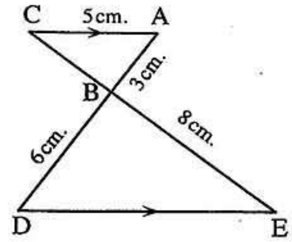
4 [a] **In the opposite figure :**

$\overline{AC} \parallel \overline{ED}$, $\overline{AD} \cap \overline{CE} = \{B\}$, AC = 5 cm.

, AB = 3 cm. and BD = 6 cm.

, BE = 8 cm.

Prove that : Δ ABC $\sim$ Δ DBE



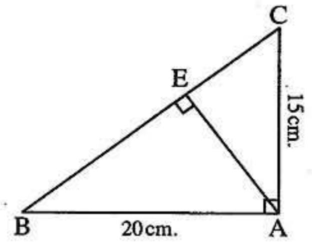
[b] **In the opposite figure :**

ABC is a right-angled triangle at A , $\overline{AE} \perp \overline{BC}$

Find :

1 The length of the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$

2 The length of $\overline{EC}$

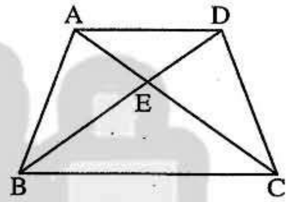


5 [a] Determine the type of triangle ABC according to its angles where AB = 7 cm. , BC = 6 cm. , AC = 9 cm.

[b] **In the opposite figure :**

The area of Δ AEB = the area of Δ DEC

Prove that : $\overline{AD} \parallel \overline{BC}$



3 Cairo Governorate

El-Zeiton Zone
Talaea Gaber El Ansary Language School

Answer the following questions :

1 **Choose the correct answer :**

1 The lengths of two adjacent sides in a parallelogram are 6 cm. , 7 cm. and the greater height is 5 cm. , then its area = ... cm^2

(a) 30 (b) 35 (c) 42 (d) 49

2 A trapezium whose middle base length is 8 cm. , then the lengths of the parallel bases may be ....

(a) 4 cm. , 4 cm. (b) 3 cm. , 5 cm. (c) 4 cm. , 6 cm. (d) 6 cm. , 10 cm.

3 A perimeter of a square is 20 cm. , then its area = ... cm.

(a) 400 (b) 80 (c) 25 (d) 20

4 In Δ ABC , if $(AB)^2 > (BC)^2 + (AC)^2$, then angle C is ....

(a) acute. (b) obtuse. (c) right. (d) straight.

[5] ABC is a right-angled triangle at B , $\overline{BD} \perp \overline{AC}$, then the projection of $\overline{BD}$ on $\overleftrightarrow{AC}$ is ....

(a) A (b) B (c) C (d) D

[6] The ratio between the lengths of two corresponding sides of two similar polygons is 3 : 5 , then the ratio between their perimeters is ....

(a) 3 : 5 (b) 5 : 3 (c) 2 : 5 (d) 9 : 25

2 Complete the following :

[1] The two polygons are similar , if their corresponding side lengths are .... and their corresponding angles are ....

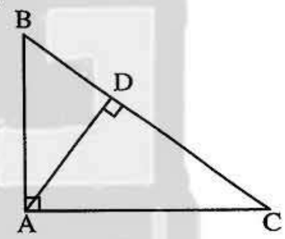
[2] The area of rhombus is 12 cm^2. , if the length of one of its diagonals is 4 cm. , then the length of the other diagonal = ... cm.

[3] The complement of an angle of measure 25° equals ....

[4] XYZ is a triangle in which $(XY)^2 = (XZ)^2 - (ZY)^2$, then m (∠ ....) = 90°

[5] A rectangle whose perimeter is 28 cm. , and its length is 8 cm. , then the length of its diagonal = ... cm.

3 [a] In the opposite figure :



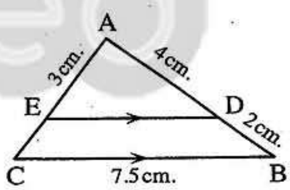
m (∠ BAC) = 90° , $\overline{AD} \perp \overline{BC}$

Complete :

[1] $(AB)^2$ = ... × ....

[2] $(AD)^2$ = ... × ....

[b] In the opposite figure :

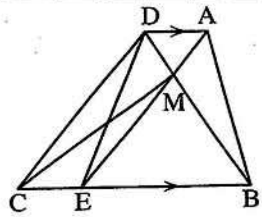


$\overline{DE}$ // $\overline{BC}$, AD = 4 cm. , AE = 3 cm. , BD = 2 cm. and BC = 7.5 cm.

[1] **Prove that :** Δ ADE ~ Δ ABC

[2] **Find :** the length of $\overline{ED}$

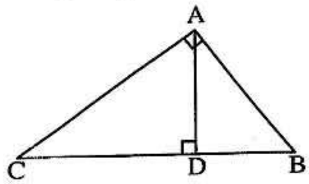
4 [a] In the opposite figure :



$\overline{AD}$ // $\overline{BC}$, $\overline{AE} \cap \overline{BD}$ = {M} , the area of Δ AMB = the area of Δ EMC

Prove that : $\overline{ME}$ // $\overline{DC}$

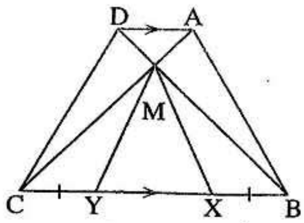
[b] In the opposite figure :



$\overline{AD} \perp \overline{CB}$, $\overline{AC} \perp \overline{AB}$

Prove that : Δ ABC ~ Δ DBA ~ Δ DAC

5 **[a] In the opposite figure :**



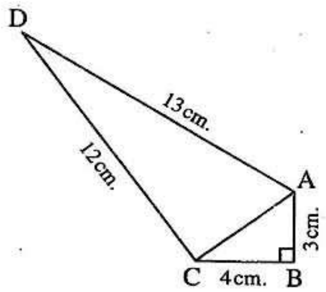
$\overline{AD} \parallel \overline{BC}$, $\overline{AC} \cap \overline{BD} = \{M\}$

, BX = CY

Prove that :

The area of the figure ABXM = The area of a figure DCYM

[b] In the opposite figure :



BC = 4 cm. , AD = 13 cm.

, AB = 3 cm. , DC = 12 cm. , m (∠ B) = 90°

1 **Find :** The length of $\overline{AC}$

2 **Prove that :** m (∠ ACD) = 90°

4 Giza Governorate

Omrania Directorate
El-Sadat Governmental Language School

Answer the following questions :

1 **Choose the correct answer :**

1 In Δ ABC , if $(AB)^2 > (BC)^2 + (AC)^2$, then ∠ C is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

2 A rhombus whose diagonal lengths are 6 cm. , 10 cm. its area = ... cm^2.

(a) 60 (b) 30 (c) 15 (d) 10

3 If the length of the middle base of a trapezium is 8 cm. and its surface area is 56 cm^2. , then its height = ... cm.

(a) 32 (b) 24 (c) 448 (d) 7

4 If the ratio of an enlargement between two triangles equals 1 , then the two triangles are ....

(a) congruent. (b) enlargement. (c) coincide. (d) reduction.

5 Any triangle has at least two .... angles.

(a) right. (b) obtuse. (c) acute. (d) straight.

6 The isosceles triangle has .... axis of symmetry.

(a) zero (b) one (c) two (d) three

2 **Complete each of the following :**

1 The median of a triangle divides it into two triangles .... in area.

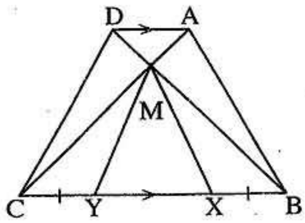
2 Two triangles are similar if their corresponding side lengths are ....

[3] ABCD is a rectangle , the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is ....

[4] Surfaces of two parallelograms with common base and between two parallel straight lines one is carrying this base are ....

[5] ABCD is a parallelogram its area = 36 cm^2 , E $\in \overline{AD}$, then the area of Δ EBC = ... cm^2

3 **[a] In the opposite figure :**

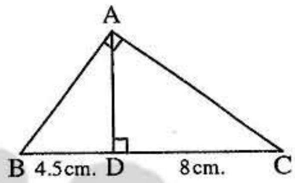


$\overline{AD} // \overline{BC}$, $\overline{AC} \cap \overline{BD} = \{M\}$

, X $\in \overline{BC}$, Y $\in \overline{BC}$ such that BX = CY

Prove that : the area of shape ABXM = the area of shape DCYM

[b] In the opposite figure :

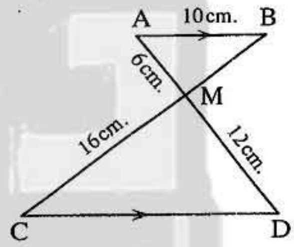


ABC is a triangle in which : m ($\angle$ A) = 90° , $\overline{AD} \perp \overline{BC}$

, BD = 4.5 cm. , DC = 8 cm.

Find : the length of each of $\overline{AC}$, $\overline{AB}$, $\overline{AD}$

4 **[a] In the opposite figure :**



$\overline{AB} // \overline{DC}$, $\overline{AD} \cap \overline{BC} = \{M\}$

, AB = 10 cm. , AM = 6 cm.

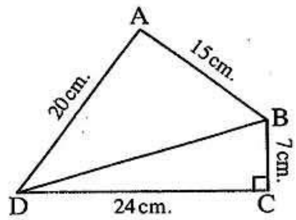
, MD = 12 cm. , MC = 16 cm.

[1] **Prove that :** Δ AMB ~ Δ DMC

[2] **Find :** the length of $\overline{CD}$, $\overline{MB}$

[b] Identify the type of Δ ABC according to the measures of its angles where AB = 5 cm. , BC = 6 cm. , AC = 7 cm.

5 **[a] In the opposite figure :**

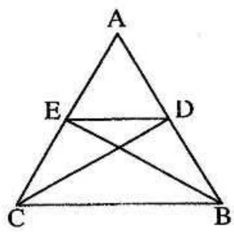


m ($\angle$ BCD) = 90° , AB = 15 cm.

, BC = 7 cm. , CD = 24 cm. , AD = 20 cm.

Prove that : m ($\angle$ BAD) = 90°

[b] In the opposite figure :



If the area of Δ ADC = the area of Δ AEB

Prove that : $\overline{DE} // \overline{BC}$

5 Giza Governorate

Abo El-Nomoros Directorate
Inspection of maths

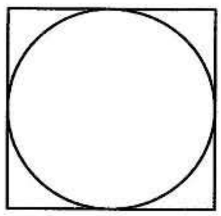
Answer the following questions :

1 Complete each of the following :

1. The two triangles are similar if their corresponding sides are ....
2. In Δ ABC , if $(AC)^2 + (BC)^2 = (AB)^2$, then m (∠ ....) = 90°
3. A square its side length is 5 cm. , then its diagonal length = ... cm.
4. If $\overline{AB} \perp \overline{BC}$, then the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is ....
5. A rhombus its diagonal lengths are 8 cm. and 7 cm. , then its area = ... cm^2.

2 Choose the correct answer :

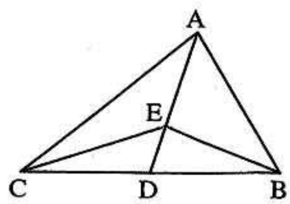
1. If the two similar triangles are congruent , then the ratio of enlargement = ...
   (a) 1 (b) 2 (c) 0.5 (d) 0.25
2. A triangle its side lengths are 3 cm. , 4 cm. and 5 cm. , then its area = ... cm^2.
   (a) 12 (b) 6 (c) 10 (d) 15
3. A parallelogram its area is 27 cm^2. , if its base length three times its height , then its height = ... cm.
   (a) 27 (b) 9 (c) 3 (d) 6
4. If the measure of the smallest angle of a triangle is 20° , then the possible measure of the greatest angle in this triangle = ...°
   (a) 90 (b) 140 (c) 159 (d) 160
5. The suitable unit to measure the height of a house is ....
   (a) cm. (b) dm. (c) m. (d) km.
6. If the area of the opposite square = 36 cm^2.
   , then the area of the circle = ... cm^2.
   (a) 6 π (b) 9 π
   (c) 36 π (d) 9



3 [a] In the opposite figure :

ABC is a triangle in which
, $\overline{AD}$ is median , E ∈ $\overline{AD}$

Prove that : the area of Δ ABE = the area of Δ ACE



[b] Determine the type of Δ ABC according to its angles if

AB = 7 cm. , BC = 3 cm. and AC = 5 cm.

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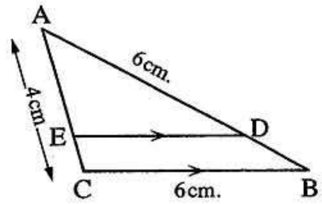
4 **[a] In the opposite figure :**



$\overline{DE}$ // $\overline{BC}$, AD = BC = 6 cm.

, AB = 8 cm. , AC = 4 cm.

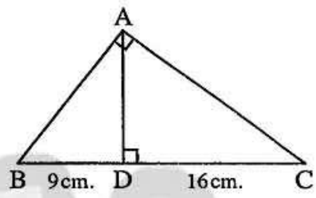
1 **Prove that :** Δ ADE ~ Δ ABC

2 **Find :** the lengths of $\overline{AE}$ and $\overline{DE}$

[b] Find the area of a trapezium if the lengths of its parallel bases are 5 cm. , 9 cm. and its height is 4 cm.

---

5 **[a] In the opposite figure :**

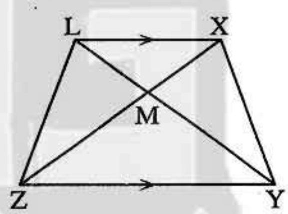


ABC is a right-angled triangle at A

, $\overline{AD} \perp \overline{BC}$, BD = 9 cm. , CD = 16 cm.

Find : the length of $\overline{AC}$, $\overline{AB}$ and $\overline{AD}$

[b] In the opposite figure :



XYZL is a trapezium in which

, $\overline{XL}$ // $\overline{YZ}$, $\overline{XZ} \cap \overline{LY}$ = {M}

Prove that : the area of Δ XMY = the area of Δ LMZ

6 Alexandria Governorate

West Educational zone
Inspectorate of Mathematics

Answer the following questions :

1 **Complete each of the following :**

1 The area of the rhombus whose diagonal lengths are12 cm. , 8 cm. equals .... cm^2.

2 If $\overline{AD} \perp \overline{BC}$, then the length of projection of $\overline{AD}$ on $\overleftrightarrow{BC}$ equals .... cm.

3 Δ ABC is a right-angled triangle at B in which AB = 5 cm. , BC = 12 cm. , then AC = ... cm.

4 In Δ ABC : AB = 8 cm. , BC = 9 cm. and AC = 6 cm. , then the type of this triangle according to its angles is ....

5 The number of axes of symmetry of an isosceles triangle equals ....

2 Choose the correct answer :

1 The diagonals of an isosceles trapezium are ....

(a) congruent. (b) perpendicular.
(c) bisect each other. (d) parallel.

2 If the ratio between two corresponding sides of two similar triangles is 1 : 2 , and the measure of an angle of the first triangle equals 60° , then the measure of its corresponding angle in the other triangle equals ....

(a) 30° (b) 120° (c) 60° (d) 62°

3 The image of point (2 , 0) is itself by reflection on ....

(a) $\mathcal{X}$-axis (b) y-aixs
(c) origin point. (d) $\mathcal{X}$-axis followed by y-axis

4 The perpendicular segment drawn from the right angle of a triangle to the hypotenuse divides it into two .... triangles.

(a) obtuse angle (b) acute angle
(c) equelateral (d) similar

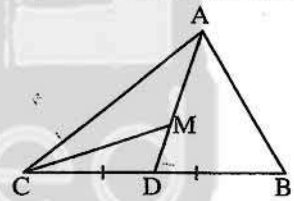
5 The measure of the complementary angle of an angle whose measure $\mathcal{X}°$ equals ....

(a) 90° (b) $90° - \mathcal{X}°$ (c) $\mathcal{X}° - 90°$ (d) $90\,\mathcal{X}°$

6 ABCD is a parallelogram , E ∈ BC , then the area of ▱ ABCD = ... area of Δ EAD

(a) the same (b) half (c) twice (d) third

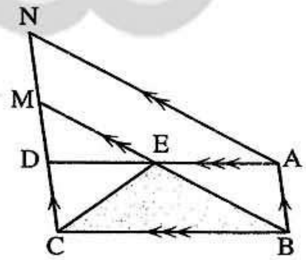
3 [a] In the opposite figure :



Δ ABC , M is the point of concurrence

Prove that : the area of Δ AMC = $\frac{1}{3}$ the area of Δ ABC

[b] In the opposite figure :

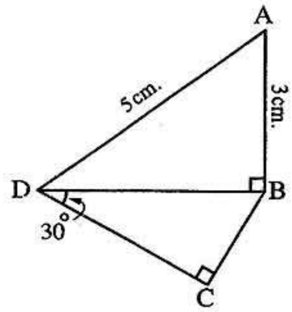


ABCD and ABMN are two parallelograms

Prove that :

The area of Δ EBC = $\frac{1}{2}$ the area of ▱ ABMN

4 [a] In the opposite figure :



ABCD is a quadrilateral in which :

m (∠ ABD) = 90° , m (∠ BCD) = 90°

, m (∠ BDC) = 30° , AB = 3 cm. , AD = 5 cm.

Find : the lengths of $\overline{BD}$ and $\overline{BC}$

[b] In the opposite figure :

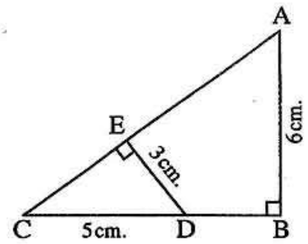
ABC is a right-angled triangle at B

, $\overline{DE} \perp \overline{AC}$, AB = 6 cm.

, ED = 3 cm. , CD = 5 cm.

1 **Prove that :** $\Delta ABC \sim \Delta DEC$

2 **Find :** the length of $\overline{AC}$



5 [a] The area of a trapezium is 88 cm^2 , its height is 8 cm. and the length of one of the two parallel bases is 10 cm. find the length of the other base.

[b] In the opposite figure :

m (∠ ABD) = 90° , $\overline{CE} \perp \overline{AB}$

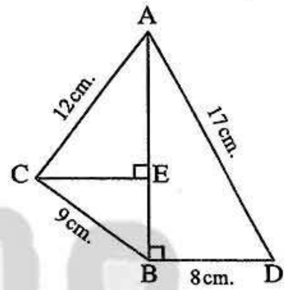
, CB = 9 cm. , AD = 17 cm.

, BD = 8 cm. , AC = 12 cm.

1 **Find :** the length of $\overline{AB}$

2 **Prove that :** m (∠ ACB) = 90°

3 **Find :** the length of the projection of $\overline{AC}$ on $\overleftrightarrow{AB}$



7 Alexandria Governorate

El Montazah Directorate
Brilliance Language School

Answer the following questions :

1 Complete each of the following :

1 If the area of a rhombus equals 30 cm^2 and the length of one of its diagonals equals 6 cm. , then the length of the other diagonal equals .... cm.

2 Surface area of two parallelograms with common base and between two parallel straight lines , one is carrying this base , are ....

3 If $\overline{AB} \perp \overleftrightarrow{CD}$, then the length of the projection of $\overline{AB}$ on $\overleftrightarrow{CD}$ equals .... cm.

4 In Δ ABC , AB = 8 cm. , BC = 9 cm. and AC = 6 cm. , then its type according to its angles is ....

5 If two straight lines intersect , then each two vertically opposite angles are ....

2 Choose the correct answer :

1 The median of triangle divides its surface into two triangles ....

(a) congruent. (b) equal in area. (c) similar. (d) coincident.

2 The isosceles trapezium has .... axis of symmetry.

(a) 1 (b) 2 (c) 3 (d) 0

3 Area of a parallelogram = 24 cm^2 and its base length is 6 cm.
, then its corresponding height = ... cm.

(a) 8 (b) 4 (c) 48 (d) 12

4 If the ratio of enlargement between two similar triangles equals 1 , then the two triangles are ....

(a) congruent. (b) different. (c) right-angled. (d) parallel.

5 The number of diagonals of any triangle = ...

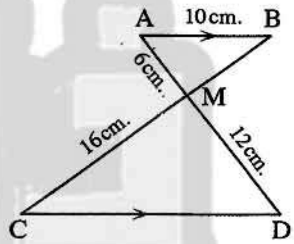
(a) 4 (b) 0 (c) 2 (d) 1

6 If Δ ABC ~ Δ XYZ , then m (∠ B) = m (∠ ....)

(a) C (b) Z (c) X (d) Y

3 [a] **In the opposite figure :**

$\overline{AB} // \overline{DC}$, $\overline{AD} \cap \overline{BC} = \{M\}$
, AB = 10 cm. , AM = 6 cm.
, MD = 12 cm. , MC = 16 cm.

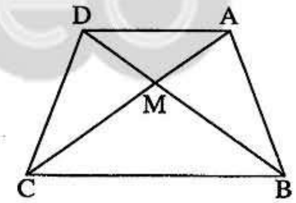


1 **Prove that :** Δ AMB ~ Δ DMC

2 **Find :** the length of $\overline{CD}$

[b] **In the opposite figure :**

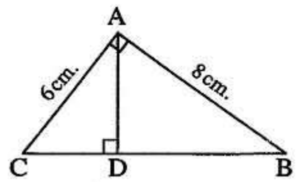
ABCD is a quadrilateral
, its diagonals intersect at M
and the area of Δ ABM = the area of Δ DCM



Prove that : $\overline{AD} // \overline{BC}$

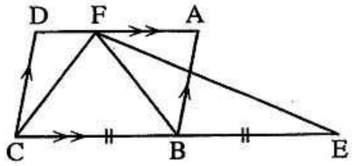
4 [a] **In the opposite figure :**

ABC is a triangle in which :
m (∠ BAC) = 90° , $\overline{AD} \perp \overline{BC}$
, AB = 8 cm. and AC = 6 cm.



Find : the lengths of $\overline{CB}$, $\overline{CD}$ and $\overline{AD}$

[b] In the opposite figure :



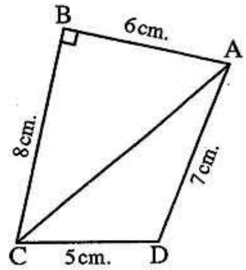
ABCD is a parallelogram

, $E \in \overrightarrow{CB}$ where BC = BE

Prove that : the area of Δ FEC = the area of ▱ ABCD

5 [a] Find the height of a trapezium with area of 450 cm^2 and the two parallel base lengths are 24 cm. and 12 cm.

[b] In the opposite figure :



ABCD is a quadrilateral in which : m (∠ B) = 90°

, AB = 6 cm. , BC = 8 cm. , AD = 7 cm.

and DC = 5 cm.

Determine the type of the angle which has the greatest measure in Δ ACD

8 El-Kalyoubia Governorate

Math's Inspection

Answer the following questions :

1 **Choose the correct answer :**

1 ABCD is a parallelogram in which m (∠ A) = 70° , then m (∠ B) = ... °

(a) 70 (b) 110 (c) 180 (d) 360

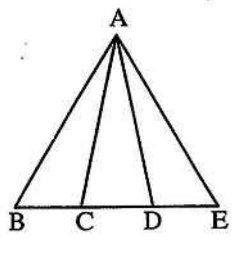
2 A rhombus whose diagonal lengths are 6 cm. and 10 cm. and its area = ... cm^2

(a) 60 (b) 30 (c) 15 (d) 10

3 A square of perimeter 20 cm. , then its area = ... cm^2

(a) 20 (b) 25 (c) 50 (d) 100

4 The number of the triangles in the opposite figure = ...



(a) 3 (b) 4

(c) 5 (d) 6

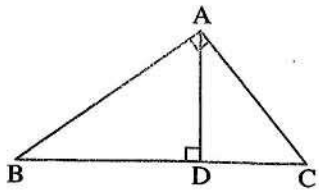
5 All the .... are similar.

(a) squares (b) triangles (c) rectangles (d) parallelograms

6 If Δ ABC ~ Δ XYZ , then m (∠ Y) = m (∠ ....)

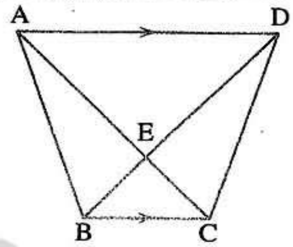
(a) A (b) B (c) C (d) X

2 Complete each of the following :



1. In the opposite figure
   AB × .... = BC × AD
2. In Δ ABC , if $(AC)^2 + (BC)^2 = (AB)^2$, then m (∠ ....) = 90°
3. If the point A ∈ the straight line L , then the projection of the point A on the straight line L is ....
4. The area of the triangle = $\frac{1}{2}$ × .... × the corresponding height.
5. The diagonals of an isosceles trapezium are .... in the length.

3 [a] In the opposite figure :

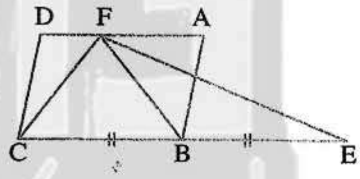


ABCD is a quadrilateral in which
, $\overline{AD} \parallel \overline{BC}$, $\overline{AC} \cap \overline{BD} = \{E\}$

Prove that : the area of Δ ABE = the area of Δ DCE

[b] The side lengths of one of two similar triangles are 3 cm. , 4 cm. , 5 cm. and the perimeter of the other triangle is 36 cm. Find the side lengths of the other triangle.

4 [a] In the opposite figure :

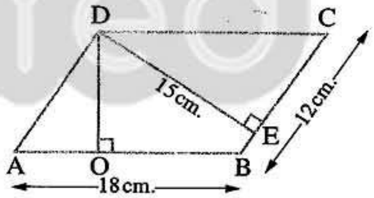


ABCD is a parallelogram , E ∈ $\overrightarrow{CB}$, where BC = BE

Prove that :

The area of Δ FCE = the area of parallelogram ABCD

[b] In the opposite figure :



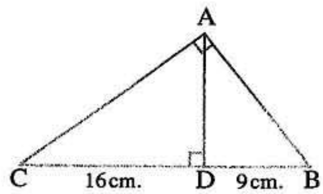
ABCD is a parallelogram in which :

AB = 18 cm. , DE = 15 cm. , and BC = 12 cm.

, $\overline{DE} \perp \overline{BC}$, $\overline{DO} \perp \overline{AB}$

Calculate the area of ▱ ABCD , and find the length of $\overline{DO}$

5 [a] In the opposite figure :



ABC is a right-angled triangle at A

, $\overline{AD} \perp \overline{BC}$, if BD = 9 cm. and DC = 16 cm.

Find : the lengths of $\overline{AB}$ and $\overline{AD}$

[b] In Δ ABC , if AB = 3 cm. , BC = 7 cm. , AC = 5 cm.

Determine the type of Δ ABC according to its angles.

9 El-Sharkia Governorate

Dep. of formal L. Schools

Answer the following questions :

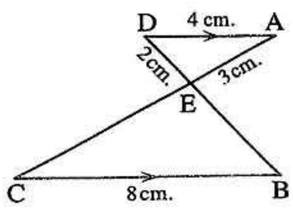
1 Complete each of the following :

1. The median of a triangle divides its surface into ....
2. The area of a trapezium whose parallel base lengths are 10 cm. , 6 cm. and its height is 5 cm. is .... cm^2.
3. Area of parallelogram = ... × ....
4. The area of the rhombus of perimeter 20 cm. and height 3 cm. = ...
5. The area of a triangle is equal to .... the area of a parallelogram if they have a common base lying on one of two parallel straight lines including them.

2 Choose the correct answer :

1. ABC is a triangle in which $(AB)^2 > (AC)^2 + (BC)^2$, then ∠ C is ....
   (a) acute. (b) right. (c) obtuse. (d) straight.
2. If $\overline{AB} // \overleftrightarrow{XY}$, then the length of the projection of $\overline{AB}$ on $\overleftrightarrow{XY}$ .... length of $\overline{AB}$
   (a) < (b) > (c) = (d) ≥
3. A square of diagonal length 12 cm. , then its area = ... cm^2.
   (a) 24 (b) 36 (c) 48 (d) 72
4. The two base angles of the isosceles trapezium are ....
   (a) congruent. (b) complementary. (c) supplementary. (d) parallel.
5. The ratio between the lengths of corresponding sides of two similar triangles is 3 : 5 , if the perimeter of the greater triangle is 60 cm. , then the perimeter of the smaller triangle is ....
   (a) 24 (b) 36 (c) 40 (d) 100
6. A triangle whose area is 15 cm^2 and its height is 3 cm. , then its base length = ... cm.
   (a) 5 (b) 10 (c) 15 (d) 3

3 [a] In the opposite figure :



$\overline{AD} // \overline{BC}$, AD = 4 cm. , BC = 8 cm. , AE = 3 cm. and ED = 2 cm.

1. **Prove that :** Δ AED ~ Δ CEB
2. **Find :** The perimeter of Δ EBC

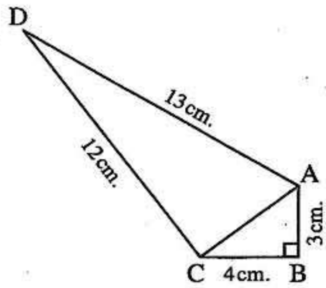
[b] In the opposite figure :

AB = 3 cm. , BC = 4 cm.

, AD = 13 cm. , CD = 12 cm.

, m (∠ B) = 90°

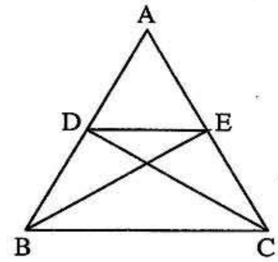
Prove that : m (∠ ACD) = 90°



4 **[a] In the opposite figure :**

If the area of Δ ACD = the area of Δ ABE

Prove that : $\overline{ED}$ // $\overline{BC}$



[b] In Δ ABC , if AB = 8 cm. , BC = 10 cm. and CA = 7 cm.

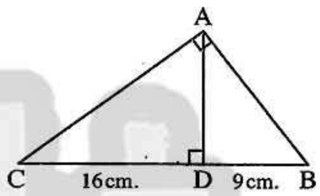
What is the type of Δ ABC according to its angles ?

5 **[a] In the opposite figure :**

ABC is a right-angled triangle at A

, $\overline{AD}$ ⊥ $\overline{BC}$, BD = 9 cm. , CD = 16 cm.

Find: The length of each of $\overline{AB}$, $\overline{AC}$, $\overline{AD}$

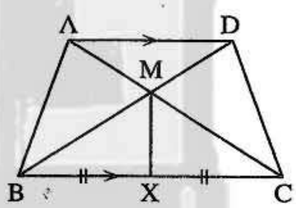


[b] In the opposite figure :

$\overline{AD}$ // $\overline{BC}$, X is the midpoint of $\overline{BC}$

Prove that :

The area of the figure ABXM = the area of the figure DCXM



10 El-Dakahlia Governorate

Directorate of Education
Maths Supervision

Answer the following questions :

1 **Choose the correct answer :**

1. The area of a rhombus whose diagonal lengths are 6 cm. and 10 cm. is .... cm^2.
   (a) 60 (b) 120 (c) 30 (d) 15
2. In Δ XYZ , if $(XZ)^2 = (XY)^2 - (ZY)^2$, then ∠ Z is .... angle.
   (a) an acute. (b) a straight. (c) an obtuse. (d) a right.
3. If the perimeter of a square is 20 cm. , then its area = ... cm^2.
   (a) 400 (b) 10 (c) 25 (d) 12.5
4. If Δ ABC ~ Δ XYZ , then m (∠ ACB) = m (∠ ....)
   (a) XYZ (b) YXZ (c) ZYX (d) XZY

5 If the diameter length of a circle = 14 cm. , then its area = ... cm^2.
(a) 154 (b) 44 (c) 616 (d) 22

6 In Δ ABC , m (∠ B) = 90° , $\overline{BD} \perp \overline{AC}$, then $(AB)^2$ = AD × ....
(a) AC (b) CD (c) BC (d) AD

2 Complete each of the following :

1 If the area of a square is 50 cm^2. , then its diagonal length = ...

2 Two polygons are similar if the corresponding sides are ....

3 If A ∈ straight line L , then its projection on straight line L is ....

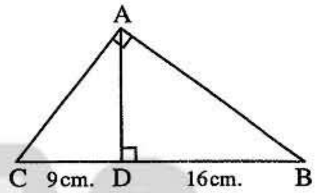
4 ABCD is a parallelogram , if m (∠ A) = 80° , then m (∠ B) = ...

5 If the perimeter of an equilateral triangle is 30 cm. and its height is 5 cm. , then its area = ... cm^2.

3 [a] In the opposite figure :

m (∠ A) = 90° , $\overline{AD} \perp \overline{BC}$, CD = 9 cm. and DB = 16 cm.

Find : AC and AD

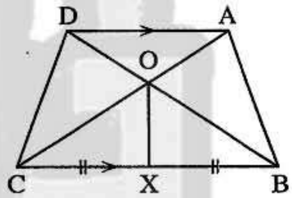


[b] In the opposite figure :

$\overline{DA}$ // $\overline{CB}$, XB = XC

Prove that :

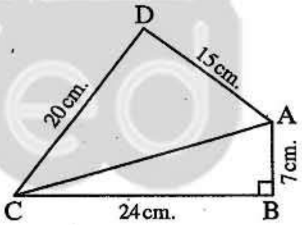
the area of the figure ABXO = the area of the figure DCXO



4 [a] In the opposite figure :

m (∠ B) = 90° , AB = 7 cm. BC = 24 cm. and AD = 15 cm. , CD = 20 cm.

Prove that : m (∠ D) = 90°



[b] In Δ ABC , AB = 5 cm. , BC = 8 cm. and AC = 10 cm.
What is the type of the triangle according to its angles ? (write steps)

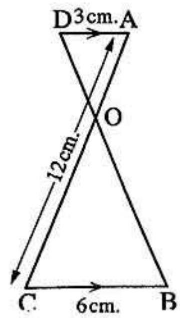
5 [a] A trapezium , the lengths of the two parallel bases are 4 cm. and 10 cm. , if its height is 5 cm. Calculate its middle base and its area.

[b] In the opposite figure :

$\overline{DA}$ // $\overline{CB}$, AD = 3 cm. and BC = 6 cm. , AC = 12 cm.

1 **Prove that :** Δ AOD ~ Δ COB

2 **Find :** the length of $\overline{AO}$



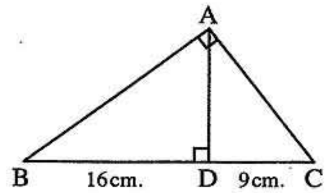
11 Suez Governorate

Directorate of Education
Inspection of Mathematics

Answer the following questions :

1 Choose the correct answer :

1. ABCD is a parallelogram in which m (∠ A) = 70° , then m (∠ B) = ...
   (a) 70° (b) 110° (c) 180° (d) 540°
2. In Δ ABC , if $(AB)^2 > (BC)^2 + (AC)^2$, then angle C is ....
   (a) acute. (b) right. (c) obtuse. (d) straight.
3. **In the opposite figure :**
   m (∠ BAC) = 90° , $\overline{AD} \perp \overline{BC}$
   , DC = 9 cm. , DB = 16 cm.
   , then AD = ... cm.



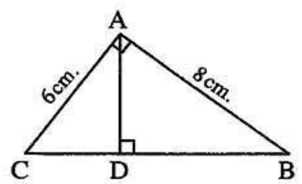
   (a) 144 (b) 25 (c) 50 (d) 12
4. The area of the square whose diagonal length is 10 cm. = ...
   (a) 100 cm^2 (b) 50 cm^2 (c) 40 cm^2 (d) 20 cm^2
5. A trapezium whose lengths of two parallel bases are 6 cm. and 8 cm. , then the length of its middle base equals .... cm.
   (a) 48 (b) 24 (c) 14 (d) 7
6. The ratio between the lengths of two corresponding sides of two similar polygons is 3 : 5 , then the ratio between their perimeters is ....
   (a) 2 : 5 (b) 5 : 3 (c) 3 : 5 (d) 1 : 2

2 Complete each of the following :

1. The median of a triangle divides its surface into two triangular surfaces equals ....
2. In Δ ABC , if $(AC)^2 + (BC)^2 = (AB)^2$, then m (∠ ....) = 90°
3. All the regular polygons that have the same number of sides are ....
4. The area of rhombus is 24 cm^2 , if the length of one of its diagonals 8 cm. , then the length of the other diagonal is ....
5. If the point A ∈ the striaght line L , then the projection of the point A on the line L is ....

3 [a] In the opposite figure :

m (∠ CAB) = 90° , $\overline{AD} \perp \overline{BC}$
, AC = 6 cm. , AB = 8 cm.



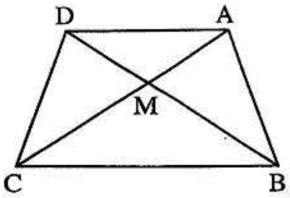
Find : the length of the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$

[b] In the opposite figure :

ABCD is a quadrilateral ,

the area of Δ AMB = the area of Δ DMC

Prove that : $\overline{AD}$ // $\overline{BC}$



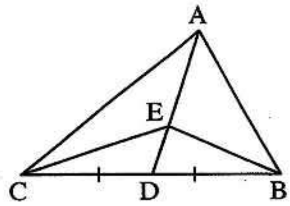
4 **[a]** Determine the type of the greatest angle in Δ ABC where AB = 9 cm. , BC = 10 cm. , AC = 12 cm.

[b] In the opposite figure :

$\overline{AD}$ is the median of Δ ABC , E ∈ $\overline{AD}$

Prove that :

The area of Δ ABE = the area of Δ ACE



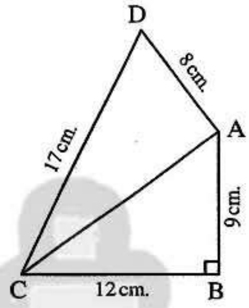
5 **[a] In the opposite figure :**

m (∠ B) = 90° , AB = 9 cm.

, BC = 12 cm. , AD = 8 cm.

, DC = 17 cm.

Prove that : m (∠ DAC) = 90°

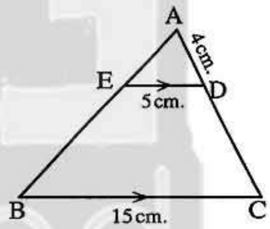


[b] In the opposite figure :

$\overline{ED}$ // $\overline{BC}$, AD = 4 cm.

, ED = 5 cm. , BC = 15 cm.

Find with proof : The length of $\overline{DC}$



12 Port Said Governorate

East Educational Directorate
mathematics Inspection

Answer the following questions :

1 **Choose the correct answer :**

1 In Δ ABC , if $(AB)^2 > (BC)^2 + (AC)^2$, then the angle C is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

2 If the lengths of the diagonals of a rhombus are 6 cm. and 8 cm. , then its perimeter = ... cm.

(a) 24 (b) 28 (c) 14 (d) 20

3 If ABCD is a parallelogram of area 20 cm^2 and E ∈ $\overline{AD}$, then the area of Δ EBC = ... cm^2

(a) 10 (b) 5 (c) 20 (d) 40

4 If the projection of a line segment on a straight line is a point , then the line segment .... straight line.

(a) // (b) ⊥ (c) ≡ (d) ⊂

5 The two triangles drawn on a common base and their vertices located on straight line parallel to the base are ....

(a) congruent. (b) similar.

(c) equal in perimeter. (d) equal in area.

6 ABCD is a parallelogram in which , AB = 5 cm. , BC = 10 cm. and its smaller height is 4 cm. , then its greater height = ... cm.

(a) 2 (b) 4 (c) 8 (d) 10

2 Complete each of the following :

1 If ∠ A complements ∠ B and ∠ B supplements ∠ C , if m (∠ A) = 30° , then m (∠ C) = ...°

2 In Δ ABC , if $(AB)^2 = (BC)^2 + (AC)^2$, then m (∠ ....) = 90°

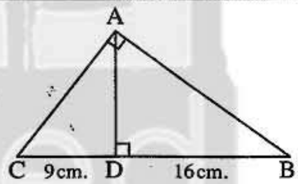
3 If two polygons are similar and the ratio between the lengths of two corresponding sides is 2 : 5 , then the ratio between their perimeters is ....

4 A trapezium whose base lengths are 4 cm. and 6 cm. , then the length of its middle base = ... cm.

5 The rectangle is a parallelogram in which one of its angles is ....

3 [a] In the opposite figure :

Find : the length of $\overline{AB}$, $\overline{AC}$ and $\overline{AD}$



[b] ABCD is a trapezium in which $\overline{AD}$ // $\overline{BC}$, if BC = 2 AD = 20 cm. and its area = 180 cm². Find its height.

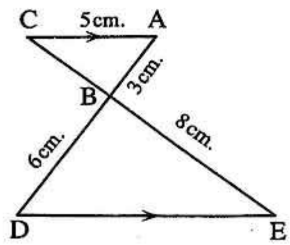
4 [a] In the opposite figure :

$\overline{AC}$ // $\overline{ED}$, AB = 3 cm. , BD = 6 cm.

, AC = 5 cm. , BE = 8 cm.

Prove that : Δ ABC ~ Δ DBE

, then find : the length of $\overline{ED}$, $\overline{BC}$

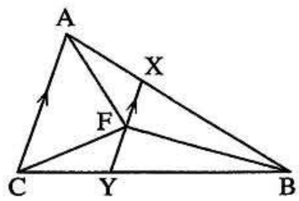


[b] In the opposite figure :

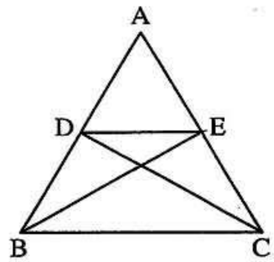
$\overline{AC}$ // $\overline{XY}$ and F is the midpoint of $\overline{XY}$

Prove that :

The area of Δ ABF = the area of Δ CBF



5 **[a] In the opposite figure :**



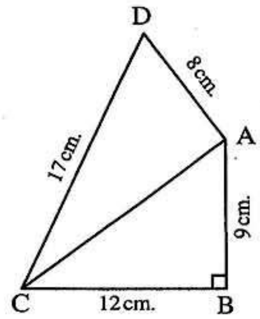
ABC is a triangle in which :

$D \in \overline{AB}$ and $E \in \overline{AC}$

, such that the area of Δ ABE = the area of Δ ACD

Prove that : $\overline{DE} \parallel \overline{BC}$

[b] In the opposite figure :



ABCD is a quadrilateral

, m ($\angle$ B) = 90° , AB = 9 cm.

, BC = 12 cm. , CD = 17 cm.

and DA = 8 cm.

Prove that : m ($\angle$ DAC) = 90°

13 Damietta Governorate

Damietta Inspection of Mathematic
Official Language Schools

Answer the following questions :

1 **Choose the correct answer :**

1. If the lengths of two adjacent sides in a parallelogram are 8 cm. and 10 cm. and the smaller height is 5 cm. , then its area is …………… cm^2.

   (a) 40 (b) 50 (c) 80 (d) 20

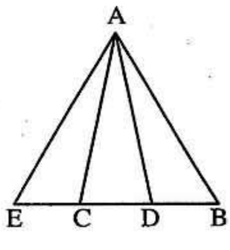
2. If the area of a square is 72 cm^2. , then its diagonal length = …………… cm.

   (a) 6 (b) 8 (c) 12 (d) 36

3. All …………… are similar.

   (a) squares (b) triangles (c) rectangles (d) parallelograms

4. **In the opposite figure :**

   The number of the triangles = ……………

   (a) 3 (b) 4

   (c) 5 (d) 6

5. In Δ ABC , if $(AC)^2 + (BC)^2 = (AB)^2 - 5$, then $\angle$ C is …………… angle.

   (a) acute (b) right (c) obtuse (d) straight

6. If the projection of a line segment on a straight line is a point , then the line segment …………… straight line.

   (a) // (b) $\perp$ (c) $\equiv$ (d) $\subset$

2 Complete each of the following :

1 The area of a triangle is equal to .... the area of the parallelogram if they have a common base lying on one of two parallel sraight lines including them.

2 If Δ ABC ~ Δ DEH where m (∠ A) = 70° , m (∠ E) = 50° , then m (∠ C) = ...°

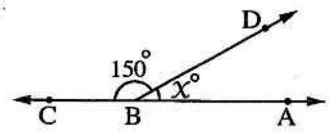
3 ABCD is a parallelogram in which m (∠ A) + m (∠ C) = 140° , then (∠ B) = ...°

4 The length of the projection of a line segment parallel to a given straight line on this straight line is .... the length of the original line segment.

5 **In the opposite figure :**

If B ∈ $\overleftrightarrow{AC}$

, then x = ...°



3 [a] Find the height of a trapezium with area of 450 cm^2. and the two parallel bases lengths are 24 cm. and 12 cm.

[b] In the opposite figure :

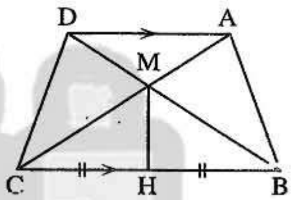
$\overline{AD}$ // $\overline{BC}$, $\overline{AC} \cap \overline{BD}$ = {M}

and H is the midpoint of $\overline{BC}$

Prove that :

1 The area of Δ AMB = the area of Δ DMC

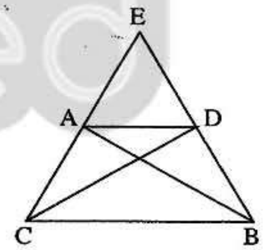
2 The area of the figure ABHM = The area of the figure DCHM



4 [a] In the opposite figure :

The area of Δ EAB = the area of Δ EDC

Prove that : $\overline{AD}$ // $\overline{BC}$



[b] Determine the type of Δ ABC according to its angles if

AB = 7 cm. , BC = 3 cm. , AC = 6 cm.

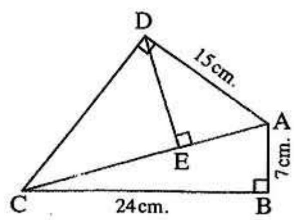
5 [a] In the opposite figure :

m (∠ ADC) = m (∠ ABC) = 90° , AB = 7 cm.

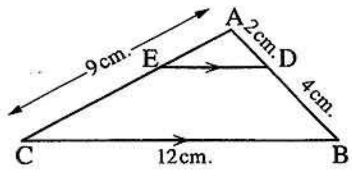
, BC = 24 cm. , AD = 15 cm.

Find : 1 The length of $\overline{AC}$ and $\overline{DC}$

2 The length of the projection of $\overline{AD}$ on $\overleftrightarrow{AC}$



[b] In the opposite figure :



ABC is a triangle , AD = 2 cm. , DB = 4 cm.

, BC = 12 cm. , AC = 9 cm. and $\overline{DE}$ // $\overline{BC}$

[1] **Prove that :** Δ ADE ~ Δ ABC

[2] **Find :** The length of $\overline{DE}$ and $\overline{AE}$

14 Beni Suef Governorate

Directorate of official Language Schools
Education administration

Answer the following questions :

1 Choose the correct answer :

[1] If the lengths of two adjacent sides of a parallelogram are 8 cm. and 10 cm. and its greater height is 5 cm. , then its area equals .... cm^2.

(a) 18 (b) 40 (c) 50 (d) 80

[2] The median of the triangle divides its surface into two triangles ....

(a) congruent. (b) equal in area. (c) isosceles. (d) right-angled.

[3] If $\overline{AB}$ // $\overleftrightarrow{XY}$, then the length of the projection of $\overline{AB}$ on $\overleftrightarrow{XY}$ .... the length of $\overline{AB}$

(a) < (b) > (c) = (d) ≤

[4] In Δ ABC , $(AC)^2 = (BC)^2 - (AB)^2$, then ∠ A is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

[5] Any triangle has at least two .... angles.

(a) acute (b) obtuse (c) right (d) reflex

[6] The number of axes of symmetry of an isosceles triangle = ...

(a) zero (b) 1 (c) 2 (d) 3

2 Complete each of the following :

[1] If two triangles are equal in area and drawn on the same base and on one side of it , then their vertices ....

[2] If two polygons are similar , then the corresponding .... are equal in measure.

[3] A triangle whose side lengths are 9 cm. , 12 cm. and 16 cm. , then its type according to its angles is ....

[4] In Δ XYZ , if $(XZ)^2 + (YZ)^2 > (XY)^2$, then the type of the angle Z is ....

[5] If a straight line intersects two parallel straight lines , then every two alternate angles are ....

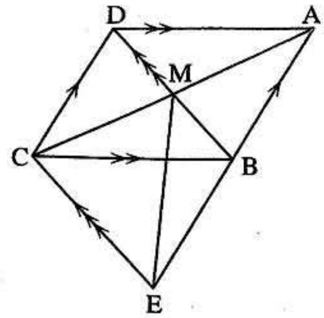
3 **[a] In the opposite figure :**

ABCD and BECD are two parallelograms

where $\overline{AC} \cap \overline{BD} = \{M\}$

Prove that :

The area of Δ ABD = the area of Δ MEC



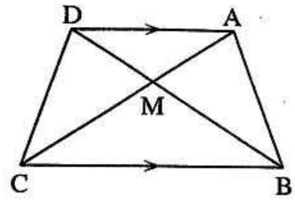
[b] In the opposite figure :

ABCD is a quadrilateral , $\overline{AD} // \overline{BC}$

and $\overline{AC} \cap \overline{BD} = \{M\}$

Prove that :

The area of Δ ABM = the area of Δ DCM



4 **[a]** Two pieces of land have equal areas , one of them has the shape of a rhombus whose diagonals lengths are 18 m. and 24 m. , and the other one has the shape of a trapzium whose height is 12 m. Find the length of its middle base.

[b] In the opposite figure :

Δ ABC , D ∈ $\overline{AB}$, E ∈ $\overline{AC}$

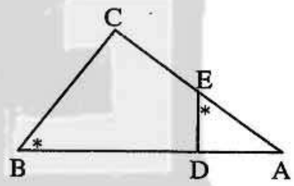
, AE = 4 cm. , EC = 5 cm.

, BC = 7.5 cm. , AD = 3 cm.

and m (∠ AED) = m (∠ B)

1 **Prove that :** Δ AED ~ Δ ABC

2 **Find the length of :** $\overline{BD}$



5 **In the opposite figure :**

$\overline{DE} \perp \overline{AB}$, m (∠ ABC) = 90°

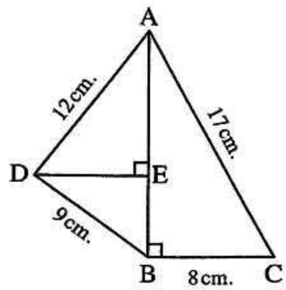
, AD = 12 cm. , AC = 17 cm.

, BC = 8 cm. , DB = 9 cm.

1 **Prove that :** m (∠ ADB) = 90°

2 **Find :** the length of $\overline{DE}$

3 **Find :** the length of the projection of $\overline{AD}$ on $\overleftrightarrow{AB}$



15 Qena Governorate

Qena Directorate
Governmental Language Schools Department

Answer the following questions :

1 Choose the correct answer :

1. If ABC is a right-angled triangle at A and $\overline{AD} \perp \overline{BC}$, then $(AC)^2 = \dots\dots$
   (a) AB × BC (b) BD × DC (c) BD × BC (d) CD × BC
2. If Δ ABC ~ Δ XYZ and AB : XY = 2 : 3 , the perimeter of Δ ABC = 14 cm. , then the perimeter of Δ XYZ = ... cm.
   (a) 14 (b) 21 (c) 7 (d) 30
3. If the area of a square is 50 cm^2 , then its diagonal length = ... cm.
   (a) 10 (b) 5 (c) 15 (d) 25
4. The length of the projection of a line segment .... the length of the line segment.
   (a) > (b) = (c) ≤ (d) ≥
5. If ABCD is a parallelogram in which AB = 7 cm. , BC = 8 cm. and its smaller height is 5 cm. , then its area = ... cm^2
   (a) 28 (b) 40 (c) 35 (d) 56
6. The number of axes of symmetry of the isosceles triangle is ....
   (a) 0 (b) 1 (c) 2 (d) 4

2 Complete each of the following :

1. If two triangles drawn on a common base and their vertices on a straight line parallel to this base , then they are ....
2. If two polygons are similar to a third polygon , then they are ....
3. If area of a trapezium is 24 cm^2 and its height is 4 cm. , then the length of the middle base is .... cm.
4. In the triangle ABC , if $(AC)^2 - (BC)^2 > (AB)^2$, then ∠ B is .... angle.
5. Two triangles are similar if their corresponding side lengths are ....

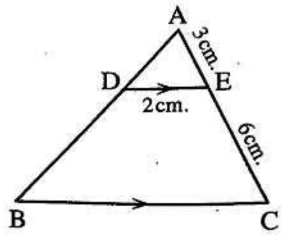
3 **[a]** A rhombus with diagonal lengths are 12 cm. and 10 cm. and its height 8 cm. find its perimeter.

[b] In the opposite figure :

ABC is a triangle in which $D \in \overline{AB}$

, $E \in \overline{AC}$, where $\overline{DE} \parallel \overline{BC}$

, AE = 3 cm. , EC = 6 cm. , DE = 2 cm.



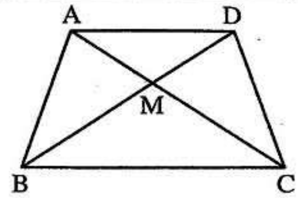
1 **Prove that :** $\Delta ADE \sim \Delta ABC$

2 **Find :** The length of $\overline{BC}$

4 **[a] In the opposite figure :**

If the area of ΔAMB = the area of ΔCMD

Prove that : $\overline{AD} \parallel \overline{BC}$

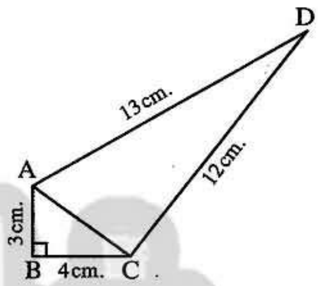


[b] In the opposite figure :

ABCD is a quadrilateral in which m (∠ B) = 90°

, AB = 3 cm. , BC = 4 cm.

, CD = 12 cm. , AD = 13 cm.



1 **Prove that :** m (∠ ACD) = 90°

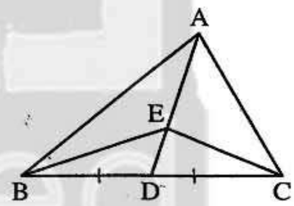
2 **Find :** the area of the figure ABCD

5 **[a] In the opposite figure :**

ABC is a triangle in which :

D is the midpoint of $\overline{BC}$, $E \in \overline{AD}$

Prove that : the area of ΔABE = the area of ΔACE

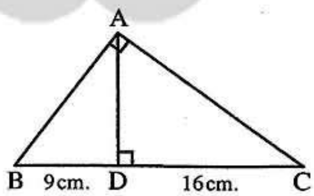


[b] In the opposite figure :

ABC is a right-angled triangle at A

, $\overline{AD} \perp \overline{BC}$, BD = 9 cm.

, CD = 16 cm.



Find : 1 The perimeter of the triangle ABC

2 The area of the triangle ABC

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A group of supervisors

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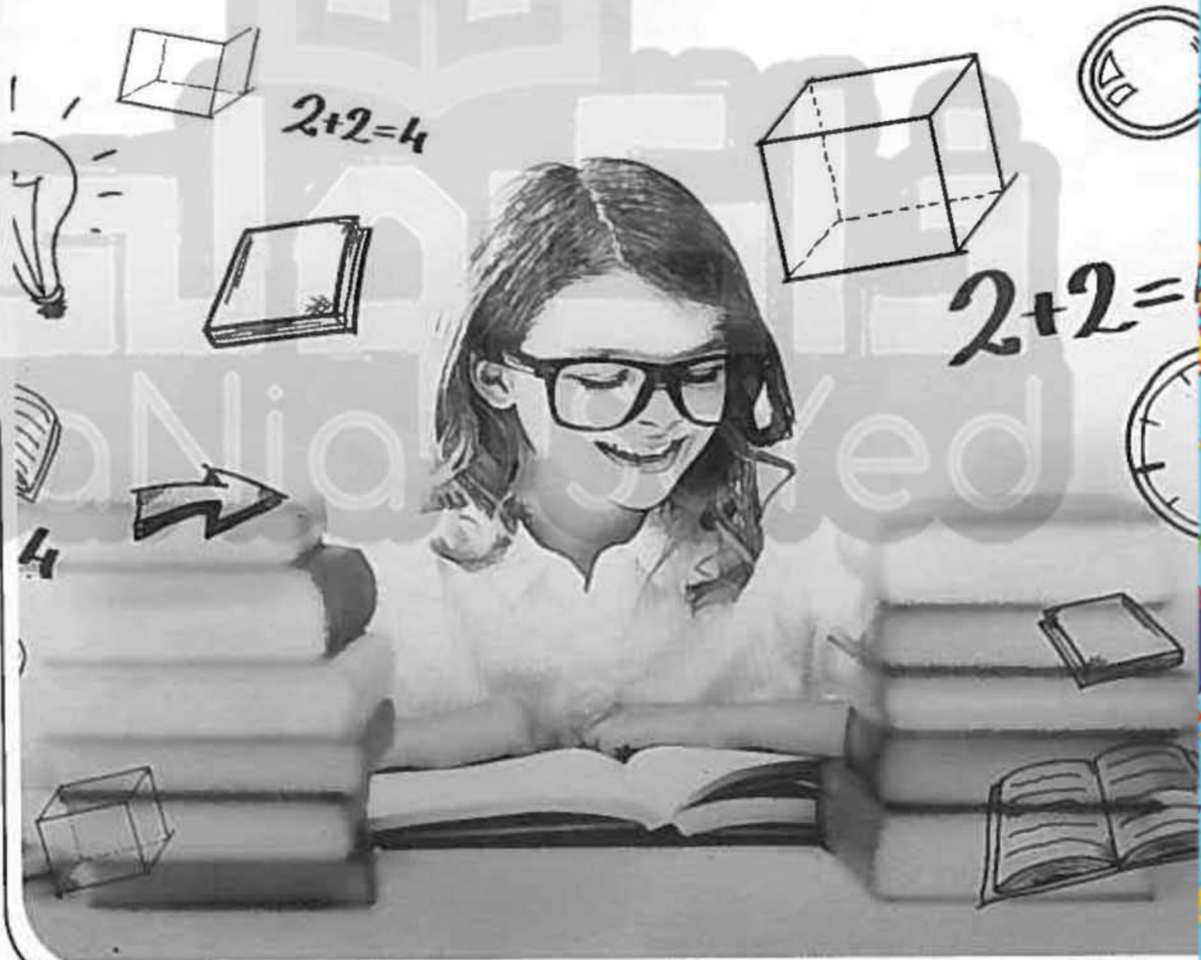
of Algebra and Statistics Exercises



Answers of Unit One

Answers of Exercise 1

1

1. 5 , 6
2. – 2 , – 6
3. – 3 , 6
4. – 15 , 1

2

1. $(x+5)(x+3)$
2. $(x+1)(x+10)$
3. $(x-3)(x-4)$
4. $(x-2)(x-15)$
5. $(x-2)(x+7)$
6. $(x+6)(x-2)$
7. $(x-8)(x+2)$
8. $(x-5)(x+2)$

3

1. $(x+2y)(x+3y)$
2. $(a+5b)(a+6b)$
3. $(b-2c)(b+5c)$
4. $(a+24b)(a-2b)$
5. $(x-12y)(x-3y)$
6. $(a-6b)(a-7b)$
7. $(x-8y)(x+3y)$
8. $(x+2y)(x-9y)$

4

1. $(a+17)(a-2)$
2. $(a+25)(a-3)$
3. $(x+5)(x-2)$
4. $(x-3)(x-7)$

5

1. $(x^2+6)(x^2+3)$
2. $(x^2-5)(x^2-3)$
3. $(\ell^3-10)(\ell^3+4)$
4. $(a^2-7b^2)(a^2+8b^2)$

6

1. $5(x^2-2x-3)=5(x-3)(x+1)$
2. $2(a^2+14a+48)=2(a+8)(a+6)$
3. $y(y^2+y-6)=y(y+3)(y-2)$
4. $x(x^2-3x-28)=x(x-7)(x+4)$
5. $3(x^2-5x-14)=3(x-7)(x+2)$
6. $3x(x^2-5x+6)=3x(x-2)(x-3)$
7. $-2(x^2+x-20)=-2(x+5)(x-4)$
8. $-(x^2-2x-63)=-(x-9)(x+7)$
9. $b^2(a^2-24a+143)=b^2(a-11)(a-13)$
10. $2(a^4-12a^2b^2-13b^4)=2(a^2-13b^2)(a^2+b^2)$

7

1. $x^2+7x+10=(x+5)(x+2)$
2. $x^2-4x-3x+6=x^2-7x+6=(x-6)(x-1)$
3. $a^2-16b^2+6ab=a^2+6ab-16b^2$
   $=(a-2b)(a+8b)$
4. $x^3-23x^2+60x=x(x^2-23x+60)$
   $=x(x-3)(x-20)$
5. $x^2-13x+36-2x-10=x^2-15x+26$
   $=(x-13)(x-2)$

8

1. $c=2$, $(x+5)(x-3)$
2. $c=10$, $(x-2)(x-5)$
3. $c=30$, $(y-29)(y-1)$
4. $c=12$, $(a+4)(a-3)$

Try to find other values to the number c

9

1. 9 , 2
2. $x+3$
3. $12x$, 7 , x
4. $x-6$
5. 3
6. 4

10

1. b
2. b
3. d
4. c
5. a
6. c
7. b

11

The width $=(x+2)$ cm.

The perimeter $=2\big((x+4)+(x+2)\big)$

$=2(2x+6)=(4x+12)$ cm.

12

1. $\big((x-1)-4\big)\big((x-1)+2\big)=(x-5)(x+1)$
2. $-\big((a+b)^2-3(a+b)-10\big)$
   $=-\big((a+b)-5\big)\big((a+b)+2\big)=-(a+b-5)(a+b+2)$

13

$x^2-2xy-3y^2=(x+y)(x-3y)$

$\therefore 7=1\times(x-3y)$ $\therefore x-3y=7$

Answers of Exercise 2

1

1. $(5x-7)(x+1)$
2. $(3x+4)(x+2)$
3. $(2x-5)(3x+2)$

4 $(x-2)(3x-1)$

5 $(3x-2)(x+3)$

6 $(2x-3)(x+2)$

7 $2x^2 - xy - 6y^2 = (2x+3y)(x-2y)$

8 $5x^2 - 3xy - 2y^2 = (x-y)(5x+2y)$

2

1 $(2x+1)(x+1)$

2 $(3a+1)(a+2)$

3 $(5z-2)(z-1)$

4 $(3x+1)(x-5)$

5 $(5x-6)(x+2)$

6 $(3x+4)(x+2)$

7 $(3x-1)(2x-3)$

8 $(5a-8)(a-2)$

9 $(3y-2)(y+3)$

10 $(2z-1)(4z+3)$

11 $(4y-7)(y+3)$

12 $(4a-3)(3a+2)$

3

1 $(x-2y)(2x-y)$

2 $(3x+y)(x-7y)$

3 $(3a+b)(2a+b)$

4 $(2y-x)(y+x)$

5 $(10a-9b)(a+2b)$

6 $(6x+7y)(x-9y)$

7 $(7x^2+30y)(x^2-y)$

4

1 $3(2x^2-7x+6) = 3(2x-3)(x-2)$

2 $4(2x^2-7x-15) = 4(2x+3)(x-5)$

3 $5(3m^2+5m-2) = 5(3m-1)(m+2)$

4 $x(8x^2-27x-20) = x(8x+5)(x-4)$

5 $2x(3x^2+7x+4) = 2x(3x+4)(x+1)$

6 $3x(6x^4+11x^2-10) = 3x(3x^2-2)(2x^2+5)$

7 $-3x^2y^2(5y^2-2y-7) = -3x^2y^2(5y-7)(y+1)$

8 $4(c+d)(3x^2+17x+20)$
$= 4(c+d)(3x+5)(x+4)$

5

1 $2x^2+6x+13x+24 = 2x^2+19x+24$
$= (2x+3)(x+8)$

2 $12x^2+28xy-5y^2 = (6x-y)(2x+5y)$

3 $5y^2-28xy-12x^2 = (5y+2x)(y-6x)$

4 $25b^2-20b+4-4b-5$
$= 25b^2-24b-1 = (25b+1)(b-1)$

6

1 $5x-7$

2 $2x+3$

7

1 $c = 2$, $(2x-5)(x+3)$

2 $c = 2$, $(2x-1)(x-6)$

Try to find other values.

8

The two dimensions are $(2x+5)$ cm. $(x+7)$ cm.
when $x = 3$ then
the two dimensions are 11 cm. and 10 cm.
the perimeter $= 2(11+10) = 42$ cm.

9

1 $-(4(a+b)+1)((a+b)-3)$

2 $(3(2x+3y)+2(x-y))((2x+3y)-(x-y))$
$= (8x+7y)(x+4y)$

Answers of Exercise 3

1

Each expression of numbers 3 , 5 , 8 , 9 and 12 is a perfect square expression.

2

1 $(m-1)^2$

2 $(x+y)^2$

3 $(3x+2)^2$

4 $(5b-1)^2$

5 $(3a+b)^2$

6 $(2x-y)^2$

7 $(4a-5b)^2$

8 $(1+7x)^2$

9 $(6-5k)^2$

10 $(1-5a^2)^2$

3

1 $2(9y^2-6y+1) = 2(3y-1)^2$

2 $3(4x^2+12xy+9y^2) = 3(2x+3y)^2$

3 $6(4a^4+4a^2+1) = 6(2a^2+1)^2$

4 $6(a^4-2a^2b^2+b^4) = 6(a^2-b^2)^2$

5 $5a(4y^2-12y+9) = 5a(2y-3)^2$

6 $6x(4+4x+x^2) = 6x(2+x)^2$

7 $3z(1+14z^3+49z^6) = 3z(1+7z^3)^2$

8 $b(4b^2+4bc+c^2) = b(2b+c)^2$

9 $-(36a^2-60ab+25b^2) = -(6a-5b)^2$

10 $(c-d)(1+2x+x^2) = (c-d)(1+x)^2$

4

1 $\left(\frac{1}{2}y-2\right)^2$

2 $\left(\frac{1}{4}a+\frac{1}{5}\right)^2$

3 $\left(\frac{2}{5}x-\frac{1}{8}\right)^2$

4 $(0.1x-1)^2$

5

1 $49x^2-70xy+25y^2 = (7x-5y)^2$

2 $9a^2+25b^2-30ab = (3a-5b)^2$

3 $4x^2-28xy+49y^2 = (2x-7y)^2$

4 $m^2-22mn+121n^2 = (m-11n)^2$

5 $x^2-2xy+y^2+4xy$
$= x^2+2xy+y^2 = (x+y)^2$

6
1. $\pm 4x$ 2. $\pm 36a$ 3. $\pm 24ab$
4. $\pm \frac{1}{5}xy$ 5. $\pm 36x^2y^2$ 6. $\pm 14z^2l$
7. 9 8. 49 9. $9b^2$
10. $4n^2$ 11. y^4 12. $9a^2$

7
1. 12 2. 80 3. $\frac{1}{16}$ 4. 4
5. 9 6. 4 7. $\frac{1}{2}$ 8. 7

8
1. c 2. d 3. d 4. c
5. b 6. c

9
1. $(87+13)^2=(100)^2=10\,000$
2. $(99-98)^2=1^2=1$
3. $(7.3+2.7)^2=(10)^2=100$
4. $(20.7-0.7)^2=(20)^2=400$
5. $(997)^2+2\times3\times997+(3)^2=(997+3)^2$
$=(1000)^2=1000\,000$
6. $(99+1)^2=(100)^2=10\,000$
7. $(5)^2-2\times5\times9+(9)^2=(5-9)^2=16$

10
$\because$ The area of the square $=9x^2+30x+m$
$\therefore$ $(9x^2+30x+m)$ is a perfect square
$\therefore m=\frac{(30x)^2}{4\times9x^2}=\frac{900x^2}{36x^2}=25$
$\therefore$ The area of the square $=9x^2+30x+25$
$=(3x+5)^2$
$\therefore$ the side length of the square $=3x+5$
when $x=2$ $\therefore$ the side length $=3\times2+5=11$ cm.
$\therefore$ the perimeter $=11\times4=44$ cm.

11
1. $(y+(x+1))^2=(y+x+1)^2$
2. $((a+b)-2c^2)^2=(a+b-2c^2)^2$
3. $(3(x+2)+2(y-3))^2=(3x+2y)^2$

Answers of Exercise 4

1
1. $(x+2)(x-2)$ 2. $(a+5)(a-5)$
3. $(4x+3)(4x-3)$ 4. $(7y+1)(7y-1)$
5. $(x+2y)(x-2y)$ 6. $(15x+y)(15x-y)$
7. $(7x+8y)(7x-8y)$ 8. $(25a+9b)(25a-9b)$
9. $(3+y)(3-y)$
10. $25-9x^2=(5-3x)(5+3x)$
11. $(ab+1)(ab-1)$ 12. $(a+bc^2)(a-bc^2)$
13. $(x^2+10)(x^2-10)$ 14. $(4a^3+b^3)(4a^3-b^3)$
15. $\left(x+\frac{1}{4}\right)\left(x-\frac{1}{4}\right)$ 16. $\left(\frac{1}{3}y+\frac{3}{2}\right)\left(\frac{1}{3}y-\frac{3}{2}\right)$
17. $\left(\frac{a}{5}+\frac{2b}{7}\right)\left(\frac{a}{5}-\frac{2b}{7}\right)$
18. $\left(\frac{1}{5}x+\frac{1}{2}y\right)\left(\frac{1}{5}x-\frac{1}{2}y\right)$

2
1. $(x^2+1)(x^2-1)=(x^2+1)(x-1)(x+1)$
2. $(x^2+4y^2)(x^2-4y^2)=(x^2+4y^2)(x+2y)(x-2y)$
3. $(x^{50}+1)(x^{50}-1)=(x^{50}+1)(x^{25}+1)(x^{25}-1)$

3
1. $2(x^2-16)=2(x+4)(x-4)$
2. $x(x^2-25)=x(x+5)(x-5)$
3. $x^2(x^2-1)=x^2(x+1)(x-1)$
4. $2(4x^2-25)=2(2x+5)(2x-5)$
5. $xy(x^2-y^4)=xy(x+y^2)(x-y^2)$
6. $3x(9x^2-16y^6)=3x(3x+4y^3)(3x-4y^3)$
7. $\frac{1}{3}(x^2-9)=\frac{1}{3}(x+3)(x-3)$
8. $3\left(x^2-\frac{1}{16}\right)=3\left(x+\frac{1}{4}\right)\left(x-\frac{1}{4}\right)$
9. $\frac{1}{2}\left(x^2-\frac{1}{9}y^2\right)=\frac{1}{2}\left(x+\frac{1}{3}y\right)\left(x-\frac{1}{3}y\right)$
10. $(2a-b)(4b^2-25a^2)$
$=(2a-b)(2b+5a)(2b-5a)$

4
1. $((a+b)+2)((a+b)-2)$
$=(a+b+2)(a+b-2)$
2. $((2x-3)+1)((2x-3)-1)$
$=(2x-2)(2x-4)=4(x-1)(x-2)$
3. $(1+(a-1))(1-(a-1))$
$=(1+a-1)(1-a+1)=a(2-a)$
4. $(3a+(2a+b))(3a-(2a+b))$
$=(5a+b)(a-b)$
5. $(ab+(ab-1))(ab-(ab-1))=(2ab-1)$
6. $((x+1)+(x-1))((x+1)-(x-1))$
$=2x\times2=4x$
7. $(3(m-1)+5(m+1))(3(m-1)-5(m+1))$
$=(3m-3+5m+5)(3m-3-5m-5)$
$=(8m+2)(-2m-8)=-4(4m+1)(m+4)$

8 $((x+y+5)+(x-y-5))\ ((x+y+5)-(x-y-5))$
$= 2x(2y+10) = 4x(y+5)$

9 $a^2-4b^2-5b^2 = a^2-9b^2 = (a-3b)(a+3b)$

5

1 $(77+23)(77-23) = 100 \times 54 = 5400$
2 $(78+77)(78-77) = 155 \times 1 = 155$
3 $(11.6+1.6)(11.6-1.6) = 13.2 \times 10 = 132$
4 $(8.27+1.73)(8.27-1.73) = 10 \times 6.54 = 65.4$
5 $(95+5)(95-5) = 100 \times 90 = 9000$
6 $(999+1)(999-1) = 1000 \times 998 = 998000$
7 $2\ [(25.87)^2-(24.13)^2]$
$= 2(25.87+24.13)(25.87-24.13)$
$= 2 \times 50 \times 1.74 = 174$

6

1 $(30+1)(30-1) = 900-1 = 899$
2 $(100+3)(100-3) = (100)^2-(3)^2 = 10000-9$
$= 9991$

7

The expression $= ((x+y)+(x-y))\ ((x+y)-(x-y))$
$= 2x \times 2y = 4xy = 4 \times 8 = 32$

8

1 $3y, 2x, 9y^2$ 2 $5x, 5x, 9\ m.^2$
3 $16.8x, 8x$ 4 6 5 2
6 3 7 9 8 1
9 9 10 28

9

1 c 2 a 3 c 4 d
5 c 6 a 7 a 8 b

10

Let the length of the other side $= x$
$\therefore x^2 = (41)^2-(40)^2 = (41-40)(41+40) = 1 \times 81 = 81$
$\therefore x = \sqrt{81} = 9$ cm.
$\therefore$ The length of the other side = 9 cm.

11

1 $(a-b)^2-c^2 = ((a-b)+c)\ ((a-b)-c)$
$= (a-b+c)(a-b-c)$

2 $(2a+3b)^3-4a^2(2a+3b)$
$= (2a+3b)\ ((2a+3b)^2-4a^2)$
$= (2a+3b)\ ((2a+3b)-2a)\ ((2a+3b)+2a)$
$= 3b(2a+3b)(4a+3b)$

12

$x = \frac{(225)^2-(25)^2}{25 \times 20} = \frac{(225+25)(225-25)}{25 \times 20}$
$= \frac{250 \times 200}{25 \times 20} = 100$

13

$(x-y)^2 = 4$ taking the square root of the two sides
$\therefore x-y = 2$ where $x > y$
$\therefore x^2-y^2 = (x+y)(x-y) = 8 \times 2 = 16$

Answers of exams on the first part of unit one

Model 1

1
1 d 2 c 3 a
4 d 5 b 6 c

2
1 81 2 $2, 9a^2, 4$ 3 $5, 2x^2, 7x$
4 19 5 $(x+5)$

3
1 $(x+1)(x-8)$ 2 $(x-4)(3x+2)$
3 $2x(2x-y)(2x+y)$ 4 $(2x+3)^2$

4
1 4 2 5600

5
The width $= (x+4)$ cm.
, the perimeter $= (4x+20)$ cm.

Model 2

1
1 c 2 c 3 d
4 b 5 d 6 b

2
1 20 2 35 3 $x+5$
4 $5y, 3x, 25y^2$ 5 $x-2$

3
1 $(x-3)(x-7)$ 2 $(5a^2-1)^2$
3 $2(2a-5)(2a+5)$ 4 $(5z-2)(z-1)$

4
The two dimensions are : $(a-2)$ cm. , $(5a-8)$ cm.
, the perimeter = 40 cm.

5
1 $y(2x-y)^2$ 2 $2a(2b+6) = 4a(b+3)$

Answers of Exercise 5

1

1. $(x+2)(x^2-2x+4)$
2. $(x-1)(x^2+x+1)$
3. $(4x+3)(16x^2-12x+9)$
4. $(2x-5)(4x^2+10x+25)$
5. $(5+a)(25-5a+a^2)$
6. $(7-3m)(49+21m+9m^2)$
7. $(m+4n)(m^2-4mn+16n^2)$
8. $(8x-y)(64x^2+8xy+y^2)$
9. $(xy+3)(x^2y^2-3xy+9)$
10. $(3xy-4)(9x^2y^2+12xy+16)$
11. $\left(\frac{1}{2}a-2b\right)\left(\frac{1}{4}a^2+ab+4b^2\right)$
12. $\left(\ell-\frac{1}{5}\right)\left(\ell^2+\frac{1}{5}\ell+\frac{1}{25}\right)$
13. $(2a+0.1)(4a^2-0.2a+0.01)$
14. $(0.3m-n)(0.09m^2+0.3mn+n^2)$
15. $(1+5b^2)(1-5b^2+25b^4)$
16. $(2x-7y^2)(4x^2+14xy^2+49y^4)$
17. $(x^2+y^2)(x^4-x^2y^2+y^4)$
18. $(x^3-8)(x^3+8)$
    $=(x-2)(x^2+2x+4)(x+2)(x^2-2x+4)$

2

1. $2(x^3+8)=2(x+2)(x^2-2x+4)$
2. $3(x^3-27)=3(x-3)(x^2+3x+9)$
3. $\ell(\ell^3+64)=\ell(\ell+4)(\ell^2-4\ell+16)$
4. $m(\ell^3-27m^3)=m(\ell-3m)(\ell^2+3\ell m+9m^2)$
5. $3x(x^3+1)=3x(x+1)(x^2-x+1)$
6. $2x^2(x^3-27)=2x^2(x-3)(x^2+3x+9)$
7. $2(8x^3+125y^3)$
   $=2(2x+5y)(4x^2-10xy+25y^2)$
8. $2b(8a^3+343b^3)$
   $=2b(2a+7b)(4a^2-14ab+49b^2)$
9. $2xy^2(27x^3-8y^3)$
   $=2xy^2(3x-2y)(9x^2+6xy+4y^2)$
10. $4x^5y^2(125x^3-64y^3)$
    $=4x^5y^2(5x-4y)(25x^2+20xy+16y^2)$
11. $\frac{1}{2}(x^3+8)=\frac{1}{2}(x+2)(x^2-2x+4)$
12. $\frac{1}{3}(x^3-27)=\frac{1}{3}(x-3)(x^2+3x+9)$

3

1 a	2 a	3 b	4 c
5 d	6 b	7 b	8 c

4

1. x^2+x+1
2. $(2a+5)(4a^2-10a+25)$
3. $(x^4+y^5)(x^8-x^4y^5+y^{10})$
4. $8a^3-27=(2a-3)(4a^2+6a+9)$
5. x^2+3x+9
6. $2a+1$

5

$x^2-y^2=(x-y)(x+y)$

$20=2(x+y)$ $\quad\therefore x+y=10$

$\therefore x^3+y^3=(x+y)(x^2-xy+y^2)=10\times 28=280$

6

1. $((a+b)-b)((a+b)^2+b(a+b)+b^2)$
   $=a(a^2+2ab+b^2+ba+b^2+b^2)$
   $=a(a^2+3ab+3b^2)$
2. $((x+5)-5)((x+5)^2+5(x+5)+25)$
   $=x(x^2+10x+25+5x+25+25)$
   $=x(x^2+15x+75)$
3. $((m-2n)-2n)\times((m-2n)^2+2n(m-2n)+4n^2)$
   $=(m-4n)(m^2-4mn+4n^2+2mn-4n^2+4n^2)$
   $=(m-4n)(m^2-2mn+4n^2)$
4. $(2(m+n)-n)(4(m+n)^2+2n(m+n)+n^2)$
   $=(2m+2n-n)$
   $\times(4m^2+8mn+4n^2+2mn+2n^2+n^2)$
   $=(2m+n)(4m^2+10mn+7n^2)$
5. $2(1-(x-1)^3)=2(1-(x-1))(1+(x-1)+(x-1)^2)$
   $=2(2-x)(1+x-1+x^2-2x+1)$
   $=2(2-x)(x^2-x+1)$
6. $((x+5)+(x-5))$
   $\times((x+5)^2-(x+5)(x-5)+(x-5)^2)$
   $=2x(x^2+10x+25-x^2+25+x^2-10x+25)$
   $=2x(x^2+75)$
7. $((x+y)-(x-y))$
   $\times((x+y)^2+(x+y)(x-y)+(x-y)^2)$
   $=2y(x^2+2xy+y^2+x^2-y^2+x^2-2xy+y^2)$
   $=2y(3x^2+y^2)$

8 $(m-n)\left(1+(m-n)^3\right)$

$=(m-n)(1+(m-n))\left(1-(m-n)+(m-n)^2\right)$

$=(m-n)(m-n+1)\left(1-m+n+m^2-2mn+n^2\right)$

9 $(x^6-4)-4=x^6-8=(x^2-2)(x^4+2x^2+4)$

10 $x^3-27+28=x^3+1=(x+1)(x^2-x+1)$

7

1 $(m^3-1)(m^3-2)=(m-1)(m^2+m+1)(m^3-2)$

2 $(y^3-1)(y^3+27)$

$=(y-1)(y^2+y+1)(y+3)(y^2-3y+9)$

3 $(x^3-1)(x^3-27)$

$=(x-1)(x^2+x+1)(x-3)(x^2+3x+9)$

4 $(x^3+1)(x^3-8)$

$=(x+1)(x^2-x+1)(x-2)(x^2+2x+4)$

8

$(x+5)^4-(x+5)=(x+5)\left((x+5)^3-1\right)$

$=(x+5)\left((x+5)-1\right)\left((x+5)^2+(x+5)+1\right)$

$=(x+5)(x+4)(x^2+10x+25+x+5+1)$

$=(x+5)(x+4)(x^2+11x+31)$

9

$x-y=1$ squaring the two sides.

$\therefore (x-y)^2=1$

$\therefore x^2-2xy+y^2=1$ $\quad\therefore x^2+y^2=1+2xy$

$\because xy=2$ $\quad\therefore x^2+y^2=1+4=5$

$\therefore x^3-y^3=(x-y)(x^2+xy+y^2)$

$=1\times(5+2)=1\times7=7$

10

Let the two integers be x and y

$\because x+y=2$ $\quad\therefore (x+y)^2=4$

$\because x^2+2xy+y^2=4$ $\quad,\because x^2+y^2=34$

$\therefore 34+2xy=4$ $\quad\therefore xy=-15$

$\therefore x^3+y^3=(x+y)(x^2-xy+y^2)$

$=2(34+15)=98$

Answers of Exercise 6

1

1 $x(a+b)+y(a+b)=(a+b)(x+y)$

2 $b(a-d)+h(a-d)=(a-d)(b+h)$

3 $x(a+y)+(a+y)=(a+y)(x+1)$

4 $a(m-n)+(m-n)=(m-n)(a+1)$

5 $x(a-c)+y(a-c)=(a-c)(x+y)$

6 $m(x-y)-n(x-y)=(x-y)(m-n)$

7 $y(x+5)+7(x+5)=(x+5)(y+7)$

8 $7(x-4)+a(x-4)=(x-4)(7+a)$

9 $5(\ell-2m)-a(\ell-2m)=(\ell-2m)(5-a)$

10 $a(3x-1)-2b(3x-1)=(3x-1)(a-2b)$

2

1 $c(c+d)+h(c+d)=(c+d)(c+h)$

2 $2m(3m+1)-n(3m+1)=(3m+1)(2m-n)$

3 $2m(4n-m)+3\ell(4n-m)=(4n-m)(2m+3\ell)$

4 $x(x-2z)-2y(x-2z)=(x-2z)(x-2y)$

5 $(a+b)^2-c^2=(a+b-c)(a+b+c)$

6 $(5x-1)^2-y^2=(5x-1-y)(5x-1+y)$

7 $1-(x+2y)^2=(1-x-2y)(1+x+2y)$

8 $(x-y)(x+y)+4(x+y)=(x+y)(x-y+4)$

9 $(x-2y)(x+2y)-5(x-2y)$

$=(x-2y)(x+2y-5)$

10 $(3x+y)^2-4a^2=(3x+y-2a)(3x+y+2a)$

11 $2x(xy+a)-y(xy+a)=(xy+a)(2x-y)$

12 $bx(ax+1)-(ax+1)=(ax+1)(bx-1)$

3

1 $a^2(a+1)+(a+1)=(a+1)(a^2+1)$

2 $x^2(x-3)+6(x-3)=(x-3)(x^2+6)$

3 $(a+b)(a^2-ab+b^2)-(a+b)$

$=(a+b)(a^2-ab+b^2-1)$

4 $x^2(x+2)-(x+2)=(x+2)(x^2-1)$

$=(x+2)(x-1)(x+1)$

5 $a^2(a+1)-9(a+1)=(a+1)(a^2-9)$

$=(a+1)(a-3)(a+3)$

6 $x^2(3x+2)+4(3x+2)=(3x+2)(x^2+4)$

7 $(y^3+8)+6y(y+2)$

$=(y+2)(y^2-2y+4)+6y(y+2)$

$=(y+2)(y^2-2y+4+6y)$

$=(y+2)(y^2+4y+4)$

$=(y+2)(y+2)^2=(y+2)^3$

8 $a(a^3-3a^2-15+5a)$

$=a\left(a^2(a-3)+5(a-3)\right)$

$=a(a-3)(a^2+5)$

9 $a^2(a^3-2)+(a^3-2)=(a^3-2)(a^2+1)$

10 $x^2(y^3+8)-(y^3+8)=(y^3+8)(x^2-1)$

$=(y+2)(y^2-2y+4)(x-1)(x+1)$

4

1 $x^3(x^2-1)-(x^2-1)=(x^2-1)(x^3-1)$

$=(x-1)(x+1)(x-1)(x^2+x+1)$

$=(x-1)^2(x+1)(x^2+x+1)$

2 $4m^4-(9m^2-6m+1)=4m^4-(3m-1)^2$

$=(2m^2-(3m-1))(2m^2+(3m-1))$

$=(2m^2-3m+1)(2m^2+3m-1)$

$=(2m-1)(m-1)(2m^2+3m-1)$

3 $121x^4-(100x^2+20x+1)$

$=121x^4-(10x+1)^2$

$=(11x^2-10x-1)(11x^2+10x+1)$

$=(11x+1)(x-1)(11x^2+10x+1)$

5

$\because x^3+5x^2+3x+15=(x^3+5x^2)+(3x+15)$

$=x^2(x+5)+3(x+5)$

$=(x+5)(x^2+3)$

, $\because (x+5)$ cm. is the width of the rectangle

$\therefore$ length of the rectangle $=(x^2+3)$ cm.

$\therefore$ perimeter of the rectangle $=2(x+5+x^2+3)$

$=2(x^2+x+8)$ cm.

, when $x=2$

$\therefore$ perimeter of the rectangle $=2(4+2+8)=28$ cm.

6

$$BC=\frac{x(x^2+2)+2(x^2+2)}{x+2}=\frac{(x^2+2)(x+2)}{x+2}=(x^2+2)\text{ cm.}$$

7

1 $2x(x^2(x+3)-9x-27)$

$=2x(x^2(x+3)-9(x+3))$

$=2x(x+3)(x^2-9)=2x(x+3)(x-3)(x+3)$

$=2x(x-3)(x+3)^2$

2 $a^2+4ab+4b^2-9=(a+2b)^2-9$

$=(a+2b-3)(a+2b+3)$

3 $a^3-4b^3+a^2b-4ab^2=a^2(a+b)-4b^2(a+b)$

$=(a+b)(a^2-4b^2)=(a+b)(a-2b)(a+2b)$

4 $a^2(b-5)-7a(b-5)-18(b-5)$

$=(b-5)(a^2-7a-18)=(b-5)(a-9)(a+2)$

8

1 $(x-2y)^2+(x-2y)=(x-2y)(x-2y+1)$

2 $3(x^2-5x-24)-y(x-8)$

$=3(x-8)(x+3)-y(x-8)$

$=(x-8)(3(x+3)-y)=(x-8)(3x+9-y)$

3 $a^3-1+a-1=(a-1)(a^2+a+1)+(a-1)$

$=(a-1)(a^2+a+1+1)=(a-1)(a^2+a+2)$

4 a^3+8+a^2-4

$=(a+2)(a^2-2a+4)+(a-2)(a+2)$

$=(a+2)(a^2-2a+4+a-2)=(a+2)(a^2-a+2)$

Answers of Exercise 7

1

1 $x^4+4+4x^2-4x^2=(x^4+4x^2+4)-4x^2$

$=(x^2+2)^2-4x^2=(x^2+2-2x)(x^2+2+2x)$

$=(x^2-2x+2)(x^2+2x+2)$

2 $x^4+16x^2+64-16x^2$

$=(x^2+8)^2-16x^2=(x^2+8-4x)(x^2+8+4x)$

$=(x^2-4x+8)(x^2+4x+8)$

3 $x^4+4x^2y^2+4y^4-4x^2y^2=(x^2+2y^2)^2-4x^2y^2$

$=(x^2+2y^2-2xy)(x^2+2y^2+2xy)$

$=(x^2-2xy+2y^2)(x^2+2xy+2y^2)$

4 $x^4+16x^2y^2+64y^4-16x^2y^2$

$=(x^2+8y^2)^2-16x^2y^2$

$=(x^2+8y^2-4xy)(x^2+8y^2+4xy)$

$=(x^2-4xy+8y^2)(x^2+4xy+8y^2)$

5 $a^4+100a^2b^2+2500b^4-100a^2b^2$

$=(a^2+50b^2)^2-100a^2b^2$

$=(a^2+50b^2-10ab)(a^2+50b^2+10ab)$

$=(a^2-10ab+50b^2)(a^2+10ab+50b^2)$

6 $81x^4+36x^2z^2+4z^4-36x^2z^2$

$=(9x^2+2z^2)^2-36x^2z^2$

$=(9x^2+2z^2-6xz)(9x^2+2z^2+6xz)$

$=(9x^2-6xz+2z^2)(9x^2+6xz+2z^2)$

7 $4x^4+100x^2z^2+625z^4-100x^2z^2$

$=(2x^2+25z^2)^2-100x^2z^2$

$=(2x^2+25z^2-10xz)(2x^2+25z^2+10xz)$

$=(2x^2-10xz+25z^2)(2x^2+10xz+25z^2)$

8 $64x^4 + 144x^2y^2 + 81y^4 - 144x^2y^2$
$= (8x^2 + 9y^2)^2 - 144x^2y^2$
$= (8x^2 + 9y^2 - 12xy)(8x^2 + 9y^2 + 12xy)$
$= (8x^2 - 12xy + 9y^2)(8x^2 + 12xy + 9y^2)$

9 $3(4x^4 + y^4) = 3(4x^4 + 4x^2y^2 + y^4 - 4x^2y^2)$
$= 3((2x^2 + y^2)^2 - 4x^2y^2)$
$= 3(2x^2 + y^2 - 2xy)(2x^2 + y^2 + 2xy)$
$= 3(2x^2 - 2xy + y^2)(2x^2 + 2xy + y^2)$

10 $2y^2(4x^4 + 81z^4)$
$= 2y^2(4x^4 + 36x^2z^2 + 81z^4 - 36x^2z^2)$
$= 2y^2((2x^2 + 9z^2)^2 - 36x^2z^2)$
$= 2y^2(2x^2 + 9z^2 - 6xz)(2x^2 + 9z^2 + 6xz)$
$= 2y^2(2x^2 - 6xz + 9z^2)(2x^2 + 6xy + 9z^2)$

2

1 $9x^4 + 6x^2 + 1 + 2x^2 - 6x^2$
$= (3x^2 + 1)^2 - 4x^2 = (3x^2 + 1 - 2x)(3x^2 + 1 + 2x)$
$= (3x^2 - 2x + 1)(3x^2 + 2x + 1)$

2 $x^4 + 8x^2 + 16 - 28x^2 - 8x^2$
$= (x^2 + 4)^2 - 36x^2 = (x^2 + 4 - 6x)(x^2 + 4 + 6x)$
$= (x^2 - 6x + 4)(x^2 + 6x + 4)$

3 $x^4 + 18x^2 + 81 + 9x^2 - 18x^2$
$= (x^2 + 9)^2 - 9x^2 = (x^2 + 9 - 3x)(x^2 + 9 + 3x)$
$= (x^2 - 3x + 9)(x^2 + 3x + 9)$

4 $9x^4 - 24x^2 + 16 - 25x^2 + 24x^2$
$= (3x^2 - 4)^2 - x^2 = (3x^2 - 4 - x)(3x^2 - 4 + x)$
$= (3x^2 - x - 4)(3x^2 + x - 4)$
$= (3x - 4)(x + 1)(3x + 4)(x - 1)$

5 $x^4 + 4x^2y^2 + 4y^4 + 3x^2y^2 - 4x^2y^2$
$= (x^2 + 2y^2)^2 - x^2y^2$
$= (x^2 + 2y^2 - xy)(x^2 + 2y^2 + xy)$
$= (x^2 - xy + 2y^2)(x^2 + xy + 2y^2)$

6 $m^4 - 2m^2n^2 + n^4 - 11m^2n^2 + 2m^2n^2$
$= (m^2 - n^2)^2 - 9m^2n^2$
$= (m^2 - n^2 - 3mn)(m^2 - n^2 + 3mn)$
$= (m^2 - 3mn - n^2)(m^2 + 3mn - n^2)$

7 $x^4 + 10x^2y^2 + 25y^4 + x^2y^2 - 10x^2y^2$
$= (x^2 + 5y^2)^2 - 9x^2y^2$
$= (x^2 + 5y^2 - 3xy)(x^2 + 5y^2 + 3xy)$
$= (x^2 - 3xy + 5y^2)(x^2 + 3xy + 5y^2)$

8 $a^4 + 8a^2b^2 + 16b^4 + 4a^2b^2 - 8a^2b^2$
$= (a^2 + 4b^2)^2 - 4a^2b^2$
$= (a^2 + 4b^2 - 2ab)(a^2 + 4b^2 + 2ab)$
$= (a^2 - 2ab + 4b^2)(a^2 + 2ab + 4b^2)$

9 $x^4 + 2x^2y^2 + y^4 - 7x^2y^2 - 2x^2y^2$
$= (x^2 + y^2)^2 - 9x^2y^2$
$= (x^2 + y^2 - 3xy)(x^2 + y^2 + 3xy)$
$= (x^2 - 3xy + y^2)(x^2 + 3xy + y^2)$

10 $16x^4 - 24x^2y^2 + 9y^4 - 28x^2y^2 + 24x^2y^2$
$= (4x^2 - 3y^2)^2 - 4x^2y^2$
$= (4x^2 - 3y^2 - 2xy)(4x^2 - 3y^2 + 2xy)$
$= (4x^2 - 2xy - 3y^2)(4x^2 + 2xy - 3y^2)$

11 $4x^4 - 20x^2y^2 + 25y^4 - 29x^2y^2 + 20x^2y^2$
$= (2x^2 - 5y^2)^2 - 9x^2y^2$
$= (2x^2 - 5y^2 - 3xy)(2x^2 - 5y^2 + 3xy)$
$= (2x^2 - 3xy - 5y^2)(2x^2 + 3xy - 5y^2)$
$= (2x - 5y)(x + y)(2x + 5y)(x - y)$

12 $3(m^4 - 2m^2n^2 + n^4 - 16m^2n^2)$
$= 3((m^2 - n^2)^2 - 16m^2n^2)$
$= 3(m^2 - n^2 - 4mn)(m^2 - n^2 + 4mn)$
$= 3(m^2 - 4mn - n^2)(m^2 + 4mn - n^2)$

13 $2(25x^4 + 9y^4 - 34x^2y^2)$
$= 2(25x^4 - 30x^2y^2 + 9y^4 - 34x^2y^2 + 30x^2y^2)$
$= 2((5x^2 - 3y^2)^2 - 4x^2y^2)$
$= 2(5x^2 - 3y^2 - 2xy)(5x^2 - 3y^2 + 2xy)$
$= 2(5x^2 - 2xy - 3y^2)(5x^2 + 2xy - 3y^2)$
$= 2(5x + 3y)(x - y)(5x - 3y)(x + y)$

14 $2a(9b^4 - 57b^2c^2 + 64c^4)$
$= 2a(9b^4 - 48b^2c^2 + 64c^4 - 57b^2c^2 + 48b^2c^2)$
$= 2a((3b^2 - 8c^2)^2 - 9b^2c^2)$
$= 2a(3b^2 - 8c^2 - 3bc)(3b^2 - 8c^2 + 3bc)$
$= 2a(3b^2 - 3bc - 8c^2)(3b^2 + 3bc - 8c^2)$

3

1 $9x^4 - 10x^2y^2 + y^4$
$= 9x^4 - 6x^2y^2 + y^4 - 10x^2y^2 + 6x^2y^2$
$= (3x^2 - y^2)^2 - 4x^2y^2$
$= (3x^2 - y^2 - 2xy)(3x^2 - y^2 + 2xy)$
$= (3x^2 - 2xy - y^2)(3x^2 + 2xy - y^2)$
$= (3x + y)(x - y)(3x - y)(x + y)$

2 $x^4 - 19\,x^2y^2 + 25\,y^4$

$= x^4 - 10\,x^2y^2 + 25\,y^4 - 19\,x^2y^2 + 10\,x^2y^2$

$= (x^2 - 5\,y^2)^2 - 9\,x^2y^2$

$= (x^2 - 5\,y^2 - 3\,x\,y)\,(x^2 - 5\,y^2 + 3\,x\,y)$

$= (x^2 - 3\,x\,y - 5\,y^2)\,(x^2 + 3\,x\,y - 5\,y^2)$

3 $16\,x^4 - 28\,x^2y^2 + y^4$

$= 16\,x^4 + 8\,x^2y^2 + y^4 - 28\,x^2y^2 - 8\,x^2y^2$

$= (4\,x^2 + y^2)^2 - 36\,x^2y^2$

$= (4\,x^2 + y^2 - 6\,x\,y)\,(4\,x^2 + y^2 + 6\,x\,y)$

$= (4\,x^2 - 6\,x\,y + y^2)\,(4\,x^2 + 6\,x\,y + y^2)$

4 $4\,a^4 - 24\,a^2b^2 + 9\,b^4$

$= 4\,a^4 + 12\,a^2b^2 + 9\,b^4 - 24\,a^2b^2 - 12\,a^2b^2$

$= (2\,a^2 + 3\,b^2)^2 - 36\,a^2b^2$

$= \big(2\,a^2 + 3\,b^2 + 6\,a\,b\big)\big(2\,a^2 + 3\,b^2 + 6\,a\,b\big)$

$= (2\,a^2 - 6\,a\,b + 3\,b^2)\,(2\,a^2 + 6\,a\,b + 3\,b^2)$

4

1 $(x^4 - 4\,y^4)\,(x^4 + 4\,y^4)$

$= (x^2 - 2\,y^2)\,(x^2 + 2\,y^2)\,(x^4 + 4\,y^4)$

$= (x^2 - 2\,y^2)\,(x^2 + 2\,y^2)\,(x^4 + 4\,x^2y^2 + 4\,y^4 - 4\,x^2y^2)$

$= (x^2 - 2\,y^2)\,(x^2 + 2\,y^2)\,\big((x^2 + 2\,y^2)^2 - 4\,x^2y^2\big)$

$= (x^2 - 2\,y^2)\,(x^2 + 2\,y^2)\,(x^2 + 2\,y^2 - 2\,x\,y)$

$\times (x^2 + 2\,y^2 + 2\,x\,y)$

$= (x^2 - 2\,y^2)\,(x^2 + 2\,y^2)\,(x^2 - 2\,x\,y + 2\,y^2)$

$\times (x^2 + 2\,x\,y + 2\,y^2)$

2 $(x^4 - 25)\,(x^4 + 4)$

$= (x^2 - 5)\,(x^2 + 5)\,(x^4 + 4\,x^2 + 4 - 4\,x^2)$

$= (x^2 - 5)\,(x^2 + 5)\,\big((x^2 + 2)^2 - 4\,x^2\big)$

$= (x^2 - 5)\,(x^2 + 5)\,(x^2 + 2 - 2\,x)\,(x^2 + 2 + 2\,x)$

$= (x^2 - 5)\,(x^2 + 5)\,(x^2 - 2\,x + 2)\,(x^2 + 2\,x + 2)$

3 $(x^4 - 9\,y^4)\,(x^4 + 4\,y^4)$

$= (x^2 - 3\,y^2)\,(x^2 + 3\,y^2)$

$\times (x^4 + 4\,x^2y^2 + 4\,y^4 - 4\,x^2y^2)$

$= \big(x^2 - 3\,y^2\big)\,\big(x^2 + 3\,y^2\big)\,\big((x^2 + 2\,y^2)^2 - 4\,x^2y^2\big)$

$= \big(x^2 - 3\,y^2\big)\,\big(x^2 + 3\,y^2\big)$

$\times (x^2 + 2\,y^2 - 2\,x\,y)\,(x^2 + 2\,y^2 + 2\,x\,y)$

$= (x^2 - 3\,y^2)\,(x^3 + 3\,y^2)\,(x^2 - 2\,x\,y + 2\,y^2)$

$\times (x^2 + 2\,x\,y + 2\,y^2)$

4 $(81\,x^4 + 64\,y^4)\,(x^4 - y^4)$

$= (81\,x^4 + 144\,x^2y^2 + 64\,y^4 - 144\,x^2y^2)$

$\times (x^2 - y^2)\,(x^2 + y^2)$

$= \big((9\,x^2 + 8\,y^2)^2 - 144\,x^2y^2\big)\,(x - y)\,(x + y)\,(x^2 + y^2)$

$= (9\,x^2 + 8\,y^2 - 12\,x\,y)\,(9\,x^2 + 8\,y^2 + 12\,x\,y)$

$\times (x - y)\,(x + y)\,(x^2 + y^2)$

$= (9\,x^2 - 12\,x\,y + 8\,y^2)\,(9\,x^2 + 12\,x\,y + 8\,y^2)$

$\times (x - y)\,(x + y)\,(x^2 + y^2)$

Answers of General Exercises on Factorization

1 $(5\,x - 3\,y)\,(5\,x + 3\,y)$

2 $2\,x^2\,(x^3 + 27) = 2\,x^2\,(x + 3)\,(x^2 - 3\,x + 9)$

3 $(2\,y + 3)\,(y + 1)$

4 $2\,(x^4 - 9) = 2\,(x^2 - 3)\,(x^2 + 3)$

5 $2\,(x^2 - 10\,x + 24) = 2\,(x - 6)\,(x - 4)$

6 $(x + 4)^2$

7 $(2\,x + 3)\,(4\,x^2 - 6\,x + 9)$

8 $(y - 51)\,(y + 1)$

9 $(5\,x - 3)^2$

10 $(x - 9)\,(x + 9)$

11 $y\,(y^4 - 1) = y\,(y^2 - 1)\,(y^2 + 1)$

$= y\,(y - 1)\,(y + 1)\,(y^2 + 1)$

12 $(3\,x - 2)\,(x + 3)$

13 $(x - 6)\,(x - 2)$

14 $3\,x\,(x^2 + 4) + 2\,(x^2 + 4) = (x^2 + 4)\,(3x + 2)$

15 $(x - 5)\,(x^2 + 5\,x + 25)$

16 $(2\,x - 3)^2$

17 $a^2\,(a + 3) - 9\,(a + 3)$

$= (a^2 - 9)\,(a + 3) = (a - 3)\,(a + 3)\,(a + 3)$

18 $(x + 2)^3 - 4\,(x + 2) = (x + 2)\,\big((x + 2)^2 - 4\big)$

$= (x + 2)\,(x + 2 - 2)\,(x + 2 + 2) = x\,(x + 2)\,(x + 4)$

19 $-\,(2\,x^2 + 15\,x + 7) = -\,(2\,x + 1)\,(x + 7)$

20 $(x - 5)\,(x - 2)$

21 $4\,x^4 + 4\,x^2y^2 + y^4 - 4\,x^2y^2 = (2\,x^2 + y^2)^2 - 4\,x^2y^2$

$= (2\,x^2 + y^2 - 2\,x\,y)\,(2\,x^2 + y^2 + 2\,x\,y)$

$= (2\,x^2 - 2\,x\,y + y^2)\,(2\,x^2 + 2\,x\,y + y^2)$

22 $(3\,x^2 - 4\,y^2)\,(3\,x^2 + 4\,y^2)$

23 $(x^2 - 4)\,(x^2 - 5) = (x - 2)\,(x + 2)\,(x^2 - 5)$

24 $(1 - 2\,x)\,(1 + 2\,x)$

25 $(a^3 - 25\,b^3)\,(a^3 + 25\,b^3)$

26 $\big((x + y) - x\big)\,\big((x + y)^2 + x\,(x + y) + x^2\big)$

$= y\,(x^2 + 2\,x\,y + y^2 + x^2 + x\,y + x^2)$

$= y\,(3\,x^2 + 3\,x\,y + y^2)$

27 $(7\,x + 5\,y^2)^2$

28 $(5\,x + 2)\,(x - 1)$

29 $x^4 - 2x^2y^2 + y^4 - 11x^2y^2 + 2x^2y^2$
$= (x^2 - y^2)^2 - 9x^2y^2$
$= (x^2 - y^2 - 3xy)(x^2 - y^2 + 3xy)$
$= (x^2 - 3xy - y^2)(x^2 + 3xy - y^2)$

30 $3x^2(x^2 - 5x + 4) = 3x^2(x - 4)(x - 1)$

31 $(3x - 1)(x - 6)$ 32 $(2x + 7y)^2$

33 $(x^3 - 8y^3)(x^3 + 8y^3)$
$= (x - 2y)(x^2 + 2xy + 4y^2)(x + 2y) \times (x^2 - 2xy + 4y^2)$

34 $2y^3(y - 2) + 7(y - 2) = (y - 2)(2y^3 + 7)$

35 $3(5a^4 - 2a^2b - 7b^2) = 3(5a^2 - 7b)(a^2 + b)$

36 $6x^2 - 7xy + 2y^2 = (2x - y)(3x - 2y)$

37 $64x^4 + 16x^2y^2 + y^4 - 16x^2y^2$
$= (8x^2 + y^2)^2 - 16x^2y^2$
$= (8x^2 + y^2 - 4xy)(8x^2 + y^2 + 4xy)$
$= (8x^2 - 4xy + y^2)(8x^2 + 4xy + y^2)$

38 $(x^2 + 3)(x^2 - 8)$

39 $5(4x^4 + 8x^2y^2 + 9y^4)$
$= 5(4x^4 + 12x^2y^2 + 9y^4 + 8x^2y^2 - 12x^2y^2)$
$= 5((2x^2 + 3y^2)^2 - 4x^2y^2)$
$= 5(2x^2 + 3y^2 - 2xy)(2x^2 + 3y^2 + 2xy)$
$= 5(2x^2 - 2xy + 3y^2)(2x^2 + 2xy + 3y^2)$

40 $(9x^2 - 4y^2)(x^2 - y^2)$
$= (3x - 2y)(3x + 2y)(x - y)(x + y)$

Answers of Exercise 8

1

1 $x(x - 6) = 0$ $\therefore x = 0$ or $x - 6 = 0$
$\therefore x = 6$ $\therefore S.S = \{0, 6\}$

2 $(x - 4)(x + 4) = 0$
$\therefore x - 4 = 0$ then $x = 4$ or $x + 4 = 0$ then $x = -4$
$\therefore S.S = \{4, -4\}$

3 $(2x - 5)(2x + 5) = 0$ $\therefore 2x - 5 = 0$
$\therefore x = \frac{5}{2}$ or $2x + 5 = 0$ then $x = \frac{-5}{2}$
$\therefore S.S = \{\frac{5}{2}, \frac{-5}{2}\}$

4 $(x + 3)(x + 2) = 0$
$\therefore x + 3 = 0$ then $x = -3$ or $x + 2 = 0$ then $x = -2$
$\therefore S.S = \{-3, -2\}$

5 $(x - 5)(x - 3) = 0$
$\therefore x - 5 = 0$ then $x = 5$ or $x - 3 = 0$ then $x = 3$
$\therefore S.S = \{5, 3\}$

6 $(x + 9)(x - 2) = 0$
$\therefore x + 9 = 0$ then $x = -9$ or $x - 2 = 0$ $\therefore x = 2$
$\therefore S.S = \{-9, 2\}$

7 $(x - 5)(x + 4) = 0$
$\therefore x - 5 = 0$ then $x = 5$ or $x + 4 = 0$ then $x = -4$
$\therefore S.S = \{5, -4\}$

8 $(3x + 1)(2x - 3) = 0$ $\therefore 3x + 1 = 0$
$\therefore x = -\frac{1}{3}$ or $2x - 3 = 0$ then $x = \frac{3}{2}$
$\therefore S.S = \{-\frac{1}{3}, \frac{3}{2}\}$

9 $(2x - 1)(x + 4) = 0$
$\therefore 2x - 1 = 0$ then $x = \frac{1}{2}$ or $x + 4 = 0$ then $x = -4$
$\therefore S.S = \{\frac{1}{2}, -4\}$

10 $(2x + 1)(x - 3) = 0$ $\therefore 2x + 1 = 0$
$\therefore x = -\frac{1}{2}$ or $x - 3 = 0$ then $x = 3$
$\therefore S.S = \{-\frac{1}{2}, 3\}$

11 $(x + 2)^2 = 0$ then $x + 2 = 0$ $\therefore x = -2$
$\therefore S.S = \{-2\}$

12 $(3x - 1)^2 = 0$ $\therefore 3x - 1 = 0$
$\therefore x = \frac{1}{3}$ $\therefore S.S = \{\frac{1}{3}\}$

2

1 $x^2 - x = 0$ $\therefore x(x - 1) = 0$
$\therefore x = 0$ or $x - 1 = 0$ then $x = 1$
$\therefore S.S = \{0, 1\}$

2 $3x^2 - 7x = 0$ $\therefore x(3x - 7) = 0$
$\therefore x = 0$ or $3x - 7 = 0$ then $x = \frac{7}{3}$
$\therefore S.S = \{0, \frac{7}{3}\}$

3 $4x^2 - 49 = 0$ $\therefore (2x - 7)(2x + 7) = 0$
$\therefore (2x - 7) = 0$ then $x = \frac{7}{2}$ or $(2x + 7) = 0$
then $x = \frac{-7}{2}$ $\therefore S.S = \{\frac{7}{2}, -\frac{7}{2}\}$

4 $x^2 + x - 6 = 0$ $\therefore (x + 3)(x - 2) = 0$
$\therefore x + 3 = 0$ then $x = -3$ or $x - 2 = 0$ then $x = 2$
$\therefore S.S = \{-3, 2\}$

5 $x^2 - 8x + 15 = 0$ $\therefore (x - 3)(x - 5) = 0$
$\therefore x - 3 = 0$ then $x = 3$ or $x - 5 = 0$ then $x = 5$
$\therefore S.S = \{3, 5\}$

6 $x^2 - 2x - 15 = 0$ $\therefore (x - 5)(x + 3) = 0$
$\therefore x - 5 = 0$ then $x = 5$ or $x + 3 = 0$ then $x = -3$
$\therefore S.S = \{5, -3\}$

7 $2x^2-10x+12=0 \quad \therefore x^2-5x+6=0$
$\therefore (x-3)(x-2)=0$
$\therefore x-3=0$ then $x=3$ or $x-2=0$ then $x=2$
$\therefore S.S=\{3, 2\}$

8 $6x^2-x-22=0 \quad \therefore (x-2)(6x+11)=0$
$\therefore x-2=0$ then $x=2$ or $6x+11=0$ then $x=\frac{-11}{6}$
$\therefore S.S=\{2, \frac{-11}{6}\}$

9 $5x^2+12x-44=0 \quad \therefore (5x+22)(x-2)=0$
$\therefore 5x+22=0$ then $x=\frac{-22}{5}$ or $x-2=0$ then $x=2$
$\therefore S.S=\{\frac{-22}{5}, 2\}$

10 $12x^2-47x+45=0 \quad \therefore (4x-9)(3x-5)=0$
$\therefore 4x-9=0$ then $x=\frac{9}{4}$ or $3x-5=0$ then $x=\frac{5}{3}$
$\therefore S.S=\{\frac{9}{4}, \frac{5}{3}\}$

11 $x^2+3=12 \quad \therefore x^2-9=0$
$\therefore (x-3)(x+3)=0$
$\therefore x-3=0$ then $x=3$ or $x+3=0$ then $x=-3$
$\therefore S.S=\{3, -3\}$

12 $x^2-3x-5x=0 \quad \therefore x^2-8x=0$
$\therefore x(x-8)=0$
$\therefore x=0$ or $x-8=0$ then $x=8$
$\therefore S.S=\{0, 8\}$

3

1 $x^2-5x+6=0 \quad \therefore (x-3)(x-2)=0$
$\therefore x-3=0$ then $x=3$ or $x-2=0$ then $x=2$
$\therefore S.S=\{3, 2\}$

2 $x^2+3x-10=0 \quad \therefore (x+5)(x-2)=0$
$\therefore x+5=0$ then $x=-5$ or $x-2=0$ then $x=2$
$\therefore S.S=\{-5, 2\}$

3 $x^2-2x-3-5=0 \quad \therefore x^2-2x-8=0$
$\therefore (x-4)(x+2)=0$
$\therefore x-4=0$ then $x=4$ or $x+2=0$ then $x=-2$
$\therefore S.S=\{4, -2\}$

4 $x^2+5x-24-3x=0 \quad \therefore x^2+2x-24=0$
$\therefore (x+6)(x-4)=0$
$\therefore x+6=0$ then $x=-6$ or $x-4=0$ then $x=4$
$S.S=\{-6, 4\}$

5 $2x^2-10x-20+4x=0$
$\therefore 2x^2-6x-20=0 \quad \therefore x^2-3x-10=0$
$\therefore (x-5)(x+2)=0$
$\therefore x-5=0$ then $x=5$ or $x+2=0$ then $x=-2$
$\therefore S.S=\{5, -2\}$

6 $(x+3-7)(x+3+7)=0$
$\therefore (x-4)(x+10)=0$
$\therefore x-4=0$ then $x=4$ or $x+10=0$ then $x=-10$
$\therefore S.S=\{4, -10\}$

7 $4(x+5)^2-25=0$
$\therefore (2(x+5)-5)(2(x+5)+5)=0$
$\therefore (2x+5)(2x+15)=0$
$\therefore 2x+5=0$ then $x=\frac{-5}{2}$
or $2x+15=0$ then $x=\frac{-15}{2}$
$\therefore S.S=\{\frac{-5}{2}, \frac{-15}{2}\}$

8 $x^2-2x+1+x-3=0 \quad \therefore (x^2-x-2)=0$
$\therefore (x-2)(x+1)=0$
$\therefore x-2=0$ then $x=2$ or $x+1=0$ then $x=-1$
$\therefore S.S=\{2, -1\}$

9 $(x+3)(2(x+3)+7)=0$
$\therefore (x+3)(2x+13)=0$
$\therefore x+3=0$ then $x=-3$
or $2x+13=0$ then $x=\frac{-13}{2}$
$\therefore S.S=\{-3, \frac{-13}{2}\}$

10 $4x^2+4x+1=9x^2-6x+1$
$\therefore 4x^2-9x^2+4x+6x+1-1=0$
$\therefore -5x^2+10x=0 \quad \therefore x^2-2x=0$
$\therefore x(x-2)=0$
$\therefore x=0$ or $x-2=0$ then $x=2$
$\therefore S.S=\{0, 2\}$

11 $4x^2-4x+1+x^2-2x+1-10=0$
$\therefore 5x^2-6x-8=0 \quad \therefore (5x+4)(x-2)=0$
$\therefore 5x+4=0$ then $x=\frac{-4}{5}$ or $x-2=0$ then $x=2$
$\therefore S.S=\{\frac{-4}{5}, 2\}$

12 $x^2+6x+9+3x+9-10=0$
$\therefore x^2+9x+8=0 \quad \therefore (x+1)(x+8)=0$
$\therefore x+1=0$ then $x=-1$ or $x+8=0$ then $x=-8$
$\therefore S.S=\{-1, -8\}$

4

1 $2x(x^2-4)=0 \quad \therefore 2x(x-2)(x+2)=0$
$\therefore 2x=0$ then $x=0$ or $x-2=0$ then $x=2$
or $x+2=0$ then $x=-2$
$\therefore S.S=\{0, 2, -2\}$

2) $4x^3 - 9x = 0$ $\therefore x(4x^2 - 9) = 0$

$\therefore x(2x-3)(2x+3) = 0$

$\therefore x = 0$ or $2x - 3 = 0$ then $x = \frac{3}{2}$

or $2x + 3 = 0$ then $x = \frac{-3}{2}$

$\therefore$ S.S $= \{0, \frac{3}{2}, \frac{-3}{2}\}$

3) $(x^2-4)(x^2-1) = 0$

$\therefore (x-2)(x+2)(x-1)(x+1) = 0$

$\therefore x - 2 = 0$ then $x = 2$ or $x + 2 = 0$ then $x = -2$

or $x - 1 = 0$ then $x = 1$ or $x + 1 = 0$ then $x = -1$

$\therefore$ S.S $= \{2, -2, 1, -1\}$

4) $x^4 - 26x^2 + 25 = 0$

$\therefore (x^2-25)(x^2-1) = 0$

$\therefore (x-5)(x+5)(x-1)(x+1) = 0$

$\therefore x - 5 = 0$ then $x = 5$ or $x + 5 = 0$ then $x = -5$

or $x - 1 = 0$ then $x = 1$ or $x + 1 = 0$ then $x = -1$

S.S $= \{5, -5, 1, -1\}$

5) $(x^2-4)(x^2+4) = 0$

$\therefore (x-2)(x+2)(x^2+4) = 0$

$\therefore x - 2 = 0$ then $x = 2$

or $x + 2 = 0$ then $x = -2$

or $x^2 + 4 = 0$ (has no solution in $\mathbb{R}$)

$\therefore$ S.S. $= \{2, -2\}$

5

1) Multiplying the equation by 3

$\therefore 3y^2 - 7y + 4 = 0$ $\therefore (3y-4)(y-1) = 0$

$\therefore 3y - 4 = 0$ then $y = \frac{4}{3}$ or $y - 1 = 0$ then $y = 1$

$\therefore$ S.S $= \{\frac{4}{3}, 1\}$

2) Multiplying the equation by 2

$\therefore 2x^2 - 2x - 3 - 9 = 0$ $\therefore 2x^2 - 2x - 12 = 0$

$\therefore x^2 - x - 6 = 0$ $\therefore (x-3)(x+2) = 0$

$\therefore x - 3 = 0$ then $x = 3$ or $x + 2 = 0$ then $x = -2$

$\therefore$ S.S $= \{3, -2\}$

3) Multiplying the equation by x

$\therefore x^2 + 2 = 3x$ $\therefore x^2 - 3x + 2 = 0$

$\therefore (x-2)(x-1) = 0$

$\therefore x - 2 = 0$ then $x = 2$ or $x - 1 = 0$ then $x = 1$

$\therefore$ S.S $= \{2, 1\}$

4) Multiplying the equation by $2x$

$\therefore 2x^2 - 10 = x$ $\therefore 2x^2 - x - 10 = 0$

$\therefore (2x-5)(x+2) = 0$

$\therefore 2x - 5 = 0$ then $x = \frac{5}{2}$ or $x + 2 = 0$ then $x = -2$

$\therefore$ S.S $= \{\frac{5}{2}, -2\}$

5) Multiplying the equation by $5x$

$\therefore x(x-1) = 30$ $\therefore x^2 - x - 30 = 0$

$\therefore (x+5)(x-6) = 0$ $\therefore x + 5 = 0$, then $x = -5$

or $x - 6 = 0$, then $x = 6$

$\therefore$ S.S. $= \{-5, 6\}$

6

1) c	2) c	3) d	4) d
5) a	6) d	7) b	8) c

7

1) 3 2) 8, 4 3) 0, 4 4) $\{4, -\frac{1}{2}\}$

8

Multiplying both sides of the equation by x

$\therefore x^2 + 1 = 2x$ $\therefore x^2 - 2x + 1 = 0$

$\therefore (x-1)^2 = 0$ $\therefore x = 1$

$\therefore x^2 + \frac{1}{x^2} = 1^2 + \frac{1}{1^2} = 2$

Another solution :

Squaring the two sides of the equation

$\therefore (x + \frac{1}{x})^2 = 4$ $\therefore x^2 + 2 + \frac{1}{x^2} = 4$

$\therefore x^2 + \frac{1}{x^2} = 4 - 2 = 2$

9

$\because x^2 + \frac{1}{x^2} = 34$,

Adding 2 to the both sides

$\therefore x^2 + 2 + \frac{1}{x^2} = 34 + 2$ $\therefore (x + \frac{1}{x})^2 = 36$

Taking the square root to both sides

$\therefore x + \frac{1}{x} = \pm\sqrt{36} = \pm 6$

Another solution :

$(x + \frac{1}{x})^2 = x^2 + 2 + \frac{1}{x^2} = x^2 + \frac{1}{x^2} + 2$

$= 34 + 2 = 36$

Taking the square root to both sides

$\therefore x + \frac{1}{x} = \pm\sqrt{36} = \pm 6$

10

Multiply the equation by 12

$\therefore 2x(x-2)-3x(x+1)+28(x-3)-24=0$

$\therefore 2x^2-4x-3x^2-3x+28x-84-24=0$

$\therefore -x^2+21x-108=0$ $\quad \therefore x^2-21x+108=0$

$\therefore (x-9)(x-12)=0$ $\quad \therefore x-9=0$ then $x=9$

or $x-12=0$ then $x=12$ $\quad$ S.S $=\{9, 12\}$

11

$\because$ 2 , 3 are the two roots of the equation

$\therefore$ the equation is $(x-2)(x-3)=0$

i.e. $x^2-5x+6=0$ $\quad \therefore b=5, c=6$

Answers of Exercise 9

1

1 c	2 c	3 b	4 c
5 d	6 d	7 c	

2

Let the number be x $\quad \therefore x^2-5x=36$

$\therefore x^2-5x-36=0$ $\quad (x+4)(x-9)=0$

$\therefore x+4=0$, then $x=-4$ (refused)

or $x-9=0$, then $x=9$ $\quad \therefore$ The number is 9

3

Let the number be x

$\therefore 2x^2+7=135$ $\quad \therefore 2x^2-128=0$

$\therefore x^2-64=0$ $\quad \therefore (x-8)(x+8)=0$

$\therefore x-8=0$, then $x=8$ or $x+8=0$, then $x=-8$

$\therefore$ The number is 8 or -8

4

Let the number be x $\quad \therefore 4x^2=81$

$\therefore 4x^2-81=0$ $\quad \therefore (2x-9)(2x+9)=0$

$\therefore 2x-9=0$, then $x=\frac{9}{2}$,

or $2x+9=0$, then $x=\frac{-9}{2}$

$\therefore$ The number is $\frac{9}{2}$ or $-\frac{9}{2}$

5

Let the number be x $\quad \therefore x^2=6x$

$\therefore x^2-6x=0$ $\quad \therefore x(x-6)=0$

$\therefore x=0$ (refused) or $x-6=0$, then $x=6$

$\therefore$ The number is 6

6

Let the number be x

$\therefore x+x^2=12$ $\quad \therefore x^2+x-12=0$

$\therefore (x+4)(x-3)=0$

$\because x+4=0$, then $x=-4$ or $x-3=0$, then $x=3$

$\therefore$ The number is -4 or 3

7

Let the number be x $\quad \therefore x^2-2x=48$

$\therefore x^2-2x-48=0$ $\quad \therefore (x+6)(x-8)=0$

$\therefore x+6=0$, then $x=-6$ (refused)

or $x-8=0$, then $x=8$

$\therefore$ The number is 8

8

Let the first number be x

$\therefore$ The second number $=20-x$

$x(20-x)=75$ $\quad \therefore 20x-x^2-75=0$

$\therefore x^2-20x+75=0$ $\quad \therefore (x-15)(x-5)=0$

$\therefore x-15=0$, then $x=15$

or $x-5=0$, then $x=5$

$\therefore$ The numbers are 5 and 15

9

Let the first number be x

$\therefore$ The second number $=x+5$

$\therefore x^2+(x+5)^2=73$

$\therefore x^2+x^2+10x+25-73=0$

$\therefore 2x^2+10x-48=0$ $\quad \therefore x^2+5x-24=0$

$(x+8)(x-3)=0$

$\therefore x+8=0$, then $x=-8$

i.e. The two numbers are -8 and -3

or $x-3=0$ then $x=3$

i.e. The two numbers are 3 and 8

10

Let the first number be x

$\therefore$ The second number $=x+4$

$\therefore x(x+4)=45$

$\therefore x^2+4x-45=0$

$\therefore (x+9)(x-5)=0$

$\therefore x+9=0$, then $x=-9$

i.e. The two numbers are -9 and -5

or $x-5=0$, then $x=5$

i.e. The two numbers are 5 and 9

11

Let the first number be x

$\therefore$ The second number $= x+2$

$\therefore x^2+(x+2)^2=130$

$\therefore x^2+x^2+4x+4-130=0$

$\therefore 2x^2+4x-126=0$

$\therefore x^2+2x-63=0$ $\therefore (x+9)(x-7)=0$

$\therefore x+9=0$, then $x=-9$

i.e. The two numbers are -9 and -7

or $x-7=0$, then $x=7$

i.e. The two numbers are 7 and 9

12

Let the numbers be x , $x+1$, $x+2$

$\therefore x+x+1+x+2=(x+1)^2$

$\therefore 3x+3=x^2+2x+1$ $\therefore x^2-x-2=0$

$\therefore (x-2)(x+1)=0$ $\therefore x-2=0$, then $x=2$

i.e. The three numbers are 2 , 3 and 4

or $x+1=0$, then $x=-1$

i.e. The three numbers are -1 , 0 and 1

13

Let the first number be $7x$

$\therefore$ The second number is $8x$

$\therefore (7x)\times(8x)-9(8x)=80$

$\therefore 56x^2-72x-80=0$

$\therefore 7x^2-9x-10=0$ $\therefore (7x+5)(x-2)=0$

$\therefore 7x+5=0$, then $x=-\frac{5}{7}$ (refused)

or $x-2=0$, then $x=2$

i.e. The two numbers are 14 and 16

14

Let the first number be x

$\therefore 2x^2+(-x)=91$ $\therefore 2x^2-x-91=0$

$\therefore (2x+13)(x-7)=0$ $\therefore 2x+13=0$

, then $x=\frac{-13}{2}$ (refused) or $x-7=0$, then $x=7$

$\therefore$ The number is 7

15

Let the number be x

$\therefore x-\frac{1}{x}=\frac{5}{6}$ multiplying by $6x$

$\therefore 6x^2-6=5x$

$\therefore 6x^2-5x-6=0$ $\therefore (2x-3)(3x+2)=0$

$\therefore 2x-3=0$, then $x=\frac{3}{2}$ or $(3x+2)=0$

, then $x=\frac{-2}{3}$

$\therefore$ The number is $\frac{3}{2}$ or $\frac{-2}{3}$

16

Let the tens digit be x

$\therefore$ the units digit is $2x$ $\therefore x(2x)-(x+2x)=9$

$\therefore 2x^2-3x-9=0$ $\therefore (2x+3)(x-3)=0$

$\therefore 2x+3=0$, then $x=\frac{-3}{2}$ (refused)

or $x-3=0$, then $x=3$ $\therefore$ The number is 36

17

Let the age of Said now be x years

$\therefore x^2-3(x-4)=192$ $\therefore x^2-3x+12-192=0$

$\therefore x^2-3x-180=0$ $\therefore (x-15)(x+12)=0$

$\therefore x-15=0$, then $x=15$

or $x+12=0$, then $x=-12$ (refused)

$\therefore$ The age of Said now is 15 years.

18

Let the age of Hatem now be x years

$\therefore$ The age of Hanan now $=(x-4)$ years

$\therefore x^2+(x-4)^2=26$

$\therefore x^2+x^2-8x+16-26=0$

$\therefore 2x^2-8x-10=0$ $\therefore x^2-4x-5=0$

$\therefore (x-5)(x+1)=0$ $\therefore x-5=0$, then $x=5$

or $x+1=0$, then $x=-1$ (refused)

$\therefore$ The age of Hatem = 5 years and

The age of Hanan = one year.

19

Let the age of Anees now be x years

$\therefore$ the age of Kamal now $=(x+3)$ years

since 4 years

The age of Anees = $(x-4)$ years

The age of Kamal = $(x+3-4) = (x-1)$ years.

$\therefore (x-4)(x-1) = 18 \quad \therefore x^2 - 5x + 4 - 18 = 0$

$\therefore x^2 - 5x - 14 = 0 \quad \therefore (x+2)(x-7) = 0$

$\therefore x + 2 = 0$, then $x = -2$ (refused)

or $x - 7 = 0$, then $x = 7$

$\therefore$ The age of Anees now is 7 years.

The age of Kamal now is 10 years.

20

Let the width of the rectangle be x cm.

$\therefore$ The length of the rectangle is $(x+4)$ cm.

$\therefore x(x+4) = 21 \quad \therefore x^2 + 4x - 21 = 0$

$\therefore (x+7)(x-3) = 0$

$\therefore x + 7 = 0$, then $x = -7$ (refused)

or $x - 3 = 0$, then $x = 3$

$\therefore$ The width = 3 cm. and the length = 7 cm.

21

Let the width of the rectangle be x cm.

The length of the rectangle = $(x + 7.5)$ cm.

$\therefore x(x+7.5) = 46 \quad \therefore x^2 + 7.5x - 46 = 0$

$\therefore 2x^2 + 15x - 92 = 0 \quad \therefore (2x+23)(x-4) = 0$

$\therefore 2x + 23 = 0$, then $x = -\frac{23}{2}$ (refused)

or $x - 4 = 0$, then $x = 4$

$\therefore$ The width = 4 cm. The length = 11.5 cm.

The perimeter = 2 (4 + 11.5) = 31 cm.

22

Let the width of the rectangle be x cm.

$\therefore$ The length of the rectangle is $(x+5)$ cm.

The area of the rectangle = $x(x+5)$ cm^2.

The side length of the square = $(3x)$ cm.

The area of the square = $9x^2$ cm^2.

$9x^2 - x(x+5) = 57$

$\therefore 9x^2 - x^2 - 5x - 57 = 0$

$\therefore 8x^2 - 5x - 57 = 0$

$(x-3)(8x+19) = 0 \quad \therefore x - 3 = 0$

, then $x = 3$ or $8x + 19 = 0$

, then $x = -\frac{19}{8}$ (refused)

$\therefore$ The width of the rectangle = 3 cm.

and the length = 8 cm.

and the side length of the square = 9 cm.

23

m (∠ BCD) + m (∠ ACD) = 180°

$\therefore x^2 + 8x = 180° \quad \therefore x^2 + 8x - 180° = 0$

$\therefore (x - 10°)(x + 18°) = 0 \quad \therefore x - 10° = 0$, then $x = 10°$

or $x + 18° = 0$, then $x = -18°$ (refused)

24

$\because$ m (∠ A) + m (∠ B) + m (∠ C) = 180°

$\therefore x^2 + 61° + 110° - 11x + 90° - 7x = 180°$

$\therefore x^2 - 18x + 261° - 180° = 0$

$\therefore x^2 - 18x + 81° = 0 \quad \therefore (x - 9°)^2 = 0 \quad \therefore x = 9°$

$\therefore$ m (∠ A) = 142°, m (∠ B) = 11° and m (∠ C) = 27°

25

Let the length of one side of the right angle be x cm.

$\therefore$ The length of the other side = $(x-2)$ cm.

$\frac{1}{2}x(x-2) = 24$ multiplying by 2

$\therefore x(x-2) = 48 \quad \therefore x^2 - 2x - 48 = 0$

$(x-8)(x+6) = 0 \quad \therefore x - 8 = 0$, then $x = 8$

or $x + 6 = 0$, then $x = -6$ (refused)

$\therefore$ The two lengths of the sides of the right angle are 8 cm. and 6 cm.

26

$\because$ The area of the triangle = 24 cm^2.

$\therefore \frac{1}{2}(5x+3)(x+5) = 24$

$\therefore (5x+3)(x+5) = 48$

$\therefore 5x^2 + 28x + 15 - 48 = 0$

$5x^2 + 28x - 33 = 0 \quad \therefore (5x+33)(x-1) = 0$

$\therefore 5x + 33 = 0$, then $x = -\frac{33}{5}$ (refused)

or $x - 1 = 0$, then $x = 1$

$\therefore$ The lengths of the two sides of the right angle are 8 and 6 cm.

$\therefore$ The length of the hypotenuse = 10 cm.

The perimeter of the triangle = 8 + 6 + 10 = 24 cm.

27

$\because$ The triangle is right angled

$\therefore$ The hypotenuse = $(2x+1)$

$\therefore (2x)^2 + (x-11)^2 = (2x+1)^2$

$\therefore 4x^2 + x^2 - 22x + 121 = 4x^2 + 4x + 1$

$\therefore x^2 - 26x + 120 = 0 \quad \therefore (x-20)(x-6) = 0$

$\therefore x - 20 = 0$, then $x = 20$

or $x - 6 = 0 \quad \therefore x = 6$ (refused)

∴ The lengths of the sides of the triangle are 40 cm. , 41 cm. and 9 cm.

∴ The perimeter of the triangle = 40 + 41 + 9 = 90 cm.

The area of the triangle = $\frac{1}{2} \times 40 \times 9 = 180$ cm².

28

Let the width of the rectangle be x

∴ The length of the rectangle = $2x$

∴ The area = $2x^2$

∴ $(2x+1)(x-1) = 2x^2 - 7$

∴ $2x^2 - x - 1 = 2x^2 - 7$

∴ $-x + 6 = 0$ ∴ $x = 6$

∴ The width of the rectangle = 6 cm.

The length of the rectangle = 12 cm.

29

∵ Δ MCD ~ Δ MAB

∴ $\frac{MC}{MA} = \frac{CD}{AB} = \frac{MD}{MB}$ ∴ $\frac{3}{MA} = \frac{MD}{4}$

, ∵ MD = AD − MA = 7 − MA

∴ $\frac{3}{MA} = \frac{7 - MA}{4}$ ∴ $12 = 7\,MA - (MA)^2$

∴ $(MA)^2 - 7\,MA + 12 = 0$ ∴ $(MA - 4)(MA - 3) = 0$

∴ MA − 4 = 0 , then MA = 4

or MA − 3 = 0 , then MA = 3

(refused because MA > MC)

30

∵ $x \times 10 + x(13 - x) = 60$

∴ $10x + 13x - x^2 - 60 = 0$

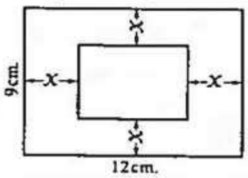
∴ $x^2 - 23x + 60 = 0$ $(x-3)(x-20) = 0$

∴ $x - 3 = 0$, then $x = 3$

or $x - 20 = 0$, then $x = 20$ (refused)

(where $x < 13$)

31



Let the width of the tape be x metre as shown in the figure

∴ the width of the carbet = $(9 - 2x)$ metre.

The length of the carbet = $(12 - 2x)$ metre.

The area of the carbet = $(12 - 2x)(9 - 2x)$

= $\frac{1}{2}$ the area of the room

∴ $(12 - 2x)(9 - 2x) = \frac{1}{2} \times 9 \times 12$

∴ $108 - 42x + 4x^2 = 54$ ∴ $4x^2 - 42x + 54 = 0$

∴ $2x^2 - 21x + 27 = 0$ ∴ $(2x - 3)(x - 9) = 0$

∴ $2x - 3 = 0$, then $x = \frac{3}{2}$ or $x - 9 = 0$, then $x = 9$ (refused) because it equals the width of the room.

∴ The width of the tape = 1.5 metre.

Answers of exams on the second part of unit one

Model 1

1
1 b 2 d 3 c
4 d 5 a 6 b

2
1 15
2 k = 8 , the other root = 4
3 $-3x$
4 $(x^2 - 5x + 25)$
5 {10 , − 10}

3
1 $(y-1)(y-3)(y+3)$
2 $(x^2 + 8y^2 - 4xy)(x^2 + 8y^2 + 4xy)$
3 $(2x - y - 4)(2x - y + 4)$
4 $(m + 3n)(m^2 - 3mn + 9n^2)$

4
1 The S.S. = {5 , 3}
2 The S.S. = $\{0 , \frac{3}{2} , \frac{-3}{2}\}$
3 The S.S. = {−2 , 4}
4 The S.S. = $\{2 , \frac{-22}{5}\}$

5
[a] The number is 3 or − 4
[b] The two dimensions are : 3 cm. , 7 cm.

Model 2

1
1 d 2 b 3 b
4 d 5 d 6 c

2
1 $(x - 3)$
2 $\{0 , \frac{1}{3}\}$
3 $x^3 + 125$
4 3
5 − 3

3
1 $(9a^2 + 2b^2 - 6ab)(9a^2 + 2b^2 + 6ab)$
2 $(x - 2y)(5 - a)$
3 $2(x - 3)(x^2 + 3x + 9)$
4 $(x^2 + 4y^2 - 2xy)(x^2 + 4y^2 + 2xy)$

4
1 The S.S. = $\{-\frac{1}{3} , -2\}$
2 The S.S. = {3 , − 3 , 1 , − 1}
3 The S.S. = {0 , 1}
4 The S.S. = {− 5 , 3}

5
[a] The number is : − 6 or 7
[b] The number is : 5

Answers of unit two

Answers of Exercise 10

1

1) $\frac{1}{3^2} = \frac{1}{9}$ 2) 4 3) $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$

4) $5^2 = 25$ 5) $\left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$

6) $\left(-\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$ 7) $\left(\frac{1}{\sqrt[3]{5}}\right)^3 = \frac{1}{5}$

8) $\left(\sqrt{5}\right)^2 = 5$ 9) $\left(\frac{1}{100}\right)^{-2} = (100)^2 = 10000$

10) $\left(\frac{2}{10}\right)^{-2} = \left(\frac{1}{5}\right)^{-2} = 5^2 = 25$

11) $\left(\frac{1}{\sqrt{2}}\right)^3 = \frac{1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$

12) $\left(\frac{3}{\sqrt{3}}\right)^5 = \left(\frac{\sqrt{3}\times\sqrt{3}}{\sqrt{3}}\right)^5 = \left(\sqrt{3}\right)^5 = 9\sqrt{3}$

2

1) $x^{\text{zero}} = 1$ 2) $x^{-4+3} = x^{-1} = \frac{1}{x}$

3) $x^{-6} \times x^{6} = x^{\text{zero}} = 1$ 4) $x^{2-3+4-1} = x^2$

5) $x^{-6-2+3+4} = x^{-1} = \frac{1}{x}$

3

1) $\left(\sqrt{2}\right)^6 = 2^3 = 8$ 2) $\left(\sqrt{7}\right)^{5-2-1} = \left(\sqrt{7}\right)^2 = 7$

3) $\left(\sqrt{2}\right)^4 \times \left(\sqrt{2}\right)^2 \times \left(\sqrt{2}\right)^{-2} = \left(\sqrt{2}\right)^{4+2-2} = \left(\sqrt{2}\right)^4 = 4$

4) $-\left(\sqrt{3}\right) \times \left(\sqrt{3}\right)^3 \times \left(\sqrt{3}\right)^4 = -\left(\sqrt{3}\right)^8 = -3^4 = -81$

5) $\left(\sqrt{5}\right)^{-4+6} = \left(\sqrt{5}\right)^2 = 5$

6) $\left(-\sqrt{5}\right)^4 = 5^2 = 25$ 7) $\left(\frac{1}{\sqrt{2}}\right)^6 = \frac{1}{\left(\sqrt{2}\right)^6} = \frac{1}{2^3} = \frac{1}{8}$

8) $\left(\sqrt{2}\right)^6 \times \left(-\sqrt{2}\right)^4 = 2^3 \times 2^2 = 2^5 = 32$

9) $\left(\frac{1}{\sqrt{3}}\right)^4 \times \left(\sqrt{2}\right)^4 = \frac{1}{9} \times 4 = \frac{4}{9}$

10) $(-5)^6 \times \left(-\frac{1}{\sqrt{5}}\right)^4 = 5^6 \times \left(\frac{1}{\sqrt{5}}\right)^4 = 5^6 \times \frac{1}{5^2} = 5^{6-2} = 5^4 = 625$

4

1) $\left(\sqrt{7}\right)^{-4-3+9} = \left(\sqrt{7}\right)^2 = 7$

2) $\left(\sqrt{3}\right)^{7+8-6} = \left(\sqrt{3}\right)^9 = 81\sqrt{3}$

3) $\left(\sqrt{3}\right)^{8+6-12} = \left(\sqrt{3}\right)^2 = 3$

4) $-\left(\sqrt{5}\right)^{10+5-11} = -\left(\sqrt{5}\right)^4 = -5^2 = -25$

5) $\frac{2^{-2} \times \left(\sqrt{7}\right)^2 \times 2^{-2}}{\left(\sqrt{7}\right)^2} = 2^{-4} = \frac{1}{16}$

6) $\frac{3^4 \times \left(\sqrt{2}\right)^4 \times \left(\sqrt{2}\right)^2}{2^2 \times \left(\sqrt{3}\right)^2} = \frac{3^4 \times \left(\sqrt{2}\right)^6}{2^2 \times 3} = \frac{3^4 \times 2^3}{2^2 \times 3} = 3^{4-1} \times 2^{3-2} = 3^3 \times 2 = 54$

7) $\frac{\left(\sqrt{3}\right)^{-4} \times \left(\sqrt{2}\right)^3 \times 3^5 \times \left(\sqrt{3}\right)^5}{3^5 \times \left(\sqrt{2}\right)^5 \times \sqrt{3}} = \left(\sqrt{3}\right)^{-4+5-1} \times \left(\sqrt{2}\right)^{3-5} = \left(\sqrt{3}\right)^{\text{zero}} \times \left(\sqrt{2}\right)^{-2} = \frac{1}{2}$

8) $\frac{\left(\sqrt{3}\right)^5 \times \left(\sqrt{3}\right)^4}{\left(\sqrt{3}\right)^3 \times \left(\sqrt{3}\right)^6} = \left(\sqrt{3}\right)^{5+4-3-6} = \left(\sqrt{3}\right)^{\text{zero}} = 1$

9) $\frac{\left(\sqrt{2}\right)^5 \times 2^6 \times 5^6}{\left(\sqrt{2}\right)^3 \times 2^3 \times 5^5} = \left(\sqrt{2}\right)^{5-3} \times 2^{6-3} \times (5)^{6-5} = \left(\sqrt{2}\right)^2 \times 2^3 \times 5 = 2 \times 2^3 \times 5 = 2^4 \times 5 = 16 \times 5 = 80$

10) $\frac{3^{-2} \times 5^{-2} \times \left(\sqrt{5}\right)^3 \times 3^3}{3^2 \times \left(\sqrt{5}\right)^{-3}} = 3^{-2+3-2} \times 5^{-2} \times \left(\sqrt{5}\right)^{3+3} = 3^{-1} \times 5^{-2} \times \left(\sqrt{5}\right)^6 = \frac{1}{3} \times \frac{1}{5^2} \times 5^3 = \frac{1}{3} \times 5^{3-2} = \frac{5}{3}$

11) $\frac{\left(\sqrt{5}\right)^6 \times \left(\sqrt{2}\right)^8}{\left(\sqrt{5\times 2}\right)^4} = \frac{\left(\sqrt{5}\right)^6 \times \left(\sqrt{2}\right)^8}{\left(\sqrt{5}\right)^4 \times \left(\sqrt{2}\right)^4} = \left(\sqrt{5}\right)^{6-4} \times \left(\sqrt{2}\right)^{8-4} = \left(\sqrt{5}\right)^2 \times \left(\sqrt{2}\right)^4 = 5 \times 4 = 20$

12) $\frac{(10)^2 \times (10)^{-7}}{(10)^{-2} \times (10)^{-3}} = (10)^{2-7+2+3} = (10)^{\text{zero}} = 1$

5

1) $\frac{2^4 \times \left(\sqrt{3}\right)^4}{\left(\sqrt{2}\right)^4} = \frac{2^4 \times 3^2}{2^2} = 2^2 \times 3^2 = 36$

2 $\frac{3^4 \times (\sqrt{2})^4}{2^4 \times (\sqrt{3})^4} = \frac{3^4 \times 2^2}{2^4 \times 3^2} = \frac{3^2}{2^2} = \frac{9}{4}$

3 $\frac{(-5)^4 \times (\sqrt{2})^4}{2^4 \times (\sqrt{5})^4} = \frac{5^4 \times 2^2}{2^4 \times 5^2} = \frac{5^2}{2^2} = \frac{25}{4}$

4 $\frac{(\sqrt{3})^2}{(\sqrt{2})^2} \times \frac{(\sqrt{2})^4}{(\sqrt{3})^4} = \frac{(\sqrt{2})^2}{(\sqrt{3})^2} = \frac{2}{3}$

6

1 $\frac{3^{2x} \times 3^{x+2}}{3^{3x}} = 3^{2x+x+2-3x} = 3^2 = 9$

2 $\frac{2^{2x} \times 3^{x-1}}{2^{2x} \times 3^x} = 3^{x-1-x} = 3^{-1} = \frac{1}{3}$

3 $\frac{2^x \times 2^{2x+2}}{2^{3x}} = 2^{x+2x+2-3x} = 2^2 = 4$

4 $\frac{(2^2 \times 3^2)^n \times 5^{2n}}{(3 \times 2 \times 5)^{2n}} = \frac{2^{2n} \times 3^{2n} \times 5^{2n}}{3^{2n} \times 2^{2n} \times 5^{2n}} = 1$

5 $\frac{2^x \times (7^2)^{x-1}}{(2 \times 7^2)^x} = \frac{2^x \times 7^{2x-2}}{2^x \times 7^{2x}} = 7^{2x-2-2x} = 7^{-2} = \frac{1}{49}$

6 $\frac{2^{2x+4} \times 3^{6+2x}}{2^{2x+3} \times 3^{2x+3}} = 2^{2x+4-2x-3} \times 3^{6+2x-2x-3}$

$= 2^1 \times 3^3 = 54$

7 $\frac{2^{2n} \times 2^{2n} \times 3^{2n}}{2^{4n} \times 3^{2n}} = 2^{2n+2n-4n} = 2^0 = 1$

8 $\frac{3^{4x} \times 2^{2x} \times 3^{2x}}{3^{6x-3} \times 2^{2x}} = 3^{4x+2x-6x+3} = 3^3 = 27$

9 $\frac{2^n \times 3^{2n+2} \times 2}{2 \times 3 \times 2^n \times 3^{2n}} = 3^{2n+2-1-2n} = 3$

10 $\frac{3 \times (2 \times 3^2)^{x+1} \times 2^x}{2 \times (2^2 \times 3^2)^x} = \frac{3 \times 2^{x+1} \times 3^{2x+2} \times 2^x}{2 \times 2^{2x} \times 3^{2x}}$

$= 3^{1+2x+2-2x} \times 2^{x+1+x-1-2x} = 3^3 \times 2^0 = 27$

11 $\frac{2^n \times 3^n \times (2^2)^{n+\frac{1}{2}}}{(2^3 \times 3)^n} = \frac{2^n \times 3^n \times 2^{2n+1}}{2^{3n} \times 3^n} = 2^{n+2n+1-3n} = 2$

12 $\frac{2^{3n-3} \times 2^{-5n}}{2^5 \times 2^{-2n}} = 2^{3n-3-5n-5+2n} = 2^{-8} = \frac{1}{2^8} = \frac{1}{256}$

13 $\frac{(\sqrt{3})^{n+2} \times 3^{n+1} \times 5^{n+1}}{(\sqrt{3})^n \times 3^n \times 5^{n+2}}$

$= (\sqrt{3})^{n+2-n} \times 3^{n+1-n} \times 5^{n+1-n-2}$

$= (\sqrt{3})^2 \times 3 \times 5^{-1} = \frac{9}{5}$

14 $\frac{2^{2x+2} \times 3^{4-2x}}{2^{2x} \times 3^{2x}} = 2^{2x+2-2x} \times 3^{4-2x-2x}$

$= 2^2 \times 3^{4-4x}$ at $x = 1$ $\therefore 2^2 \times 3^{4-4} = 2^2 \times 3^0 = 4$

15 $\frac{3^{2x-2} \times 2^{3x}}{2^{3x} \times 3^x} = 3^{2x-2-x} \times 2^{3x-3x}$

$= 3^{x-2} \times 2^0 = 3^{x-2}$ when $x = 2$

$\therefore 3^{2-2} = 3^0 = 1$

16 $2^{2x-2} \times 2^{3x+2} \times 2^{-3x} = 2^{2x}$

$, \because 2^x = 5 \quad \therefore 2^{2x} = (2^x)^2 = (5)^2 = 25$

7

1 The left side $= \frac{2 \times 2^{1-x} \times 3^{2x-1} \times 2^{4x-2}}{2^{3x} \times 3^{2x}}$

$= 2^{1+1-x+4x-2-3x} \times 3^{2x-1-2x}$

$= 2^0 \times 3^{-1} = \frac{1}{3}$ = The right side.

2 The left side $= \frac{3^{3x-3} \times 2^{3x}}{2^{2x} \times (\sqrt{2})^{2x} \times 3^{2x} \times (\sqrt{3})^{2x}}$

$= \frac{3^{3x-3} \times 2^{3x}}{2^{2x} \times 2^x \times 3^{2x} \times 3^x}$

$= 3^{3x-3-2x-x} \times 2^{3x-2x-x}$

$= 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$ = The right side.

8

1 $a^4 - b^4 = (\sqrt{3})^4 - (\sqrt{2})^4 = 3^2 - 2^2 = 9 - 4 = 5$

2 $\frac{a^4}{b^4} = \frac{(\sqrt{3})^4}{(\sqrt{2})^4} = \frac{3^2}{2^2} = \frac{9}{4}$

9

$(x^2 - y^2)^3 = ((2\sqrt{2})^2 - 3^2)^3 = (2^2 \times (\sqrt{2})^2 - 3^2)^3$

$= (4 \times 2 - 9)^3 = (8 - 9)^3 = -1$

10

$2(ab)^2 \times \left(\frac{a}{b}\right)^2 = 2\left(\frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{3}}\right)^2 \times \left(\frac{3}{\sqrt{2}} \times \frac{\sqrt{3}}{\sqrt{2}}\right)^2$

$= 2\left(\frac{3}{\sqrt{3}}\right)^2 \times \left(\frac{3\sqrt{3}}{2}\right)^2$

$= 2 \times \frac{9}{3} \times \frac{27}{4} = \frac{81}{2}$

11

$x^2 + (xzy)^2 = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{3}}\right)^2$

$= \frac{3}{4} + \left(\frac{\sqrt{2}}{4}\right)^2 = \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$

12

$\because \frac{a}{b} = \frac{3\sqrt{2}}{2} \times \frac{\sqrt{2}}{\sqrt{3}} = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$

$\therefore \left(\frac{a}{b}\right)^2 - 3\left(\frac{b}{a}\right)^2 = \left(\sqrt{3}\right)^2 - 3 \times \left(\frac{1}{\sqrt{3}}\right)^2 = 3 - 1 = 2$

13

1. $3(x+y)^4(x-y)^4 = 3\left(\left(2+\sqrt{3}\right)^4\left(2-\sqrt{3}\right)^4\right)$
$= 3\left(\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)\right)^4$
$= 3(4-3)^4 = 3$

2. $\left(\frac{x-y}{x+y}\right)^2 = \left(\frac{2-\sqrt{3}}{2+\sqrt{3}}\right)^2 = \left(\frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}\right)^2$
$= \left(\frac{4-4\sqrt{3}+3}{4-3}\right)^2 = \left(7-4\sqrt{3}\right)^2$
$= 49 - 56\sqrt{3} + 48 = 97 - 56\sqrt{3}$

14

$7a^6 + (1-b)^{-3} = 7 \times \left(\frac{1}{\sqrt{2}}\right)^6 + (1+1)^{-3}$

$= 7 \times \frac{1}{2^3} + 2^{-3} = 7 \times \frac{1}{2^3} + \frac{1}{2^3} = \frac{7}{8} + \frac{1}{8} = \frac{8}{8} = 1$

15

1. $x^{-2}y^{-4} = \frac{1}{x^2y^4} = \frac{1}{3^2 \times \left(\sqrt{2}\right)^4} = \frac{1}{9 \times 4} = \frac{1}{36}$

2. $(x^{-2} \times y^4)^{-2} = x^4y^{-8} = \frac{x^4}{y^8} = \frac{3^4}{\left(\sqrt{2}\right)^8} = \frac{81}{16}$

3. $\left(\frac{x}{y}\right)^{-3} = \left(\frac{y}{x}\right)^3 = \frac{y^3}{x^3} = \frac{\left(\sqrt{2}\right)^3}{3^3} = \frac{2\sqrt{2}}{27}$

16

1 d	2 c	3 d	4 c	5 c
6 a	7 b	8 c	9 b	10 c
11 c	12 a	13 c	14 a	15 c
16 d	17 b	18 d	19 c	20 b
21 b	22 a	23 d	24 a	25 c

17

1 3	2 1	3 zero	4 zero	5 3
6 $\frac{1}{4}$	7 $\left(-\sqrt{2}\right)^{24}$	8 3	9 $\mathbb{R} - \{5\}$	10 1
11 1	12 $\frac{36}{25}$	13 125	14 35	15 $\frac{3}{7}$

18

1. $\frac{1}{4}$ 2. $\sqrt{2}$ 3. 16 4. 2025

19

1. c 2. c 3. c 4. b 5. c

Answers of Exercise 11

1

1. $\because 5^n = 5^2$ $\therefore n = 2$
2. $\because 2^{-n} = 2^5$ $\therefore -n = 5$ $\therefore n = -5$
3. $\because 2^{n-1} = 2^5$ $\therefore n - 1 = 5$ $\therefore n = 6$
4. $\because 3^{n-2} = 3^4$ $\therefore n - 2 = 4$ $\therefore n = 6$
5. $\because 3^{n-2} = 3^{zero}$ $\therefore n - 2 = zero$ $\therefore n = 2$
6. $\because 2^{n-3} = 2^{-2}$ $\therefore n - 3 = -2$ $\therefore n = 1$
7. $\because 3^{n-2} = 3^{-2}$ $\therefore n - 2 = -2$ $\therefore n = zero$
8. $\because \left(\sqrt{3}\right)^{n-1} = \left(\sqrt{3}\right)^4$ $\therefore n - 1 = 4$ $\therefore n = 5$
9. $\because \left(\frac{2}{5}\right)^{2n-1} = \left(\frac{2}{5}\right)^3$ $\therefore 2n - 1 = 3$
$\therefore 2n = 4$ $\therefore n = 2$
10. $\because \left(\frac{3}{5}\right)^{n+2} = \left(\frac{5}{3}\right)^3$ $\therefore \left(\frac{3}{5}\right)^{n+2} = \left(\frac{3}{5}\right)^{-3}$
$\therefore n + 2 = -3$ $\therefore n = -5$
11. $\because \left(\frac{2}{3}\right)^{n-4} = \frac{9}{4}$ $\therefore \left(\frac{2}{3}\right)^{n-4} = \left(\frac{3}{2}\right)^2$
$\therefore \left(\frac{2}{3}\right)^{n-4} = \left(\frac{2}{3}\right)^{-2}$ $\therefore n - 4 = -2$ $\therefore n = 2$
12. $\because \left(\frac{2}{3}\right)^{n+5} = \left(\frac{27}{8}\right)^{-2}$
$\therefore \left(\frac{2}{3}\right)^{n+5} = \left(\left(\frac{3}{2}\right)^3\right)^{-2} = \left(\frac{3}{2}\right)^{-6}$
$\therefore \left(\frac{2}{3}\right)^{n+5} = \left(\frac{2}{3}\right)^6$ $\therefore n + 5 = 6$ $\therefore n = 1$
13. $\because 5^{2n-4} = 7^{2n-4}$ $\therefore 2n - 4 = 0$
$\therefore 2n = 4$ $\therefore n = 2$
14. either $n = 3$ or $n - 4 = 0$ $\therefore n = 4$

15 $\because 3^2 \times 3^{n-4} = 1$ $\therefore 3^{n-2} = 1$
$\therefore n - 2 = 0$ $\therefore n = 2$

16 $\because 2 \times 2^{2n+6} = \frac{1}{32}$ $\therefore 2^{2n+7} = 2^{-5}$
$\therefore 2n + 7 = -5$ $\therefore 2n = -12$ $\therefore n = -6$

17 $\because (\sqrt{2})^6 \times (\sqrt{2})^{n-3} = 1$ $\therefore (\sqrt{2})^{n+3} = 1$
$\therefore n + 3 = 0$ $\therefore n = -3$

18 $\because \frac{1}{(\sqrt{3})^6} \times (\sqrt{3})^{n+2} = 1$
$\therefore (\sqrt{3})^{-6} \times (\sqrt{3})^{n+2} = 1$ $\therefore (\sqrt{3})^{n-4} = 1$
$\therefore n - 4 = 0$ $\therefore n = 4$

2

1 $\because x^2 - 4 = 0$ $\therefore x^2 = 4$
$\therefore x = \pm 2$ $\therefore$ The S.S. = {2 , −2}

2 $\because 2^{x^2-9} = 1$ $\therefore x^2 - 9 = 0$
$\therefore x^2 = 9$ $\therefore x = \pm 3$
$\therefore$ The S.S. = {−3 , 3}

3 $\because 2^{x^2-x} = 4$ $\therefore 2^{x^2-x} = 2^2$
$\therefore x^2 - x = 2$ $\therefore x^2 - x - 2 = 0$
$\therefore (x-2)(x+1) = 0$ $\therefore x - 2 = 0$
$\therefore x = 2$ or $x + 1 = 0$
$\therefore x = -1$ $\therefore$ The S.S. = {−1 , 2}

4 $\because 5^{|x|} = 5^3$ $\therefore |x| = 3$ $\therefore x = \pm 3$
$\therefore$ The S.S. = {3 , −3}

5 $\because 2^{5x-15} = 2^{6x+3}$ $\therefore 5x - 15 = 6x + 3$
$\therefore 5x - 6x = 15 + 3$ $\therefore -x = 18$
$\therefore x = -18$ $\therefore$ The S.S. = {−18}

6 $\because (\sqrt{3})^{2x-6} = (\sqrt{3})^{x+5}$ $\therefore 2x - 6 = x + 5$
$\therefore x = 11$ $\therefore$ The S.S. = {11}

7 $\because 3^{n-1} \times 2^{-(n-1)} = \left(\frac{3}{2}\right)^3$
$\therefore \frac{3^{n-1}}{2^{n-1}} = \left(\frac{3}{2}\right)^3$ $\therefore \left(\frac{3}{2}\right)^{n-1} = \left(\frac{3}{2}\right)^3$
$\therefore n - 1 = 3$ $\therefore n = 4$ $\therefore$ The S.S. = {4}

8 $\because \frac{3^{x-1}}{5^{x-1}} = \frac{9}{25}$ $\therefore \left(\frac{3}{5}\right)^{x-1} = \left(\frac{3}{5}\right)^2$
$\therefore x - 1 = 2$ $\therefore x = 3$ $\therefore$ The S.S. = {3}

3

1 $\because \frac{2^n \times 3^{2n+2}}{2^n \times 3^{2n}} = 3^n$ $\therefore 3^{2n+2-2n} = 3^n$
$\therefore 3^2 = 3^n$ $\therefore n = 2$

2 $\because \frac{2^{3n} \times 9^n}{2^n \times 9^n} = 64$ $\therefore 2^{3n-n} = 64$
$\therefore 2^{2n} = 64$ $\therefore 2^{2n} = 2^6$
$\therefore 2n = 6$ $\therefore n = 3$

3 $\because \frac{6^{2n-3}}{(2 \times 3)^{n-1}} = 6$ $\therefore \frac{6^{2n-3}}{6^{n-1}} = 6$
$\therefore 6^{2n-3-n+1} = 6$ $\therefore 6^{n-2} = 6^1$ $\therefore n = 3$

4 $\because \frac{3^{n-1} \times 2^{2n-2}}{2^{n-1} \times 3^{n-1}} = 1$ $\therefore 2^{2n-2-n+1} = 2^0$
$\therefore n - 1 = 0$ $\therefore n = 1$

5 $\because \frac{3^n \times 2^{3n}}{3^{n+1} \times 2^{2n+2}} = \frac{1}{3}$
$\therefore 3^{n-n-1} \times 2^{3n-2n-2} = 3^{-1}$
$\therefore 3^{-1} \times 2^{n-2} = 3^{-1}$ $\therefore 2^{n-2} = 1$ $\therefore 2^{n-2} = 2^0$
$\therefore n - 2 = 0$ $\therefore n = 2$

6 $\because \frac{2^{4n} \times 3^{2n}}{3^{2n} \times 2^{2n}} = \frac{1}{2^4}$ $\therefore 2^{2n} = 2^{-4}$ $\therefore 2n = -4$
$\therefore n = -2$

7 $\because \frac{2^{2n-2} \times 2^{n+3}}{2^{3n}} = 2n^2$ $\therefore 2^{2n-2+n+3-3n} = 2n^2$
$\therefore 2 = 2n^2$ $\therefore n^2 = 1$ $\therefore n = \pm 1$

8 $\because \frac{2^{2n} \times 7^{2n} \times 2^{2n+2}}{2^2 \times 7^n \times 2^{4n}} = 7^2$
$\therefore 2^{2n+2n+2-2-4n} \times 7^{2n-n} = 7^2$
$\therefore 2^0 \times 7^n = 7^2$ $\therefore 7^n = 7^2$ $\therefore n = 2$

4

1 $\because (x-4)^5 = 2^5$ $\therefore x - 4 = 2$ $\therefore x = 6$
$\therefore$ The S.S. = {6}

2 $\because \frac{1}{(x+9)^4} = \frac{1}{10000}$ $\therefore \frac{1}{(x+9)^4} = \frac{1}{(10)^4}$
$\therefore (x+9)^4 = (10)^4$ $\therefore x + 9 = \pm 10$
$\therefore x = 1$ or $x = -19$ $\therefore$ The S.S. = {1 , −19}

3 $\because (x^2 - x)^5 = 2^5$ $\therefore x^2 - x = 2$
$\therefore x^2 - x - 2 = 0$ $\therefore (x+1)(x-2) = 0$
$\therefore$ either $x + 1 = 0$ $\therefore x = -1$ or $x - 2 = 0$
$\therefore x = 2$ $\therefore$ The S.S. = {−1 , 2}

4 $\because x^2 - x = 0$ $\therefore x(x-1) = 0$
$\therefore x = 0$ or $x - 1 = 0$ $\therefore x = 1$
$\therefore$ The S.S. = {0 , 1}

5 $\because 5^{x^2-5x}=\frac{16}{10000}=\left(\frac{2}{10}\right)^4$

$\therefore 5^{x^2-5x}=\left(\frac{10}{2}\right)^{-4}=5^{-4}$

$\therefore x^2-5x=-4$ $\quad \therefore x^2-5x+4=0$

$\therefore (x-4)(x-1)=0$ $\quad \therefore x-4=0$

$\therefore x=4$ $\quad$ or $x-1=0$

$\therefore x=1$ $\quad \therefore$ The S.S. $=\{1, 4\}$

6 $\because 5^{x^2}=5^{2x+8}$ $\quad \therefore x^2=2x+8$

$\therefore x^2-2x-8=0$

$\therefore (x-4)(x+2)=0$ either $x-4=0$ then $x=4$

or $x+2=0$ then $x=-2$ $\quad \therefore$ The S.S. $=\{4, -2\}$

7 $\because 3^{2x^2-2}=\frac{1}{3^{3x}}$ $\quad \therefore 3^{2x^2-2}=3^{-3x}$

$\therefore 2x^2-2=-3x$ $\quad \therefore 2x^2+3x-2=0$

$\therefore (2x-1)(x+2)=0$ either $2x-1=0$

$\therefore x=\frac{1}{2}$ or $x+2=0$ $\quad \therefore x=-2$

$\therefore$ The S.S. $=\left\{-2, \frac{1}{2}\right\}$

5

$\because \frac{2^{2n}\times 3^{2n}\times 2^{2n}}{2^{4n}\times 3^{2n+4}}=3^{-2x}$

$\therefore 2^{2n+2n-4n}\times 3^{2n+2n-4}=3^{-2x}$

$\therefore 2^0\times 3^{-4}=3^{-2x}$ $\quad \therefore 3^{-4}=3^{-2x}$

$\therefore -2x=-4$ $\quad \therefore x=2$

6

$\because \frac{3^{4x}\times 2^{2x}}{2^{2x}\times 3^{2x}\times 3^{2x}}=3^{4x-2x-2x}=1$

$\therefore 3^{y-1}=1$ $\quad \therefore y-1=0$ $\quad \therefore y=1$

7

$\because \frac{3^{3x-3}\times 2^{3x}}{2^{2x}\times(\sqrt{2})^{2x}\times 3^{2x}\times(\sqrt{3})^{2x}}=3^x$

$\therefore \frac{3^{3x-3}\times 2^{3x}}{2^{2x}\times 2^x\times 3^{2x}\times 3^x}=3^x$

$\therefore \frac{3^{3x-3}\times 2^{3x}}{2^{3x}\times 3^{3x}}=3^x$ $\quad \therefore 3^{3x-3-3x}=3^x$

$\therefore 3^{-3}=3^x$ $\quad \therefore x=-3$

8

$\frac{7^x\times 2^x\times 3^x}{2^x\times 7^x}=3^{2-y}$ $\quad \therefore 3^x=3^{2-y}$

$\therefore x=2-y$ $\quad \therefore x+y=2$

9

$\because \left(\sqrt{\frac{2}{3}}\right)^{-x}=\left(\sqrt{\frac{2}{3}}\right)^4$ $\quad \therefore -x=4$ $\quad \therefore x=-4$

$\therefore \left(\frac{3}{2}\right)^{x+1}=\left(\frac{3}{2}\right)^{-4+1}=\left(\frac{3}{2}\right)^{-3}=\left(\frac{2}{3}\right)^3=\frac{8}{27}$

10

$\because \frac{7^{2n}\times 5^{4n}\times 3^{4n}}{7^{-n}\times 3^{4n}\times 5^{4n}}=7^3$

$\therefore 7^{2n+n}\times 5^{4n-4n}\times 3^{4n-4n}=7^3$ $\quad \therefore 7^{3n}=7^3$

$\therefore 3n=3$ $\quad \therefore n=1$ $\quad \therefore 6^{2n}=6^2=36$

11 $3^x=3^3$ $\quad \therefore x=3$ $\quad \therefore 4^{3+y}=1$

$\therefore 3+y=0$ $\quad \therefore y=-3$

12

1 c	2 d	3 a	4 b	5 d
6 b	7 b	8 c	9 b	10 b
11 a	12 a	13 b	14 a	15 b

13

1 zero or 1	2 −5	3 2	4 7
5 1	6 −1	7 3	8 2
9 −1	10 1	11 −1	12 2
13 4 , 5			

14

$\because$ The number of the small squares in the chess board of side length 4 cm. = 64 squares.

$\therefore$ The area of one small square $=4\times 4=16$ cm.^{2}

$\therefore$ The area of the chess board $=16\times 64=2^n$

$\therefore 2^4\times 2^6=2^n$ $\quad \therefore 2^{10}=2^n$ $\quad \therefore n=10$

15

1 $\because x^{x+2}=4^{x+2}$ $\quad \therefore x=\pm 4$

(because : $x+2=4+2=6$ "even number"

or : $x+2=-4+2=-2$ "even number"

or $x+2=0$ $\quad \therefore x=-2$ $\quad \therefore x=\pm 4$ or -2

2 $\because a^{x+3}-1=(a^2-1)(a^2+1)(a^4+1)$

$a^{x+3}-1=(a^4-1)(a^4+1)$

$\therefore a^{x+3}-1=a^8-1$ $\quad \therefore a^{x+3}=a^8$

$\therefore x+3=8$ $\quad \therefore x=5$

Answers of Exercise 12

1

1) 2   2) 18   3) $-\frac{1}{2}$ 4) $-\frac{2}{9}$

2

1) $\left(\sqrt{5}\right)^5 \div \left(\sqrt{5}\right)^3 + 2\sqrt{3} \times \sqrt{3}$

$= \left(\sqrt{5}\right)^{5-3} + 2 \times \left(\sqrt{3}\right)^{1+1} = \left(\sqrt{5}\right)^2 + 2\left(\sqrt{3}\right)^2$

$= 5 + 6 = 11$

2) $2^3 \times \left(\sqrt{3}\right)^3 \times \sqrt{3} - \left(\sqrt{2}\right)^7 \div \left(\sqrt{2}\right)^5$

$= 2^3 \times \left(\sqrt{3}\right)^{3+1} - \left(\sqrt{2}\right)^{7-5}$

$= 2^3 \times \left(\sqrt{3}\right)^4 - \left(\sqrt{2}\right)^2 = 8 \times 9 - 2 = 70$

3) $\left(\sqrt{3}\right)^{-3} \times \left(\sqrt{3}\right)^3 + \left(\sqrt{3}\right)^{-4} \times \left(\sqrt{3}\right)^{10}$

$= \left(\sqrt{3}\right)^{-3+3} + \left(\sqrt{3}\right)^{-4+10}$

$= 1 + \left(\sqrt{3}\right)^6 = 1 + 27 = 28$

4) $2^4 \times \left(\sqrt{5}\right)^4 - \left(\sqrt{5}\right)^{-3} \times \left(\sqrt{5}\right)^6 \div \left(\sqrt{5}\right)^3$

$= 2^4 \times \left(\sqrt{5}\right)^4 - \left(\sqrt{5}\right)^{-3+6-3} = 16 \times 25 - 1 = 399$

3

1) $\dfrac{\left(\sqrt{3}\right)^7 \times \left(\sqrt{3}\right)^{-5} - \left(\sqrt{3}\right)^2}{\left(\sqrt{3}\right)^7 \times \left(\sqrt{3}\right)^{-5} + \left(\sqrt{3}\right)^2} = \dfrac{\left(\sqrt{3}\right)^2 - \left(\sqrt{3}\right)^2}{\left(\sqrt{3}\right)^2 + \left(\sqrt{3}\right)^2}$

$= \dfrac{0}{3+3} = \text{zero}$

2) $\dfrac{2\left(\sqrt{3}\right)^5 \div \left(\sqrt{3}\right)^3}{2\sqrt{3} + 3 - 2\sqrt{3} + 1} = \dfrac{2\left(\sqrt{3}\right)^2}{4} = \dfrac{3}{2}$

3) $\dfrac{2^3 \times \left(\sqrt{2}\right)^3 \times 3 \times \sqrt{2}}{6 + 2\sqrt{12} + 2 - 2\sqrt{12}} = \dfrac{8 \times 3 \times \left(\sqrt{2}\right)^4}{8} = 3 \times 4 = 12$

4

1) $\dfrac{b^4 - a^4}{b^2 + a^2} = \dfrac{(b^2 - a^2)(b^2 + a^2)}{(b^2 + a^2)} = b^2 - a^2$

$= \left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2 = 3 - 2 = 1$

2) $\dfrac{a^3 + b^3}{a + b} = \dfrac{(a+b)(a^2 - ab + b^2)}{(a+b)}$

$= a^2 - ab + b^2$

$= \left(\sqrt{2}\right)^2 - \sqrt{2} \times \sqrt{3} + \left(\sqrt{3}\right)^2$

$= 2 - \sqrt{6} + 3 = 5 - \sqrt{6}$

5

1) a   2) d   3) a   4) a   5) d

6

1) $\because$ The total area of the cube $= 6\,\ell^2$

$\therefore 6\,\ell^2 = 3.375 \times 10^2 \qquad \therefore \ell^2 = \dfrac{3.375 \times 10^2}{6}$

$\therefore \ell = \sqrt{\dfrac{3.375 \times 10^2}{6}}$

Using the calculator $\ell = 7.5$ cm.

2) $\because$ The volume of the cube $= \ell^3$

$\therefore$ The volume of the cube $= (7.5)^3$

Using the calculator ,

the volume of the cube $= 421.875$ cm³.

7

$\because$ The volume of the sphere $= \frac{4}{3}\pi r^3$

$\therefore 3.8808 \times 10^4 = \frac{4}{3}\pi r^3$

$\therefore r^3 = \dfrac{3.8808 \times 10^4 \times 3}{4\pi} \qquad \therefore r = \sqrt[3]{\dfrac{3.8808 \times 10^4 \times 3}{4 \times \frac{22}{7}}}$

Using the calculator $r = 21$ cm.

8

$\because$ The volume of the cone $= \frac{1}{3}\pi r^2 h$

$\therefore 7.7 \times 10^2 = \frac{1}{3}\pi \times 7^2 \times h$

$\therefore 7.7 \times 10^2 = \frac{49}{3} \times \pi \times h$

$\therefore h = \dfrac{7.7 \times 10^2 \times 3}{49 \times \frac{22}{7}}$ Using the calculator $h = 15$ cm.

9

$c = 2.5 \times 10^4 (1 + 9.8 \times 10^{-2})^{12}$

Using the calculator we get:

$c = 76765.85477 \simeq 76766$ to the nearest pound.

10

1) $y = 11.7\,(1.02)^6$ Using the calculator

$y = 13.17610031 \simeq 13$ million persons

2) $y = 11.7\,(1.02)^{-5}$ Using the calculator

$y = 10.59\,705\,048 \simeq 11$ million persons

11

$\dfrac{x^7 y^8 - y}{(x+y)^9} = \dfrac{y\,(x^7 y^7 - 1)}{(x+y)^9}$

$, x \times y = \left(2 + \sqrt{3}\right)\left(2 - \sqrt{3}\right) = 4 - 3 = 1$

$, x + y = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

$\therefore \dfrac{x^7 y^8 - y}{(x+y)^9} = \dfrac{\left(2 - \sqrt{3}\right)(1^7 - 1)}{4^9} = \text{zero}$

12

$\because \ell + m = \frac{1}{2}(3^x + 3^{-x}) + \frac{1}{2}(3^x - 3^{-x})$

$= \frac{1}{2}(3^x + 3^{-x} + 3^x - 3^{-x})$

$= \frac{1}{2} \times (2 \times 3^x) = 3^x$

$, \ell - m = \frac{1}{2}(3^x + 3^{-x}) - \frac{1}{2}(3^x - 3^{-x})$

$= \frac{1}{2}(3^x + 3^{-x} - 3^x + 3^{-x})$

$= \frac{1}{2} \times (2 \times 3^{-x}) = 3^{-x}$

$\therefore \ell^2 - m^2 = (\ell - m)(\ell + m) = 3^x \times 3^{-x} = 3^{zero} = 1$

Answers of exams on unit two

Model 1

1

1 c 2 d 3 c
4 b 5 c 6 c

2

1 2 2 3 3 − 5
4 0 5 20

3

[a] $x = 2$ [b] $x = 1$

4

[a] The expression = 1

[b] $x = 3$, $y = -3$

5

[a] $\frac{4}{27}$

[b] The expression = 1

Model 2

1

1 c 2 c 3 a
4 d 5 b 6 d

2

1 4 2 $\frac{1}{5}$ 3 − 3
4 21 5 36

3

[a] 1 $\frac{1}{36}$ 2 $\frac{2\sqrt{2}}{27}$

[b] $x^2 = 25$

4

[a] The expression = 1

[b] The expression = 25

5

[a] 64

[b] The expression = $4 \times 3^{4-4x}$

, the numerical value = 4

Answers of Unit Three

Answers of Exercise 13

1

1. 0 , 1
2. [0 , 1]
3. $\frac{1}{2}$
4. $\frac{2}{5}$
5. $\frac{6}{11}$
6. $\frac{1}{5}$
7. $\frac{1}{6}$
8. $\frac{1}{3}$
9. zero
10. $\frac{1}{12}$, $\frac{11}{12}$
11. $\frac{3}{8}$
12. $\frac{1}{3}$
13. 600
14. 392 lamps

2

1. ∵ the multiples of 4 are 4 , 8 , 12 , 16 , 20 , 24 its number = 6
   ∴ the probability = $\frac{6}{24} = \frac{1}{4}$
2. ∵ the multiples of 6 are 6 , 12 , 18 , 24
   its number = 4
   ∴ the probability = $\frac{4}{24} = \frac{1}{6}$
3. ∵ the multiples of 4 and 6 together are 12 , 24
   its number = 2
   ∴ the probability = $\frac{2}{24} = \frac{1}{12}$
4. ∵ the multiples of 4 or 6 are 4 , 6 , 8 , 12 , 16 , 18 , 20 , 24 its number = 8
   ∴ the probability = $\frac{8}{24} = \frac{1}{3}$
5. ∵ the numbers which are divisible by 25 are nothing
   ∴ its number = zero.
   ∴ the probability = $\frac{0}{24} = 0$
6. ∵ the numbers from 1 to 24 are positive integer numbers and each of them is less than 25
   its number = 24
   ∴ the probability = $\frac{24}{24} = 1$

3

1. ∵ the even numbers from 1 to 40 are 2 , 4 , 6 , 8 , 10 , 12 , 14 , 16 , 18 , 20 , 22 , 24 , 26 , 28 , 30 , 32 , 34 , 36 , 38 , 40
   its number = 20 ∴ the probability = $\frac{20}{40} = \frac{1}{2}$
2. ∵ the numbers from 1 to 40 and which are divisible by 3 are 3 , 6 , 9 , 12 , 15 , 18 , 21 , 24 , 27 , 30 , 33 , 36 , 39 their number = 13
   ∴ the probability = $\frac{13}{40}$
3. ∵ the numbers from 1 to 40 and each of them is divisible by 10 are 10 , 20 , 30 , 40 its number = 4
   ∴ the remained number are not divisible by 10 their number = 36
   ∴ the probability = $\frac{36}{40} = \frac{9}{10}$
4. ∵ the numbers from 1 to 40 and each of them is even number and divisible by 3 are 6 , 12 , 18 , 24 , 30 , 36 their number = 6
   ∴ the probability = $\frac{6}{40} = \frac{3}{20}$
5. ∵ the numbers from 1 to 20 and each of them is a prime number are 2 , 3 , 5 , 7 , 11 , 13 , 17, 19
   their number = 8
   ∴ the probability = $\frac{8}{40} = \frac{1}{5}$

4

1. ∵ the numbers from 1 to 25 and each of them is divisible by 4 are 4 , 8 , 12 , 16 , 20 , 24
   their number = 6
   ∴ the probability = $\frac{6}{25}$
2. ∵ the numbers from 1 to 25 and each of them is divisible by 5 are 5 , 10 , 15 , 20 , 25
   their number = 5
   ∴ the probability = $\frac{5}{25} = \frac{1}{5}$
3. ∵ the numbers from 1 to 25 and each of them is divisible by 4 or 5 are 4 , 5 , 8 , 10 , 12 , 15 , 16 , 20 , 24 , 25
   their number = 10
   ∴ the probability = $\frac{10}{25} = \frac{2}{5}$
4. ∵ the numbers from 1 to 25 and each of them is divisible by 4 and 5 is 20 and
   its number = 1
   ∴ the probability = $\frac{1}{25}$
5. ∵ the numbers from 1 to 25 and each of them is greater than or equals 20 are 20 , 21 , 22 , 23 , 24 , 25
   their number = 6
   ∴ the probability = $\frac{6}{25}$
6. ∵ the numbers from 1 to 25 and each of them is not divisible by 5 or 4 are 1 , 2 , 3 , 6 , 7 , 9 , 11 , 13 , 14 , 17 , 18 , 19 , 21 , 22 , 23
   their number = 15
   ∴ the probability = $\frac{15}{25} = \frac{3}{5}$

5

1. the probability of getting an even number less than or equal to 4 $= \frac{2}{6} = \frac{1}{3}$
2. the probability of getting a number between 0 and $10 = \frac{6}{6} = 1$
3. the probability of getting a number divisible by $7 = \frac{0}{6} =$ zero
4. the probability of getting a number not divisible by $2 = \frac{3}{6} = \frac{1}{2}$

6

∵ the total number of marbles $= 12 + 18 + 20 = 50$

1. the probability that the marble is white $= \frac{18}{50} = \frac{9}{25}$
2. the probability that the marble is red $= \frac{12}{50} = \frac{6}{25}$
3. the probability that the marble is yellow $= \frac{0}{50} =$ zero
4. the probability that the marble is not red $= \frac{50-12}{50} = \frac{38}{50} = \frac{19}{25}$
5. the probability that the marble is red or blue $= \frac{20+12}{50} = \frac{16}{25}$

7

Number of those don't play any of the previous sports $= 40 - (20 + 10 + 6) = 4$ pupils

∴ The probability that the pupil is one of those don't play any of the previous sports $= \frac{4}{40} = \frac{1}{10}$

8

∵ the number of red marbles $= 22 - 12 = 10$ and after drawing 2 red marbles the number of marbles became 20 and the number of red marbles $= 8$

∴ the probability that the drawn marble is black $= \frac{12}{20} = \frac{3}{5}$

9

∵ the number of girls $= 20$ and the number of boys $= 30$

∴ the probability that the selected student is a boy $= \frac{30}{50} = \frac{3}{5}$

10

∵ the probability of drawing a red ball $= \frac{1}{4}$

∴ the probability that the drawn ball is blue $= 1 - \frac{1}{4} = \frac{3}{4}$

∴ the number of blue balls $= \frac{3}{4} \times 80 = 60$ balls.

11

the number of red marbles $= \frac{3}{8} \times 32 = 12$ marbles

12

∵ the probability that the first player scores a goal $= \frac{18}{21} = \frac{6}{7} \approx 0.86$

the probability that the second player scores a goal $= \frac{25}{32} \approx 0.78$

∴ $0.86 > 0.78$

∴ we select the first player because his probability to score a goal is the greater.

13

∵ the number of matches predicted to win $= 0.6 \times 30 = 18$ matches

∵ the probability of loss the match $= 1 - (0.6 + 0.3) = 0.1$

∴ the number of matches predicted to loss $= 0.1 \times 30 = 3$ matches.

14

∵ the probability that the student read books $= \frac{53}{100} = 0.53$

∴ the probability that the student don't read books $= 1 - 0.53 = 0.47$

∴ the number of students who don't read books $= 0.47 \times 400 = 188$ students.

15

the predicted number to the cars which makes accidents $= 0.004 \times 7000 = 28$ cars.

the predicted number that the company pays $= 28 \times 2000 =$ L.E. 56000

16

∵ the probability of defective units $= \frac{20}{1000} = \frac{1}{50}$

∴ the number of defective units $= \frac{1}{50} \times 6000 = 120$ units.

17

∵ the ratio of fruits which is suitable to be exported $= 100\% - 30\% = 70\%$

∴ the amount of exported fruits $= 70\% \times 20 = 14$ tons.

∴ the amount of exported fruits within 10 days $= 14 \times 10 = 140$ tons.

18

1 the number of defective production in the sample $= 0.06 \times 200 = 12$ units.

2 $\because$ the probability of fit units $= 1 - 0.06 = 0.94$

$\therefore$ the number of fit units $= 0.94 \times 1500 = 1410$ units.

19

1 death probability $= \frac{67}{10\,000} = 0.0067$

2 because these results help these companies to set the suitable insurance system for each category according to the age.

3 The estimated number of death cases in one year $= 0.0067 \times 50\,000 = 335$

20

1 the probability that the pupil uses bus $= \frac{16}{48} = \frac{1}{3}$

2 the probability that the pupil goes on foot $= \frac{12}{48} = \frac{1}{4}$

3 the probability that the pupil doesn't use the bicycle $= \frac{36}{48} = \frac{3}{4}$

21

1 the probability of getting a score of excellent $= \frac{6}{50} = \frac{3}{25}$

2 the probability of getting a score of good $= \frac{11}{50}$

3 the probability of getting a score of fail $= \frac{8}{50} = \frac{4}{25}$

4 the probability of getting a score of less than good $= \frac{16+8}{50} = \frac{12}{25}$

22

1 Probability that the call of duration is less than 2 minutes $= \frac{120}{150} = \frac{4}{5}$

2 Probability that the call of duration is from 2 to 4 minutes $= \frac{15}{150} = \frac{1}{10}$

3 Probability that the call of duration is more than 6 minutes $= \frac{5}{150} = \frac{1}{30}$

4 Probability that the call of duration is 2 minutes and more $= \frac{15+10+5}{150} = \frac{1}{5}$

23

1 (a) The probability that the student prefers practicing football $= \frac{44}{100} = 0.44$

(b) The probability that the student prefers practicing handball $= \frac{27}{100} = 0.27$

(c) The probability that the student prefers practicing athletics $= \frac{12}{100} = 0.12$

(d) The probability that the student prefers practicing tennis $= \frac{4}{100} = 0.04$

(e) The probability that the student prefers practicing hockey $= \frac{13}{100} = 0.13$

2 The number of students $= 0.13 \times 600 = 78$ students.

24

1 The total number of sales from the first type $= 39 + 82 + 34 + 22 + 53 = 230$ shirts.

The total number of sales from the second type $= 61 + 18 + 66 + 78 + 47 = 270$ shirts.

The probability of sales in the first type $= \frac{230}{500} = 0.46$

The probability of sales of the second type $= \frac{270}{500} = 0.54$

$\therefore$ The second type is the more request and we advise the company to increase production of the second type.

2 The number of shirts of the first type $= 0.46 \times 4000 = 1840$ shirts.

25

1 The probability that the unit be a defective $= \frac{18}{300} = 0.06$

2 The probability that the unit be a functional $= \frac{300-18}{300} = 0.94$

3 No

4 $0.06 + 0.94 = 1$

we observe that : the sum of the probabilities = 1

5 The number of the functional units $= 0.94 \times 1600 = 1504$ units.

26

1 (a) The probability of choosing the weight (125) $= \frac{120}{300} = 0.4$

(b) The probability of choosing the weight (250) $= \frac{45}{300} = 0.15$

(c) The probability of choosing the weight (375)
$= \frac{96}{300} = 0.32$

(d) The probability of choosing the weight (500)
$= \frac{39}{300} = 0.13$

2 We advice the manager of this company to increase production of powder of weight 125 gm.

27

1 (a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{4}{8} = \frac{1}{2}$

2 The probability that the pointer doesn't stop at the red colour = the probability that the pointer stops at green colour or yellow colour $= \frac{7}{8}$

28

1 d	2 d	3 b	4 c
5 b	6 c	7 b	8 b
9 d	10 c	11 b	12 d
13 b	14 c	15 d	

29

$\because$ The probability of drawing a red ball $= \frac{2}{3}$

$\therefore$ The probability of drawing a white ball $= 1 - \frac{2}{3} = \frac{1}{3}$

$\therefore$ The total number of balls $= 3 \times 5 = 15$ balls

30

$\because$ The probability that the drawn card carries a number less than or equal to $8 = 1 - \frac{1}{3} = \frac{2}{3}$

$\therefore$ The number of cards $= 8 \times \frac{3}{2} = 12$ cards

$\therefore n = 12$

Answers of exam on unit three

1

1 d	2 a	3 d
4 b	5 a	6 d

2

1 1 , 0	2 14	3 $\frac{1}{5}$
4 [0 , 1]	5 $\frac{1}{2}$	

3

[a]	1 $\frac{3}{10}$	2 $\frac{2}{5}$	3 $\frac{3}{5}$
[b]	1 $\frac{1}{4}$	2 0	3 $\frac{2}{3}$

4

1 $\frac{1}{2}$	2 1	3 $\frac{1}{3}$
4 0		

5

[a] 45 balls [b] $\frac{5}{8}$

Answers of accumulative basic skills

1

1 a	2 c	3 d	4 c	5 c
6 a	7 c	8 c	9 c	10 b
11 c	12 b	13 a	14 b	15 d

2

1 8	2 16	3 $\frac{4}{13}$	4 zero	5 500
6 x^2	7 27	8 1	9 14	10 4
11 16	12 25	13 25	14 -4	15 2

Guide Answers

of Geometry Exercises

Answers of Unit Four

Answers of Exercise 1

1

1. equal in area
2. the rectangle
3. the length of the base × its corresponding height
4. equal

2

Fig. (1) The base $\overline{BC}$ or $\overline{AD}$ and the corresponding altitude $\overline{AE}$ and the base $\overline{CD}$ or $\overline{AB}$ and the corresponding height $\overline{AF}$

Fig. (2) The base $\overline{DC}$ or $\overline{AB}$ and the corresponding altitude $\overline{FE}$ and the base $\overline{BC}$ or $\overline{AD}$ and the corresponding height $\overline{DZ}$

Fig. (3) The base $\overline{AB}$ or $\overline{CD}$ and the corresponding altitude $\overline{CE}$ and the base $\overline{AD}$ or $\overline{BC}$ and the corresponding height $\overline{DF}$

Fig. (4) The base $\overline{AD}$ or $\overline{BC}$ and the corresponding altitude $\overline{AB}$ or $\overline{DC}$ and the base $\overline{AB}$ or $\overline{DC}$ and the corresponding height $\overline{AD}$ or $\overline{BC}$

3

1. 48
2. 24
3. 48
4. 3.4
5. 60 , 50
6. $6\frac{2}{3}$

4

1. d
2. b
3. b
4. c
5. c
6. d

5

$\because m(\angle ABC) = 150°$ $\therefore m(\angle ABE) = 30°$

$\because m(\angle E) = 90°$ $\therefore AE = \frac{1}{2} AB$

$\therefore AE = 4$ cm.

$\therefore$ The area of ▱ ABCD $= 12 \times 4 = 48$ cm^2 (The req.)

6

$\because$ ▱ ABCD , ▱ EBCF are common in the base $\overline{BC}$,

$\because \overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$

$\therefore$ The area of ▱ ABCD = the area of ▱ EBCF

subtracting the figure BEDC from both sides we deduce that the area of ▱ ABCD – the area of the figure BEDC = The area of ▱ EBCF – the area of the figure BEDC

$\therefore$ The area of Δ ABE = the area of Δ DCF (Q.E.D.)

7

$\because$ The rectangle ABCD and the parallelogram AEFD have the common base $\overline{AD}$

$\because \overleftrightarrow{AD} \parallel \overline{BF}$

$\therefore$ the area of the rectangle ABCD = the area of ▱ AEFD subtracting the area of Δ AMD from both sides we deduce that the area of the figure ABCM = the area of the figure DMEF (Q.E.D.)

8

$\because$ The rectangle XDEY and the parallelogram ABED have the common base $\overline{DE}$

, $\because \overleftrightarrow{AY} \parallel \overline{DE}$

$\therefore$ the area of ▱ ABED = the area of the rectangle XDEY ,

$\because$ the area of the rectangle XDEY $= 12 \times 24 = 288$ cm^2

$\therefore$ the area of ▱ ABED $= 288$ cm^2 (First req.)

$\therefore$ the length of the perpendicular from B to $\overline{AD}$

$= \frac{\text{the area of ▱ ABED}}{AD}$

$= \frac{288}{30} = 9.6$ cm (Second req.)

9

Δ CXY $\equiv$ Δ XCB

$\therefore$ we deduce that the area of Δ XCY = the area of Δ XCB ,

$\because$ the area of Δ XCY $= 15$ cm^2

$\therefore$ the area of ▱ XBCY $= 30$ cm^2

$\because$ ▱ ABCD and ▱ XBCY have the common base $\overline{BC}$, $\overleftrightarrow{AY} \parallel \overline{BC}$

$\therefore$ the area of ▱ ABCD $= 30$ cm^2 (The req.)

10

$\because$ AEBD is a parallelogram

$\therefore \overline{AE} \parallel \overline{DB}$ $\because \overline{EF} \parallel \overline{AB}$

$\therefore$ AEFB is a parallelogram (Q.E.D. 1)

$\because$ ▱ ABCD , ▱ AEBD have the common base $\overline{AD}$, $\overleftrightarrow{AD} \parallel \overleftrightarrow{EC}$

$\therefore$ the area of ▱ ABCD = the area of ▱ AEBD (1)

$\because$ ▱ AEFB and ▱ AEBD have the commen base $\overline{AE}$, $\overleftrightarrow{AE} \parallel \overleftrightarrow{FD}$

$\therefore$ the area of ▱ AEFB = the area of ▱ AEBD (2)

from (1) and (2)

$\therefore$ the area of ▱ ABCD = the area of ▱ AEFB (Q.E.D. 2)

11

∵ ▱ DBFE , ▱ DFCE have the common base $\overline{DE}$, $\overline{DE} // \overline{BC}$

∴ the area of ▱ DBFE = the area of ▱ DFCE adding the area of Δ ADE to both sides we deduce that the area of the figure ABFE = the area of the figure ADFC (Q.E.D.)

12

1 ∵ ▱ AXYD and ▱ ABCD have the common base $\overline{AD}$ and $\overline{AD} // \overleftrightarrow{XC}$

∴ the area of ▱ AXYD = the area of ▱ ABCD (1)

∵ ▱ ABCD and ▱ DEFC have the common base $\overline{DC}$, $\overline{DC} // \overleftrightarrow{AF}$

∴ the area of ▱ ABCD = the area of ▱ DEFC (2)

from (1) and (2)

∴ the area of ▱ AXYD = the area of ▱ ABCD
= the area of ▱ DEFC

(Q.E.D.)

2 ∵ ▱ ADCB and ▱ FDCE have the common base $\overline{DC}$, $\overline{DC} // \overleftrightarrow{AE}$

∴ the area of ▱ ADCB = the area of ▱ FDCE (1)

∵ ▱ FDCE and ▱ XYCE have the common base $\overline{CE}$, $\overline{CE} // \overleftrightarrow{XD}$

∴ the area of ▱ FDCE = the area of ▱ XYCE (2)

from (1) , (2)

∴ the area of ▱ ADCB = the area of ▱ FDCE
= the area of ▱ XYCE

(Q.E.D.)

3 ∵ ▱ DXYF and ▱ DCEF have the common base $\overline{DF}$, $\overline{DF} // \overleftrightarrow{XE}$

∴ the area of ▱ DXYF = the area of ▱ DCEF (1)

∵ ▱ DCEF , ▱ DCBA have the common base $\overline{DC}$, $\overline{DC} // \overleftrightarrow{AE}$

∴ the area of ▱ DCEF = the area of ▱ DCBA (2)

from (1) and (2)

∴ the area of ▱ DXYF = the area of ▱ DCEF
= the area of ▱ DCBA

(Q.E.D.)

13

∵ ▱ ABCD , ▱ ABEC have the common base $\overline{AB}$, $\overline{AB} // \overleftrightarrow{DE}$

∴ the area of ▱ ABCD = the area of ▱ ABEC (1)

∵ ▱ ABCD , ▱ BFCD have the common base $\overline{CD}$, $\overline{CD} // \overleftrightarrow{AF}$

∴ the area of ▱ ABCD = the area of ▱ BFCD (2)

from (1) and (2)

∴ the area of ▱ BFCD = the area of ▱ ABEC

(Q.E.D.)

14

∵ ABCD is a parallelogram ∴ $\overline{AB} // \overline{DC}$

∵ E ∈ $\overleftrightarrow{DC}$ ∴ $\overline{AB} // \overline{NE}$ (1)

∵ EC = DN and adding CN to both sides

∴ EN = DC ∴ EN = AB (2)

from (1) and (2)

∴ the figure ABEN is a parallelogram.

∵ ▱ ABEN and ▱ ABCD have the common base $\overline{AB}$, $\overline{AB} // \overline{DE}$

∴ the area of ▱ ABCD = the area of ▱ ABEN (3)

∵ M ∈ $\overleftrightarrow{AN}$ ∴ $\overline{AM} // \overline{BE}$

∵ ▱ BEMN and ▱ ABEN have the common base $\overline{BE}$, $\overline{BE} // \overline{AM}$

∴ the area of ▱ BEMN = the area of ▱ ABEN (4)

from (3) and (4)

∴ the area of ▱ ABCD = the area of ▱ BEMN

(Q.E.D.)

15

Yes , the area of piece number 15
= the area of piece number 16
and the area of piece number 18
= the area of piece number 17
= the area of piece number 16

Because they are parallelograms included between two parallel straight lines and their bases on these straight lines are equal in length.

∵ the area of piece number 16 = the area of piece number 15

∴ Area of piece number 15 = the area of piece number 16 = the area of piece number 17
= the area of piece number 18 (Q.E.D.)

16

Let $\ell_1 = 5x$ $\therefore h_1 = 3x$

$\therefore$ the area of ▱ ABCD $= \ell_1 h_1 = 5x \times 3x = 15x^2$

$\therefore 15x^2 = 240$ $x^2 = 16$ $\therefore x = 4$

$\therefore \ell_1 = 5x = 20$ cm , $h_1 = 3x = 12$ cm.

$\because \ell_1 : \ell_2 = 4 : 3$ $\therefore \frac{20}{\ell_2} = \frac{4}{3}$

$\therefore \ell_2 = \frac{20 \times 3}{4} = 15$ cm.

$\therefore$ the area of ▱ ABCD = 240 cm^2.

$\therefore \ell_2 \times h_2 = 240$ $\therefore 15 h_2 = 240$

$\therefore h_2 = 16$ cm. (The req.)

17

$\frac{\text{The area of parallelogram ABCD}}{\text{The area of rectangle XBYD}} = \frac{24}{12}$

$\therefore \frac{AB \times XD}{XB \times XD} = 2$

$\therefore AB = 2\,XB$ $\therefore$ X is the midpoint of $\overline{AB}$

$\therefore BX = AX = 2$ cm.

$\because$ The area of the rectangle = 12 cm^2.

$\therefore BX \times DX = 12$ $\therefore 2\,DX = 12$

$\therefore DX = 6$ cm.

$\therefore$ The perimeter of the rectangle XBYD

$= 2\,(DX + BX) = 2\,(6 + 2) = 2 \times 8 = 16$ cm.

Answers of Exercise 2

1

1 b	2 b	3 c	4 a	5 d
6 b	7 c	8 c	9 b	

2

1 $\frac{1}{2}$ 2 40

3

1 6 2 12

4

1 48 2 24

5

The area of Δ ABC = $\frac{1}{2}$ the base length × the corresponding height

$= \frac{1}{2} \times AB \times AC = \frac{1}{2} \times 4 \times 3 = 6$ cm^2. (First req.)

$\because$ Δ ABC which is right-angled at A

$\because (BC)^2 = (AB)^2 + (AC)^2$

$\therefore BC = \sqrt{16 + 9} = \sqrt{25} = 5$ cm.

$\because$ the area of Δ ABC $= \frac{1}{2} \times BC \times AD$

$\therefore 6 = \frac{1}{2} \times 5 \times AD$

$\therefore AD = 2.4$ cm. (Second req.)

6

The area of Δ ABC = $\frac{1}{2}$ the base length × the corresponding height

$\therefore \frac{1}{2} \times AC \times BD = \frac{1}{2} \times 6 \times 5 = 15$ cm^2. (First req.)

$\because$ the area of Δ ABC $= \frac{1}{2} \times BC \times AE$

$\therefore 15 = \frac{1}{2} \times 6.5 \times AE$

$\therefore AE = 4\frac{8}{13}$ cm. (Second req.)

7

$\because$ The area of Δ ABC = $\frac{1}{2}$ × the base length × the corresponding height $= \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 10 \times 8 = 40$ cm^2. (First req.)

$\because$ the area of Δ ABC $= \frac{1}{2} \times AC \times BE$

$\therefore 40 = \frac{1}{2} \times 16 \times BE$ $\therefore BE = 5$ cm. (Second req.)

8

$\because$ Δ ABC has the common base $\overline{BC}$ with the rectangle ABCD , $A \in \overleftrightarrow{AD}$

$\therefore$ the area of Δ ABC = $\frac{1}{2}$ the area of ▱ ABCD (1)

$\because$ Δ DAE has the common base $\overline{AD}$ with the rectangle ABCD , $E \in \overleftrightarrow{BC}$

$\therefore$ the area of Δ DAE = $\frac{1}{2}$ the area of ▱ ABCD (2)

from (1) and (2)

we deduce that the area of Δ DAE = the area of Δ ABC (Q.E.D.)

9

$\because$ Δ EBC has the common base $\overline{BC}$ with the ▱ ABCD , $E \in \overline{AD}$

$\therefore$ the area of Δ EBC = $\frac{1}{2}$ the area of ▱ ABCD

but the area of ▱ ABCD = the area of ▱ ABMN

(have a common base $\overline{AB}$ and between two parallel straight lines $\overleftrightarrow{AB}$, $\overleftrightarrow{CN}$)

$\therefore$ the area of Δ EBC = $\frac{1}{2}$ the area of ▱ ABMN (Q.E.D.)

10

The area of ▱ ABCD = AB × BC = 4 × 10 = 40 cm^2

∵ the area of ▱ ABEF

= the area of ▱ ABCD (they have the common base $\overline{AB}$ and they are between two parallel straight line $\overleftrightarrow{AB}$, $\overleftrightarrow{CF}$)

∴ the area of ▱ ABEF = 40 cm^2 (Frist req.)

∵ Δ XAF has the common base $\overline{AF}$ with ▱ ABEF , X ∈ $\overline{BE}$

∴ the area of Δ XAF = $\frac{1}{2}$ the area of ▱ ABEF = 20 cm^2 (Second req.)

11

∵ ABCD and BECD are two parallelograms having the common base $\overline{DC}$, $\overleftrightarrow{AE}$ // $\overline{DC}$

∴ the area of ▱ ABCD = the area of ▱ BECD (1)

∵ Δ ABD and ▱ ABCD have the common base $\overline{AB}$, D ∈ $\overline{DC}$

∴ the area of Δ ABD = $\frac{1}{2}$ the area of ▱ ABCD (2)

, ∵ Δ MEC and ▱ BECD have the common base $\overline{EC}$, M ∈ $\overline{BD}$

∴ the area of Δ MEC = $\frac{1}{2}$ the area of ▱ BECD (3)

from (1) , (2) and (3)

∴ the area of Δ ABD = the area of Δ MEC (Q.E.D.)

12

∵ The two parallelograms ABCD , EBCF have the common base $\overline{BC}$, $\overleftrightarrow{AF}$ // $\overline{BC}$

∴ the area of ▱ ABCD = the area of ▱ EBCF (1)

∵ Δ ABL , ▱ ABCD have a common base $\overline{AB}$, L ∈ $\overline{DC}$

∴ the area of Δ ABL = $\frac{1}{2}$ the area of ▱ ABCD (2)

∵ Δ FCL , ▱ EBCF have a common base $\overline{CF}$, L ∈ $\overline{BE}$

∴ the area of Δ FCL = $\frac{1}{2}$ the area of ▱ EBCF (3)

from (1) , (2) and (3)

∴ the area of Δ ABL = the area of Δ FCL (Q.E.D. 1)

Adding the area of Δ LBC to both sides

∴ the area of the figure ABCL = the area of the figure FCBL (Q.E.D. 2)

13

∵ ABCD and AEFD are two parallelograms having the common base $\overline{AD}$, $\overleftrightarrow{BF}$ // $\overline{AD}$

∴ the area of ▱ ABCD = the area of ▱ AEFD (1)

∵ Δ ABX has a common base $\overline{AB}$ with ▱ ABCD , X ∈ $\overline{DC}$

∴ the area of Δ ABX = $\frac{1}{2}$ the area of ▱ ABCD (2)

∵ Δ DFX has a common base $\overline{DF}$ with ▱ AEFD, X ∈ $\overline{AE}$

∴ the area of Δ DFX = $\frac{1}{2}$ the area of ▱ AEFD (3)

from (1) , (2) and (3)

∴ the area of Δ ABX = the area of Δ DFX (Q.E.D.)

14

∵ The two parallelograms ABCD and AEFD have the common base $\overline{AD}$, $\overleftrightarrow{BF}$ // $\overline{AD}$

∴ the area of ▱ ABCD = the area of ▱ AEFD (1)

∵ Δ ABM has a common base $\overline{AB}$ with ▱ ABCD , M ∈ $\overline{CD}$

∴ the area of Δ ABM = $\frac{1}{2}$ the area of ▱ ABCD(2)

∵ Δ DMF has a common base $\overline{DF}$ with ▱ AEFD , M ∈ $\overline{AE}$

∴ the area of Δ DMF = $\frac{1}{2}$ the area of ▱ AEFD (3)

from (1) , (2) and (3)

∴ the area of Δ ABM = the area of Δ DMF (Q.E.D.)

15

∵ Δ ACD has a common base $\overline{AC}$ with ▱ ACEF , D ∈ $\overline{FE}$

∴ the area of Δ ACD = $\frac{1}{2}$ the area of ▱ ACEF (1)

∵ Δ ACD has a common base $\overline{AD}$, C ∈ $\overline{BC}$

∴ the area of Δ ACD = $\frac{1}{2}$ the area of ▱ ABCD (2)

from (1) and (2)

∴ the area of ▱ ABCD = the area of ▱ ACEF (Q.E.D.)

16

∵ The two parallelograms EBCF and ABCD have the common base $\overline{BC}$, $\overline{BC}$ // $\overline{ED}$

∴ the area of ▱ EBCF = the area of ▱ ABCD (1)

∵ Δ DXC has a common base $\overline{DC}$ with the ▱ ABCD , X ∈ $\overline{AB}$

∴ the area of Δ DXC = $\frac{1}{2}$ the area of ▱ ABCD

∵ Δ DXC has a common base $\overline{XC}$ with ▱ DXCY , D ∈ $\overline{DY}$

∴ the area of Δ DXC = $\frac{1}{2}$ the area of ▱ DXCY

∴ the area of ▱ ABCD = the area of ▱ DXCY (2)

from (1) and (2)

∴ the area of ▱ EBCF = the area of ▱ ABCD

= the area of ▱ DXCY (Q.E.D.)

17

∵ ABCD is a parallelogram , $\overline{XY}$ // $\overline{AB}$ // $\overline{DC}$

∴ ABYX , XYCD are parallelograms.

1 ∵ Δ XLY has a common base $\overline{XY}$ with ▱ ABYX , L ∈ $\overline{AB}$

∴ the area of Δ XLY = $\frac{1}{2}$ the area of ▱ ABYX (1)

∵ Δ XYC has a common base $\overline{YC}$ with ▱ XYCD , X ∈ $\overline{XD}$

∴ the area of Δ XYC = $\frac{1}{2}$ the area of ▱ XYCD (2)

Adding (1) and (2)

∴ the area of the figure XLYC = $\frac{1}{2}$ the area of ▱ ABCD (Q.E.D.)

2 ∵ Δ XLY has a common base $\overline{XY}$, L ∈ $\overline{AB}$

∴ the area of Δ XLY = $\frac{1}{2}$ the area of ▱ ABYX (1)

∵ Δ XYM has a common base $\overline{XY}$ with ▱ XYCD , M ∈ $\overline{DC}$

∴ the area of Δ XYM = $\frac{1}{2}$ the area of ▱ XYCD (2)

Adding (1) and (2)

∴ the area of the figure XLYM = $\frac{1}{2}$ the area of ▱ ABCD (Q.E.D.)

3 ∵ Δ AXY has a common base $\overline{AX}$ with ▱ ABYX , Y ∈ $\overline{BY}$

∴ the area of Δ AXY = $\frac{1}{2}$ the area of ▱ ABYX (1)

∵ Δ XYL has a common base $\overline{XY}$ with ▱ XYCD , L ∈ $\overleftrightarrow{DC}$

∴ the area of Δ XYL = $\frac{1}{2}$ the area of ▱ XYCD (2)

Adding (1) and (2)

∴ the area of Δ ALX = $\frac{1}{2}$ the area of ▱ ABCD (Q.E.D.)

4 ∵ Δ XYF has a common base $\overline{XY}$ with ▱ ABYX , F ∈ $\overleftrightarrow{AB}$

∴ the area of Δ XYF = $\frac{1}{2}$ the area of ▱ ABYX (1)

∵ Δ EXY has a common base $\overline{XY}$ with ▱ XYCD , E ∈ $\overleftrightarrow{CD}$

∴ the area of Δ EXY = $\frac{1}{2}$ the area of ▱ XYCD (2)

Adding (1) and (2)

∴ the area of Δ FYE = $\frac{1}{2}$ the area of ▱ ABCD (Q.E.D.)

5 ∵ Δ XLY has a common hase $\overline{XY}$ with ▱ ABYX , L ∈ $\overline{AB}$

∴ the area of Δ XLY = $\frac{1}{2}$ the area of ▱ ABYX (1)

∵ Δ XEY has a common base $\overline{XY}$ with ▱ DCYX , E ∈ $\overline{DC}$

∴ the area of Δ XEY = $\frac{1}{2}$ the area of ▱ DCYX (2)

subtracting (2) from (1)

∴ the area of Δ XLE = $\frac{1}{2}$ the area of ▱ ABCD (Q.E.D.)

6 ∵ Δ XEY has a common base $\overline{XY}$ with ▱ ABYX , E ∈ $\overrightarrow{AB}$

∴ the area of Δ XEY = $\frac{1}{2}$ the area of ▱ ABYX (1)

∵ Δ XMY has a common base $\overline{XY}$ with ▱ DCYX , M ∈ $\overleftrightarrow{DC}$

∴ the area of Δ XMY = $\frac{1}{2}$ the area of ▱ DCYX (2)

subtracting (2) from (1)

∴ the area of Δ EMY = $\frac{1}{2}$ the area of ▱ ABCD (Q.E.D.)

18

[a] ∵ ABCD is a parallelogram , $\overline{BD}$ is a diagonal

∴ the area of Δ ABD = the area of Δ BCD (1)

∵ BELF is a parallelogram , $\overline{BL}$ is a diagonal

∴ the area of Δ BEL = the area of Δ BFL (2)

subtracting (2) from (1)

∴ the area of the figure (1) = the area of the figure (2) (Q.E.D.)

[b] ∵ ABCD is a parallelogram , $\overline{BD}$ is a diagonal

∴ the area of Δ ABD = the area of Δ CBD (1)

∵ XBFM is a parallelogram , $\overline{BM}$ is a diagonal

∴ the area of Δ XBM = the area of Δ FBM (2)

∵ EMYD is a parallelogram , $\overline{MD}$ is a diagonal

∴ the area of Δ EMD = the area of Δ YMD (3)

adding (2) , (3) and subtracting the sum from (1)

∴ the area of the figure(1) = the area of the figure (2) (Q.E.D.)

19

∵ Δ LFM has a common base $\overline{LM}$ with ▱ LMNE , F ∈ $\overline{EN}$

∵ the area of Δ LFM = $\frac{1}{2}$ the area of ▱ LMNE

∴ the area of Δ LEF + the area of Δ MFN = $\frac{1}{2}$ the area of ▱ LMNE (1)

∵ Δ LEM has a common base $\overline{LM}$ with ▱ LMNE , E ∈ $\overline{EN}$ the area of Δ LEM = $\frac{1}{2}$ the area of ▱ LMNE (2)

from (1) and (2)

∴ the area of Δ LEF + the area of Δ MFN = the area of Δ LEM (Q.E.D.)

20

∵ Δ AFB has a common base $\overline{AB}$ with ▱ ABCD , F ∈ $\overline{DC}$

∴ the area of Δ AFB = $\frac{1}{2}$ the area of ▱ ABCD

∴ the area of Δ AFD + the area of Δ BFC = $\frac{1}{2}$ the area of ▱ ABCD (1)

∵ Δ BCE has a common base $\overline{BC}$ with ▱ ABCD , E ∈ $\overleftrightarrow{AD}$

∴ the area of Δ BCE = $\frac{1}{2}$ the area of ▱ ABCD (2)

from (1) and (2)

∴ the area of Δ AFD + the area of Δ BFC = the area of Δ BCE

subtracting the area of Δ BFC from both sides

∴ the area of Δ AFD = the area of Δ EFC (Q.E.D.)

21

∵ The two parallelograms DBCF , ACFX have a common base $\overline{CF}$, $\overline{BX}$ // $\overline{CF}$

∴ the area of ▱ DBCF = the area of ▱ ACFX (1)

∵ Δ EBC has a common base $\overline{BC}$ with ▱ DBCF , E ∈ $\overline{DF}$

∴ the area of Δ EBC = $\frac{1}{2}$ the area of ▱ DBCF (2)

∵ Δ EFX has a common base $\overline{XF}$ with ▱ ACFX , E ∈ $\overline{AC}$

∴ the area of Δ EFX = $\frac{1}{2}$ the area of ▱ ACFX (3)

from (1) , (2) , (3)

∴ the area of Δ EBC = the area of Δ EFX (Q.E.D. 1)

∵ the area of ▱ DBCF = 80 cm²

and from (1)

∴ the area of ▱ ACFX = 80 cm²

∴ the area of Δ EFX = $\frac{1}{2}$ the area of ▱ ACFX

∴ the area of Δ AXE + the area of Δ ECF

= $\frac{1}{2}$ the area of ▱ ACFX

= $\frac{1}{2}$ × 80 = 40 cm² (Q.E.D. 2)

22

∵ ABCD is a square whose perimeter = 48 cm.

∴ AB = $\frac{48}{4}$ = 12 cm.

∵ E is the midpoint of $\overline{AB}$

∴ AE = $\frac{1}{2}$ AB = 6 cm.

∵ BC is a height of Δ AEC corresponding to the base $\overline{AE}$

∴ the area of Δ AEC = $\frac{1}{2}$ AE × BC

= $\frac{1}{2}$ × 6 × 12 = 36 cm² (The req.)

23

∵ The perimeter of ▱ ABCD = 48 cm.

∴ AB + BC = $\frac{48}{2}$ = 24 cm.

∵ BC = 2 AB ∴ AB + 2 AB = 24 ∴ 3 AB = 24

∴ AB = 8 cm. , BC = 2 × 8 = 16 cm.

∵ the area of Δ ABC = 56 cm²

∴ $\frac{1}{2}$ AB × the corresponding height of $\overline{AB}$ = 56

∴ $\frac{1}{2}$ × 8 × the corresponding height of $\overline{AB}$ = 56

∴ the corresponding height of $\overline{AB}$ = $\frac{56}{4}$ = 14 cm.

similarly $\frac{1}{2}$ × 16 × corresponding height of $\overline{BC}$ = 56

∴ the corresponding height of $\overline{BC}$ = $\frac{56}{8}$ = 7 cm. (First req.)

∵ BC = 16 cm. ∴ EC = 8 cm.

∴ The area of Δ AEC = $\frac{1}{2}$ × 8 × 7 = 28 cm² (Second req.)

24

∵ Δ EBC has a common base $\overline{BC}$ with ▱ ABCD , E ∈ $\overleftrightarrow{AD}$

∴ the area of Δ EBC = $\frac{1}{2}$ the area of ▱ ABCD (1)

∵ Δ FBC has a common base $\overline{BC}$ with ▱ ABCD , F ∈ $\overleftrightarrow{AD}$

∴ the area of Δ FBC = $\frac{1}{2}$ the area of ▱ ABCD (2)

from (1) and (2)

∴ the area of Δ EBC = the area of Δ FBC

subtracting the area of Δ BXC from both sides

$\therefore$ the area of Δ EBX = the area of Δ FCX (First req.)

$\because$ the area of ▱ ABCD = $12 \times 6 = 72$ cm^2

from (2) $\therefore$ the area of Δ FBC = $\frac{72}{2} = 36$ cm^2

$\because$ the area of Δ XBC = 12 cm^2

by subtracting

$\therefore$ the area of Δ FCX = $36 - 12 = 24$ cm^2 (Second req.)

25

$\because$ Δ FXY has a common base $\overline{XY}$ with ▱ XYZL , F $\in \overleftrightarrow{LZ}$

$\therefore$ the area of Δ FXY = $\frac{1}{2}$ the area of ▱ XYZL (1)

$\because$ Δ EYZ has a common base $\overline{YZ}$ with ▱ XYZL , E $\in \overleftrightarrow{XL}$

$\therefore$ the area of Δ EYZ = $\frac{1}{2}$ the area of ▱ XYZL (2)

From (1) and (2)

$\therefore$ the area of Δ FXY = the area of Δ EYZ

Adding the area of Δ FYE to both sides

$\therefore$ the area of the figure FXYE = the area of the figure FYZE (Q.E.D.)

26

1 The area used for parking cars

= 4 × the area of the parallelogram

= $4 \times 3 \times 3 = 36$ m^2 (First req.)

2 The area used for planting flowers

= 2 × the area of the triangle

= $2 \times \frac{1}{2} \times 3 \times 3 = 9$ m^2 (Second req.)

27

$\because$ Δ ABF has a common base $\overline{AB}$ with ▱ ABCD , F $\in \overline{DC}$

$\therefore$ the area of Δ ABF = $\frac{1}{2}$ the area of ▱ ABCD

$\therefore$ the area of Δ ADF + the area of Δ FBC = $\frac{1}{2}$ the area of ▱ ABCD (1)

$\because$ Δ EBC has a common base $\overleftrightarrow{BC}$ with ▱ ABCD , E $\in \overleftrightarrow{AD}$

$\therefore$ the area of Δ EBC = $\frac{1}{2}$ the area of ▱ ABCD (2)

from (1) and (2)

$\therefore$ the area of Δ ADF + the area of Δ FBC = the area of Δ EBC

Subtracting the area of Δ FBC from both sides

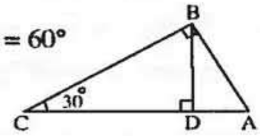
$\therefore$ the area of Δ ADF = the area of Δ EFC

Adding the area of Δ DFE to both sides

$\therefore$ the area of Δ AFE = the area of Δ DCE (Q.E.D.)

28

from Δ BDC :



m ($\angle$ DBC) = $180° - (90° + 30°) = 60°$

$\therefore$ m ($\angle$ ABD) = $90° - 60° = 30°$

from Δ ABD

$\because$ m ($\angle$ ADB) = 90° , m ($\angle$ ABD) = 30°

$\therefore$ AD = $\frac{1}{2}$ AB i.e AB = 2 AD (1)

$\because$ the area of Δ ABC = $\frac{1}{2}$ AB × BC = $\frac{1}{2}$ AC × BD

$\therefore$ BD = $\frac{AB \times BC}{AC}$ substituting from (1)

$\therefore$ BD = $\frac{2\,AD \times BC}{AC}$ (Q.E.D.)

Answers of Exercise 3

1

1 equal in area. 2 equal in area.

3 two triangles equal in surface area.

4 Δ ACD 5 twice 6 $\frac{1}{3}$

2

$\because$ $\overline{AD}$ is a median in Δ ABC

$\therefore$ The area of Δ ABD = the area of Δ ACD (1)

$\because$ $\overline{ED}$ is a median in Δ EBC

$\therefore$ the area of Δ EBD = the area of Δ ECD (2)

subtracting sides of (2) from sides of (1) then , the area of Δ ABE = the area of Δ ACE (Q.E.D.)

3

1 Δ ACB , drawn on one base and their vertices lie on one straight line parallel to this base.

2 Δ DBC, drawn on one base and their vertices lie on one straight line parallel to this base.

3 Δ CBM , the area of Δ DAC = the area of Δ DBC and the area of Δ DAC – the area of Δ DMC = the area of Δ DBC – the area of Δ DMC

4

$\because$ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

$\therefore$ $\overline{DE}$ // $\overline{BC}$

$\because$ $\Delta\Delta$ BDE , CDE have the common base $\overline{DE}$, $\overline{DE}$ // $\overline{BC}$

$\therefore$ the area of Δ BDE = the area of Δ CDE (Q.E.D.)

5

$\because \Delta\Delta$ ACE , ADE have the common base $\overline{AE}$
, $\overline{AE} // \overline{CD}$

$\therefore$ the area of Δ ACE = the area of Δ ADE
Adding the area of Δ ABE to both sides

$\therefore$ the area of Δ ABC = the area of the figure ABED (Q.E.D.)

6

$\because \Delta\Delta$ ADB , ADC have the common base $\overline{AD}$
, $\overline{AD} // \overline{BC}$

$\therefore$ the area of Δ ADB = the area of Δ ADC (1)

$\because$ A D is a median in Δ BDE

$\therefore$ the area of Δ ADE = the area of Δ ADB (2)
from (1) and (2)

$\therefore$ the area of Δ ADC = the area of Δ ADE (Q.E.D.)

7

$\because \Delta\Delta$ AXF , CYF have equal bases in lengths
, $\overline{XY} // \overline{AC}$

$\therefore$ the area of Δ AXF = the area of Δ CYF (1)

$\because \overline{BF}$ is a median in Δ BXY

$\therefore$ the area of Δ BFX = the area of Δ BFY (2)
adding (1) and (2)

$\therefore$ the area of Δ ABF = the area of Δ CBF (Q.E.D.)

8

$\because$ X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AC}$

$\therefore \overline{XY} // \overline{BC}$

$\because \Delta\Delta$ XYE , XYB have the common base $\overline{XY}$
, $\overleftrightarrow{XY} // \overleftrightarrow{BC}$

$\therefore$ the area of Δ XYE = the area of Δ XYB (1)

$\because \overline{XY}$ is a median in Δ ABY

$\therefore$ the area of Δ XYB = the area of Δ AXY (2)
from (1) and (2)

$\therefore$ the area of Δ XYE = the area of Δ AXY (Q.E.D.)

9

$\because \Delta\Delta$ ABC , DBC have the common base $\overline{BC}$
, $\overline{BC} // \overline{AD}$

$\therefore$ the area of Δ ABC = the area of Δ DBC (1)

$\because \overline{ME}$ is a median in Δ MBC

$\therefore$ the area of Δ MCE = the area of Δ MBE (2)
subtracting (2) from (1)

$\therefore$ the area of the figure ABEM
= the area of the figure DMEC (Q.E.D.)

10

$\because \Delta\Delta$ ABC , DBC have the same base $\overline{BC}$
, $\overline{BC} // \overline{AD}$

$\therefore$ the area of Δ ABC = the area of Δ DBC
subtracting the area of Δ MBC from both sides

$\therefore$ the area of Δ AMB = the area of Δ DMC (1)

$\because \Delta\Delta$ MBX , MCY have equal bases in length and on one straight line and they are common in the vertex M

$\therefore$ the area of Δ MBX = the area of Δ MCY (2)
Adding (1) and (2)

$\therefore$ the area of the figure ABXM = the area of the figure DCYM (Q.E.D.)

11

$\because \Delta\Delta$ ABD , ACD have the same base $\overline{AD}$
, $\overline{AD} // \overline{BC}$

$\therefore$ the area of Δ ABD = the area of Δ ACD
subtracting the area of Δ AMD from both sides

$\therefore$ the area of Δ ABM = the area of Δ DCM (1)

$\because \overline{AB}$ is a median in Δ AYM

$\therefore$ the area of Δ ABM = the area of Δ AYB (2)

$\because \overline{DC}$ is a median in Δ DMX

$\therefore$ the area of Δ DMC = the area of Δ DCX (3)
from (1) , (2) and (3)

$\therefore$ the area of Δ AYB = the area of Δ DCX (Q.E.D.)

12

$\because \Delta\Delta$ ADB , ADC have the same base $\overline{AD}$, $\overline{BC} // \overline{AD}$

$\therefore$ the area of Δ ADB = the area of Δ ADC
subtracting the area of Δ AMD from both sides

$\therefore$ the area of Δ AMB = the area of Δ DMC (1)

$\because \overline{MD}$ is a median in Δ EMC

$\therefore$ the area of Δ MDE = the area of Δ DMC (2)
from (1) and (2)

$\therefore$ the area of Δ MDE = the area of Δ AMB (Q.E.D.)

13

∵ Δ Δ ABD , ACD have the common base $\overline{AD}$, $\overline{AD}$ // $\overline{BC}$

∴ the area of Δ ABD = the area of Δ ACD

subtracting the area of Δ AMD from both sides

∴ the area of Δ ABM = the area of Δ DMC (1)

∵ $\overline{ME}$ is a median in Δ ABM

∴ the area of Δ AEM = $\frac{1}{2}$ the area of Δ ABM (2)

∵ $\overline{DN}$ is a median in Δ DMC

∴ the area of Δ DNC = $\frac{1}{2}$ the area of Δ DMC (3)

from (1) , (2) and (3)

∴ the area of Δ AEM = the area of Δ DNC (Q.E.D.)

14

∵ Δ Δ ABE , DCE have equal bases in length , $\overline{AD}$ // $\overline{BC}$

∴ the area of Δ ABE = the area of Δ DCE

Adding the area of Δ EBC to both sides

∴ the area of the figure ABCE

= the area of the figure DEBC (Q.E.D.)

15

∵ Δ ABC has a common base $\overline{BC}$ with ▱ ABCD

∴ the area of Δ ABC = $\frac{1}{2}$ the area of ▱ ABCD (1)

∵ $\overline{AE}$ is a median in Δ ABC

∴ the area of Δ ABE = $\frac{1}{2}$ the area of Δ ABC (2)

from (1) and (2)

∴ the area of Δ ABE = $\frac{1}{2} \times \frac{1}{2}$ the area of ▱ ABCD

= $\frac{1}{4}$ the area of ▱ ABCD

16

∵ Δ BCE has the common base $\overline{BC}$ with ▱ ABCD , E ∈ $\overline{AD}$

∴ the area of Δ BCE = $\frac{1}{2}$ the area of ▱ ABCD

∴ the area of Δ ABE + the area of Δ DEC

= $\frac{1}{2}$ the area of ▱ ABCD = $\frac{1}{2} \times 48 = 24$ cm²

∵ Δ Δ ABE , DEC have equal bases in length , $\overline{BC}$ // $\overline{AD}$

∴ the area of Δ ABE = the area of Δ DEC

= $\frac{1}{2} \times 24 = 12$ cm² (Q.E.D.)

17

∵ Δ XFN has a common base $\overline{FN}$ with the square DEFN , X ∈ $\overline{ED}$

∴ the area of Δ XFN = $\frac{1}{2}$ the area of the square DEFN

= $\frac{1}{2} \times 12 \times 12 = 72$ cm²

∵ $\overline{YF}$ is a median in Δ XFN

∴ the area of Δ XYF = $\frac{1}{2}$ the area of Δ XFN

= $\frac{1}{2} \times 72 = 36$ cm²

18

∵ $\overline{DE}$ is a median in Δ ADC

∴ the area of Δ DEC = $\frac{1}{2}$ the area of Δ ADC

∴ the area of Δ ADC = 2 × 5 = 10 cm²

∵ $\overline{AD}$ is a median in Δ ABC

∴ the area of Δ ADC = $\frac{1}{2}$ the area of Δ ABC

∴ the area of Δ ABC = 2 × 10 = 20 cm² (Q.E.D.)

19

∵ $\overline{BE}$ is a median in Δ ABC

∴ The area of Δ ABE = the area of Δ CBE (1)

, ∵ $\overline{DE}$ is a median in Δ ADC

∴ The area of Δ ADE = the area of Δ CDE (2)

adding (1) and (2)

∴ The area of Δ ABD = the area of Δ CBD

∴ The area of Δ BCD = $\frac{1}{2} \times 8 \times 6 = 24$ cm²

∴ The area of the figure ABCD = 24 + 24 = 48 cm²

(Q.E.D.)

20

∵ Δ Δ AXM , DYM have equal bases in length and on one straight line and they have the same vertex M

∴ the area of Δ AXM = the area of Δ DYM (1)

∵ Δ Δ ABC , DCB have the same base $\overline{BC}$, $\overline{BC}$ // $\overline{AD}$

∴ the area of Δ ABC = the area of Δ DCB

subtracting the area of Δ BMC from both sides

∴ the area of Δ ABM = the area of Δ DCM (2)

Adding (1) and (2)

∴ the area of the figure ABMX = the area of the figure DCMY (Q.E.D.)

21

∵ Δ BFC , ▱ ABCD have the common base $\overline{BC}$ and they are included between two parallel straight lines one of them carries this base

∴ the area of Δ BFC = $\frac{1}{2}$ the area of ▱ ABCD (1)

∵ $\overline{BF}$ is a median in Δ FEC

∴ the area of Δ BFC = $\frac{1}{2}$ the area of Δ FEC (2)

from (1) and (2)

∴ the area of Δ FEC = the area of ▱ ABCD (Q.E.D.)

22

∵ Δ Δ ADB , ADC have the same base $\overline{AD}$, $\overline{BC}$ // $\overline{AD}$

∴ the area of Δ ADB = the area of Δ ADC

subtracting the area of Δ AFD from both sides

∴ the area of Δ AFB = the area of Δ DFC (1)

∵ $\overline{BF}$ is a median in Δ ABE

∴ the area of Δ AFB = the area of Δ BFC (2)

from (1) and (2)

∴ the area of Δ DFC = the area of Δ BFE (Q.E.D.)

23

∵ Δ Δ XLY , XLZ have the same base $\overline{XL}$, $\overline{YZ}$ // $\overline{XL}$

∴ the area of Δ XLY = the area of Δ XLZ

subtracting the area of Δ XLM from both sides

∴ the area of Δ XMY = the area of Δ LMZ (1)

∵ $\overline{LD}$ is a median in Δ LMZ

∴ the area of Δ LMD = $\frac{1}{2}$ the area of Δ LMZ (2)

from (1) and (2)

∴ the area of Δ LMD = $\frac{1}{2}$ the area of Δ XMY (Q.E.D.)

24

∵ Δ Δ AYB , AZC have equal bases in length and on one straight line and they have the common vertex A

∴ the area of Δ AYB = the area of Δ AZC (1)

∵ Δ Δ AZC , AXC have the common base $\overline{AC}$, $\overline{ZX}$ // $\overline{AC}$

∴ the area of Δ AZC = the area of Δ AXC (2)

from (1) and (2)

∴ the area of Δ AYB = the area of Δ AXC (Q.E.D.)

25

∵ Δ Δ ADB , ADC have the common base $\overline{AD}$, $\overline{BC}$ // $\overline{AD}$

∴ the area of Δ ADB = the area of Δ ADC

subtracting the area of Δ ADE from both sides

∴ the area of Δ ABE = the area of Δ DEC (1)

∵ Δ Δ DEC , ECF have the common base $\overline{EC}$, $\overline{DF}$ // $\overline{EC}$

∴ the area of Δ DEC = the area of Δ ECF (2)

from (1) and (2)

∴ the area of Δ ABE = the area of Δ ECF (Q.E.D.)

26

∵ Δ Δ ABD , ABC have the same base $\overline{AB}$, $\overline{DC}$ // $\overline{AB}$

∴ the area of Δ ABD = the area of Δ ABC (1)

∵ Δ Δ ABC , AXC have the same base $\overline{AC}$, $\overline{AC}$ // $\overline{BX}$

∴ the area of Δ ABC = the area of Δ AXC (2)

from (1) and (2)

∴ the area of Δ ABD = the area of Δ AXC (Q.E.D.)

27

∵ ABCD is a parallelogram , $\overline{AC}$ is a diagonal in it

∴ the area of Δ ACD = the area of Δ ABC (1)

∵ Δ Δ ABC , EBD have equal bases in length and on one straight line , $\overline{CE}$ // $\overline{AD}$

∴ the area of Δ ABC = the area of Δ EBD (2)

from (1) and (2)

∴ the area of Δ EBD = the area of Δ ACD (Q.E.D.)

28

∵ Δ Δ ABC , DBC have the common base $\overline{BC}$, $\overline{BC}$ // $\overline{AD}$

∴ the area of Δ ABC = the area of Δ DBC

subtracting the area of Δ BMC from both sides

∴ the area of Δ ABM = the area of Δ DMC (1)

∵ Δ Δ DMC , EMC have the common base $\overline{MC}$, $\overline{MC}$ // $\overline{DE}$

∴ the area of Δ DMC = the area of Δ EMC (2)

from (1) and (2)

$\therefore$ the area of Δ ABM = the area of Δ DMC = the area of Δ EMC (First req.)

$\because$ the area of Δ DMC = the area of Δ EMC

Adding the area of Δ BMC to both sides

$\therefore$ the area of Δ DBC = the area of Δ EBM (Second req.)

29

$\because$ BE = FC and adding EF to both sides

$\therefore$ BF = EC

$\because$ $\Delta\Delta$ ABF , DEC have equal bases in length and on one straight line , $\overline{AD} \parallel \overline{BC}$

$\therefore$ the area of Δ ABF = the area of Δ DEC

$\therefore$ the area of the figure ABCD – the area of Δ ABF = the area of the figure ABCD – the area of Δ DEC

$\therefore$ the area of the figure AFCD = the area of the figure ABED

30

$\because$ $\Delta\Delta$ AXY , DXY have the common base $\overline{XY}$, $\overline{AD} \parallel \overline{XY}$

$\therefore$ the area of Δ AXY = the area of Δ DXY (1)

$\because$ $\Delta\Delta$ XYB , XYC have the common base $\overline{XY}$, $\overline{BC} \parallel \overline{XY}$

$\therefore$ the area of Δ XYB = the area of Δ XYC (2)

Adding (1) and (2)

$\therefore$ the area of Δ YAB = the area of Δ XDC (Q.E.D.)

31

$\because$ $\Delta\Delta$ ABC , DBC have the common base $\overline{BC}$, $\overline{BC} \parallel \overline{AD}$

$\therefore$ the area of Δ ABC = the area of Δ DBC

subtracting the area of Δ BMC from both sides

$\therefore$ the area of Δ ABM = the area of Δ DMC (First req.)

$\because$ the area of Δ ABM = 3 times the area of Δ BMC

$\therefore$ the area of Δ ABM = $3 \times 20 = 60$ cm^2

$\therefore$ the area of Δ ABC = $60 + 20 = 80$ cm^2

$\because$ Δ ABC has a common base $\overline{BC}$ with the rectangle drawn on $\overline{BC}$ and another base on $\overline{AD}$, A $\in \overleftrightarrow{AD}$

$\therefore$ the area of Δ ABC = $\frac{1}{2}$ the area of the rectangle

$\therefore$ the area of the required rectangle = $2 \times 80 = 160$ cm^2 (Second req.)

32

$\because$ Δ DEC has the common base $\overline{CD}$ with ▱ ABCD , E $\in \overleftrightarrow{AB}$

$\therefore$ the area of Δ DEC = $\frac{1}{2}$ the area of ▱ ABCD (1)

$\because$ $\overline{CF}$ is a median in Δ DEC

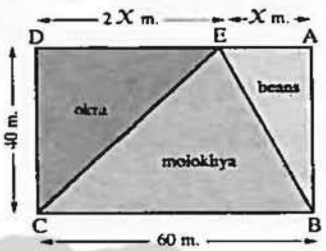
$\therefore$ the area of Δ CFD = $\frac{1}{2}$ the area of Δ DEC

substituting in (1)

$\therefore$ the area of Δ CFD = $\frac{1}{4}$ the area of ▱ ABCD

$\therefore$ the area of Δ CFD : the area of ▱ ABCD = 1 : 4 (Q.E.D.)

33



$\because$ The area of the rectangle ABCD $= 60 \times 40 = 2400$ m^2

, in Δ BCE and the rectangle ABCD :

$\because$ E $\in \overline{AD}$

$\therefore$ The area of Δ BCE = $\frac{1}{2}$ the area of the rectangle ABCD

$\therefore$ The area of Δ BCE = $\frac{2400}{2} = 1200$ m^2

$\therefore$ The planted area with molokhya = 1200 m^2

$\because$ AE = x m. , ED = $2x$ m. $\therefore$ AE = $\frac{1}{2}$ ED

, $\because$ $\Delta\Delta$ ABE , ECD have bases on one straight line , $\overline{AD} \parallel \overline{BC}$

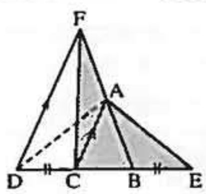
$\therefore$ The area of Δ ABE = $\frac{1}{2}$ the area of Δ ECD

, $\because$ The area of Δ ABE + the area of Δ ECD $= \frac{2400}{2} = 1200$ m^2

$\therefore$ The area of Δ ABE = the planted area with beans = 400 m^2

, the area of Δ ECD = the planted area with okra = 800 m^2 (The req).

34



Construction : Draw $\overline{AD}$

Proof :

$\because$ $\Delta\Delta$ FAC , DAC have the common base $\overline{AC}$, $\overline{FD} \parallel \overline{AC}$

$\therefore$ the area of Δ FAC = the area of Δ DAC (1)

∵ Δ Δ AEB , ADC have equal bases in length and on one straight line and they have the same vertex A

∴ the area of Δ AEB = the area of Δ ADC (2)

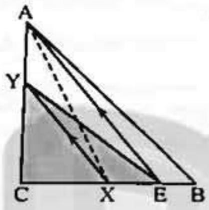
from (1) and (2)

∴ the area of Δ FAC = the area of Δ AEB

Adding the area of Δ ABC to both sides

∴ the area of Δ FBC = the area of Δ AEC (Q.E.D.)

35



Construation : Draw $\overline{AX}$

Proof :

∵ $\overline{AX}$ is a median in Δ ABC

∴ the area of Δ AXC

$= \frac{1}{2}$ the area of Δ ABC

∴ the area of Δ AXY + the area of Δ YXC = $\frac{1}{2}$ the area of Δ ABC (1)

∵ Δ Δ AXY , EXY have the common base $\overline{XY}$, $\overline{EA}$ // $\overline{XY}$

∴ the area of Δ AXY = the area of Δ EXY

substituting in (1)

∴ the area of Δ EXY + the area of Δ YXC = $\frac{1}{2}$ the area of Δ ABC

∴ the area of Δ EYC = $\frac{1}{2}$ the area of Δ ABC (Q.E.D.)

Answers of Exercise 4

1

∵ The area of Δ ABM = the area of Δ DMC

Adding the area of Δ BMC to both sides.

∴ the area of Δ ABC = the area of Δ DBC but they have the common base $\overline{BC}$ and on one side of it.

∴ $\overline{AD}$ // $\overline{BC}$ (Q.E.D.)

2

∵ The area of the figure ABCD = the area of the figure ABCE

and subtracting the area of Δ ABC from both sides.

∴ the area of Δ ADC = the area of Δ AEC but they have the common base $\overline{AC}$ and on one side of it.

∴ $\overline{DE}$ // $\overline{AC}$ (Q.E.D.)

3

∵ The area of Δ ABE = the area of Δ ACD and subtracting the area of Δ ADE from both sides.

∴ the area of Δ DEB = the area of Δ DEC

but they have the common base $\overline{DE}$ and on one side of it.

∴ $\overline{DE}$ // $\overline{BC}$ (Q.E.D.)

4

∵ AB = AC

∴ m (∠ ABC) = m (∠ ACB)

∴ Δ Δ EBC and DCB in them

{ m (∠ BEC) = m (∠ CDB) = 90°
m (∠ EBC) = m (∠ DCB)
$\overline{BC}$ is a common side.

∵ Δ EBC ≡ Δ DCB

∴ the area of Δ EBC = the area of Δ DCB

but they have the common base $\overline{BC}$ and on one side of it.

∴ $\overline{ED}$ // $\overline{BC}$ (Q.E.D. 1)

∴ Δ DBE and Δ ECD have the common base $\overline{ED}$, $\overline{ED}$ // $\overline{BC}$

∴ the area of Δ DBE = the area of Δ ECD

Then adding the area of Δ ADE to both sides.

∴ the area of Δ ADB = the area of Δ AEC (Q.E.D. 2)

5

∵ $\overline{MC}$ is a median in Δ DEC

∴ the area of Δ CME = the area of Δ CMD

but the area of Δ CME = the area of Δ AMB

∴ the area of Δ AMB = the area of Δ CMD

Adding the area of Δ AMD to both sides.

∴ the area of Δ ABD = the area of Δ ACD and they have the common base $\overline{AD}$ and on one side of it.

∴ $\overline{AD}$ // $\overline{BC}$ (Q.E.D.)

6

∵ Δ Δ ADB , ADC have the common base $\overline{AD}$, $\overline{AD}$ // $\overline{BC}$

∴ the area of Δ ADB = the area of Δ ADC

subtracting the area of Δ ADM from both sides.

∴ the area of Δ ABM = the area of Δ DCM

but the area of Δ ABM = the area of Δ ECM

∴ the area of Δ ECM = the area of Δ DCM and they have the common base $\overline{MC}$ and on one side of it.

∴ $\overline{DE}$ // $\overline{AC}$ (Q.E.D.)

7

$\because$ ABEC is a parallelogram, $\overline{BC}$ is a diagonal of it.

$\therefore$ the area of Δ ABC = the area of Δ BEC

but the area of Δ BEC = the area of DBC

$\therefore$ the area of Δ ABC = the area of Δ DBC and they have the common base $\overline{BC}$ and on one side of it.

$\therefore$ $\overline{AD}$ // $\overline{BC}$ (Q.E.D.)

8

$\because$ The area of the rectangle ABCD = 12 × 9 = 108 cm²

$\because$ the area of Δ ADC = $\frac{1}{2}$ the area of the rectangle ABCD

$\therefore$ the area of Δ ADC = $\frac{108}{2}$ = 54 cm²

$\therefore$ the area of Δ ADC = the area of Δ XAC = 54 cm²

but they have the common base $\overline{AC}$ and on one side of it.

$\therefore$ $\overline{XD}$ // $\overline{AC}$ (Q.E.D.)

9

$\because$ The area of Δ ABC = the area of Δ AME

and subtracting the area of Δ ABM from both sides.

$\therefore$ the area of Δ BMC = the area of Δ BME and they have the common base $\overline{BM}$ and on one side of it.

$\therefore$ $\overleftrightarrow{EC}$ // $\overleftrightarrow{BM}$ $\therefore$ $\overline{BD}$ // $\overline{EC}$ (1)

$\because$ ABCD is a parallelogram.

$\therefore$ $\overline{AB}$ // $\overline{DC}$ $\therefore$ $\overline{BE}$ // $\overline{DC}$ (2)

$\therefore$ The figure DBEC is a parallelogram. (Q.E.D.)

10

$\because$ Δ EBC has a common base $\overline{BC}$ with ▱ ABCD, E $\in$ $\overline{AD}$

$\therefore$ the area of Δ BEC = the area of Δ ABE + the area of Δ EDC = $\frac{1}{2}$ the area of ▱ ABCD

$\because$ the area of Δ FCE = the area of Δ AEB + the area of Δ ECD

$\therefore$ the area of Δ BEC = the area of Δ FCE and they have a common base $\overline{EC}$ and on one side of it.

$\therefore$ $\overline{BF}$ // $\overline{EC}$ (Q.E.D.)

11

$\because$ $\Delta\Delta$ ABC, BCD have the common base $\overline{BC}$, $\overline{BC}$ // $\overline{AD}$

$\therefore$ the area of Δ ABC = the area of Δ BCD

Subtracting the area of Δ BMC from both sides.

$\therefore$ the area of Δ ABM = the area of Δ DMC, but the area of Δ DMC = the area of Δ ABX

$\therefore$ the area of Δ ABM = the area of Δ ABX but they have the common base $\overline{AB}$ and on one side of it.

$\therefore$ $\overline{MX}$ // $\overline{AB}$ (Q.E.D.)

12

$\because$ $\Delta\Delta$ AXM, DYM have the equal bases in length and on the same straight line and they have a common vertex M

$\therefore$ the area of Δ AXM = the area of Δ DYM (1)

$\because$ the area of Δ ABM = the area of Δ DCM (2)

Subtracting (1) from (2) :

$\because$ the area of Δ AXB = the area of Δ DYC but their bases are equal in length and on the same straight line and the two triangles are on one side of this straight line.

$\therefore$ $\overline{AD}$ // $\overline{BC}$ (Q.E.D.)

13

$\because$ The area of Δ ABD = the area of Δ ACD and they have the common base $\overline{AD}$ and on one side of it.

$\therefore$ $\overline{BC}$ // $\overline{AD}$ $\because$ E $\in$ $\overline{BC}$ $\therefore$ $\overline{EC}$ // $\overline{AD}$

$\because$ $\Delta\Delta$ AED, ACD have the common base $\overline{AD}$

$\therefore$ the area of Δ AED = the area of Δ ACD (Q.E.D.)

14

$\because$ The area of Δ ABE = the area of Δ DEC and their bases are equal in length and on the same straight line and the two triangles are on one side of the common base.

$\therefore$ $\overline{AD}$ // $\overline{BC}$

$\therefore$ the area of Δ ABC = the area of Δ DBC, they have the common base $\overline{BC}$, $\overline{AD}$ // $\overline{BC}$

Subtracting the area of Δ MBC from the two sides.

$\therefore$ the area of Δ AMB = the area of Δ DMC (Q.E.D.)

15

$\because$ $\Delta\Delta$ ABD, ACD have the common base $\overline{AD}$, $\overline{BC}$ // $\overline{AD}$

$\therefore$ the area of Δ ABD = the area of Δ ACD (1)

$\because$ the area of Δ ABE = the area of Δ DFC (2)

Subtracting (2) from (1) :

$\because$ the area of Δ AED = the area of Δ AFD but they have the common base $\overline{AD}$ and on one side of it.

$\therefore$ $\overline{AD}$ // $\overline{EF}$ $\because$ $\overline{BC}$ // $\overline{AD}$

$\therefore$ $\overline{EF}$ // $\overline{BC}$ (Q.E.D.)

16

∴ Δ Δ ABD , ACD have the common base $\overline{AD}$, $\overline{BC}$ // $\overline{AD}$
∴ the area of Δ ABD = the area of Δ ACD
Subtracting the area of Δ ADE from both sides
∴ the area of Δ AEB = the area of Δ DEC (Q.E.D. 1)
∴ the area of Δ AFC = the area of Δ DEC
∴ the area of Δ AEB = the area of Δ AFC
Subtracting the area of Δ AFE from both sides.
∴ the area of Δ FEB = the area of Δ FEC
but they have the common base $\overline{FE}$ and on one side of it.
∴ $\overline{BC}$ // $\overline{FE}$ ∴ $\overline{AD}$ // $\overline{BC}$
∴ $\overline{FE}$ // $\overline{AD}$ (Q.E.D. 2)

17

∴ Δ Δ ABC , DBC have the common base $\overline{BC}$, $\overline{AD}$ // $\overline{BC}$
∴ the area of Δ ABC = the area of Δ DBC
∴ $\frac{1}{2}$ the area of Δ ABC = $\frac{1}{2}$ the area of Δ DBC
∴ the area of Δ XBC = the area of Δ YBC
because $\overline{CX}$ is a median in Δ DBC ,
$\overline{BY}$ is a median in Δ ABC
and they have the common base $\overline{BC}$ and on one side of it.
∴ $\overline{XY}$ // $\overline{BC}$ (Q.E.D.)

18

∴ Δ Δ ADB , ADC have the common base $\overline{AD}$, $\overline{BC}$ // $\overline{AD}$
∴ the area of Δ ADB = the area of Δ ADC
∴ $\frac{1}{2}$ the area of Δ ADB = $\frac{1}{2}$ the area of Δ ADC
∴ the area of Δ ADE = the area of Δ ADF
because $\overline{AE}$ is a median in Δ ADB ,
$\overline{DF}$ is a median in Δ ADC
and they have the common base $\overline{AD}$ and on one side of it.
∴ $\overline{AD}$ // $\overline{EF}$ ∵ $\overline{AD}$ // $\overline{BC}$
∴ $\overline{EF}$ // $\overline{BC}$ (Q.E.D.)

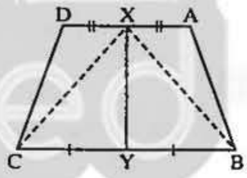
19

∴ $\overline{YX}$ is a median in Δ AYD
∴ the area of Δ XYD = $\frac{1}{2}$ the area of Δ AYD
∴ $\overline{DY}$ is a median in Δ DBC
∴ the area of Δ BYD = $\frac{1}{2}$ the area of Δ DBC
∵ the area of Δ AYD = the area of Δ DBC
∴ the area of Δ XYD = the area of Δ BYD
and they have the common base $\overline{YD}$ and on one side of it.
∴ $\overline{BX}$ // $\overline{DY}$ (Q.E.D.)

20

∴ The area of Δ XAD = the area of Δ YAD (1)
and they have the common base $\overline{AD}$ and on one side of it.
∴ $\overline{XY}$ // $\overline{AD}$ (2)
∵ $\overline{DX}$ is a median in Δ ABD
∴ the area of Δ ABD = 2 the area of Δ XAD (3)
∵ $\overline{AY}$ is a median in Δ ACD
∴ the area of Δ ACD = 2 the area of Δ YAD (4)
From (1), (3) and (4) :
∴ the area of Δ ABD = the area of Δ ACD
and they have the common base $\overline{AD}$ and on one side of it.
∴ $\overline{BC}$ // $\overline{AD}$ (5)
From (2) and (5) :
∴ $\overline{AD}$ // $\overline{BC}$ // $\overline{XY}$ (Q.E.D.)

21



Construction :
Draw $\overline{BX}$, $\overline{CX}$
Proof :
∵ $\overline{XY}$ is a median in Δ BXC
∴ the area of Δ BXY = the area of Δ CXY (1)
∴ the area of the figure ABYX = the area of the figure DCYX (2)
Subtracting (1) from (2) :
∴ the area of Δ ABX = the area of Δ DCX and the lengths of their bases are equal and on the same straight line.
and the two triangles are on the same side of the straight line.
∴ $\overline{AD}$ // $\overline{BC}$ (Q.E.D.)

22

∴ Δ Δ AMD , DMC, their bases $\overline{AM}$ and $\overline{MC}$ are on one straight line and they have a common vertex D, AM = $\frac{1}{2}$ MC

∴ the area of Δ ADM = $\frac{1}{2}$ the area of Δ DMC (1)

∵ Δ Δ ADM , AMB , their bases $\overline{DM}$ and $\overline{MB}$ are on one straight line and they have the common vertex A , DM = $\frac{1}{2}$ MB

∴ the area of Δ ADM = $\frac{1}{2}$ the area of Δ AMB (2)

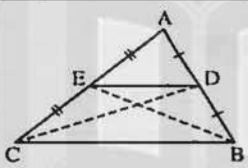
From (1) and (2) :

∴ the area of Δ DMC = the area of Δ AMB and adding the area of Δ AMD to both sides.

∴ the area of Δ ADB = the area of Δ ADC and they have one common base $\overline{AD}$ and on one side of it.

∴ $\overline{BC}$ // $\overline{AD}$ (Q.E.D.)

23



Construction : Draw $\overline{DC}$, $\overline{BE}$

Proof :

∵ $\overline{BE}$ is a median in Δ ABC

∴ the area of Δ EBC = $\frac{1}{2}$ the area of Δ ABC

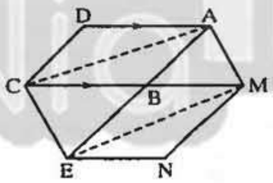
∴ $\overline{CD}$ is a median in Δ ABC

∴ the area of Δ DBC = $\frac{1}{2}$ the area of Δ ABC

∴ the area of Δ EBC = the area of Δ DBC but they have the common base $\overline{BC}$ and on one side of it.

∴ $\overline{BC}$ // $\overline{DE}$ (Q.E.D.)

24



Construction : Draw $\overline{AC}$, $\overline{ME}$

Proof :

∴ ABCD is a parallelogram and $\overline{AC}$ is a diagonal in it.

∴ the area of Δ ABC = $\frac{1}{2}$ the area of ▱ ABCD (1)

∵ BMNE is a parallelogram , $\overline{ME}$ is a diagonal in it.

∴ the area of Δ MBE = $\frac{1}{2}$ the area of ▱ BMNE (2)

From (1) and (2) :

∵ the area of ▱ ABCD = the area of ▱ BMNE

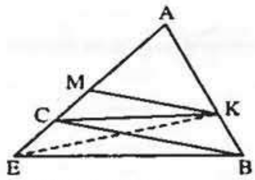
∴ the area of Δ ABC = the area of Δ MBE

Adding the area of Δ BEC to both sides

∴ the area of Δ MEC = the area of Δ ACE but they have the common base $\overline{EC}$ and on one side of it.

∴ $\overline{AM}$ // $\overline{CE}$ (Q.E.D.)

25



Construction : Draw $\overline{KE}$

Proof : In Δ AKE

∵ M is the midpoint of $\overline{AE}$

∴ the area of Δ AKE = 2 the area of Δ AKM

∵ the area of Δ ABC = 2 the area of Δ AKM

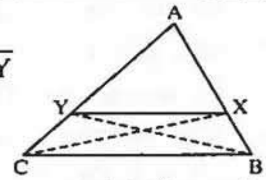
∴ the area of Δ AKE = the area of Δ ABC

Subtracting the area of Δ AKC from both sides.

∴ the area of Δ KCE = the area of Δ KCB but they have the common base $\overline{KC}$ and on one side of it.

∴ $\overline{KC}$ // $\overline{BE}$ (Q.E.D.)

26



Construction : Draw $\overline{CX}$, $\overline{BY}$

Proof :

∵ Δ Δ XBC , ABC , their bases $\overline{XB}$, $\overline{AB}$ are on one straight line and they have a common vertex C , XB = $\frac{1}{3}$ AB

∴ the area of Δ XBC = $\frac{1}{3}$ the area of Δ ABC

Similarly we can prove that :

the area of Δ YBC = $\frac{1}{3}$ the area of Δ ABC

∴ the area of Δ XBC = the area of Δ YBC but they have the common base $\overline{BC}$ and on one side of it.

∴ $\overline{XY}$ // $\overline{BC}$ (Q.E.D.)

Answers of Exercise 5

1

1. its height , the lengths of its diagonals
2. its side , the square of the length of its diagonal
3. $\frac{1}{2}$ of the sum of lengths of its two parallel bases
4. its height, the middle base.
5. Congruent (equal in measure)
6. Congruent (equal in length)

2

1. The area = 6 × 5 = 30 cm^2
2. The area = 12 × 8 = 96 cm^2
3. The area = $\frac{1}{2}$ × 8 × 10 = 40 cm^2
4. The area = $\frac{1}{2}$ × 24 × 10 = 120 cm^2
5. The area = $\frac{1}{2}$ × 10 × 10 = 50 cm^2
6. The area = $\frac{1}{2}$ × 8 × 8 = 32 cm^2

7 The area $= \frac{(6+8)}{2} \times 12 = 84$ cm^2.

8 The area $= \frac{(8+10)}{2} \times 5 = 45$ cm^2.

9 The area $= 7 \times 6 = 42$ cm^2.

10 The area $= 12 \times 8 = 96$ cm^2.

3

1 a	2 c	3 a	4 b	5 b
6 c	7 b	8 b	9 c	10 b

4

In Δ AED : $\because$ m ($\angle$ E) = 90°

$\therefore (AD)^2 = (AE)^2 + (ED)^2 = 16 + 9 = 25$

$\therefore AD = \sqrt{25} = 5$ cm.

$\therefore$ The area of shaded part

= The area of the square – the area of the triangle

$= 5^2 - \frac{1}{2} \times 4 \times 3 = 19$ cm^2.

5

The area of the rectangle $= 2 \times 9 = 18$ cm^2.

The area of the square = 18 cm^2.

$\therefore \frac{1}{2}$ (the length of the diagonal)$^2 = 18$

$\therefore$ (the length of the diagonal)$^2 = 36$

$\therefore$ the length of the diagonal = 6 cm.

6

$\because$ The area of the rhombus $= \frac{1}{2} \times 8 \times 16 = 64$ m^2.

$\therefore$ The area of the square = 64 m^2.

$\therefore$ The side length of the square = 8 m.

$\therefore$ The perimeter of the square $= 8 \times 4 = 32$ m.

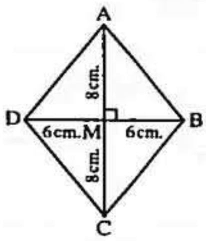
7

$\because$ The area of the rhombus $= \frac{1}{2} \times 18 \times 24 = 216$ m^2.

$\therefore$ The area of the trapezium = 216 m^2.

$\therefore$ The length of the middle base $= \frac{216}{12} = 18$ m.

8



From the figure :

$(AB)^2 = (6)^2 + (8)^2 = 100$

$\therefore$ AB = 10 cm.

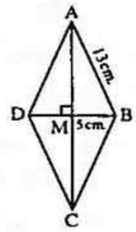
$\therefore$ The side length = 10 cm.

$\therefore$ The area of the rhombus

$= \frac{1}{2} \times 12 \times 16 = 96$ cm^2.

$\therefore$ The height $= \frac{96}{10} = 9.6$ cm.

9



$\because$ The perimeter of the rhombus = 52 cm.

$\therefore$ The side length of the rhombus $= \frac{52}{4} = 13$ cm.

Drawing the rhombus as shown in the figure such that BD = 10 cm.

$\therefore$ BM = 5 cm.

$\because \overline{AC} \perp \overline{BD}$

$\therefore (AM)^2 = (AB)^2 - (BM)^2$

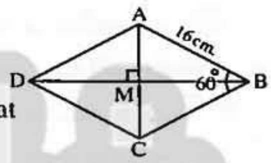
$= 169 - 25 = 144$

$\therefore$ AM = 12 cm.

$\therefore$ AC = 24 cm.

The area of the rhombus $= \frac{1}{2} \times 10 \times 24 = 120$ cm^2.

10



$\because$ The perimeter of the rhombus = 64 cm.

$\therefore$ The side length of the rhombus $= \frac{64}{4} = 16$ cm.

Drawing the rhombus as shown in the figure such that AB = 16 cm.

, m ($\angle$ B) = 60°

$\because$ The diagonal of the rhombus bisects the two angles joinning their vertices.

$\therefore$ m ($\angle$ ABM) = 30°

$\because$ The diagonals of the rhombus are perpendicular

$\therefore$ m ($\angle$ AMB) = 90°

$\therefore$ In the right-angled triangle, the length of the side opposite to the angle of measure 30° $= \frac{1}{2}$ the length of the hypotenuse.

$\therefore AM = \frac{1}{2} \times 16 = 8$ cm.

$\therefore AC = 2 \times 8 = 16$ cm.

$\because$ m ($\angle$ AMB) = 90°

$\therefore (BM)^2 = (AB)^2 - (AM)^2$

$\therefore (BM)^2 = 256 - 64 = 192$

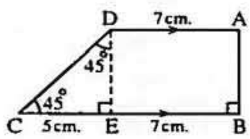
$\therefore BM = \sqrt{192} = 8\sqrt{3}$

$\therefore BD = 2 \times 8\sqrt{3} = 16\sqrt{3}$ cm.

$\therefore$ The area of the rhombus $= \frac{1}{2} \times 16 \times 16\sqrt{3}$

$= 128\sqrt{3}$ cm^2.

11



1 **Construction :**

Draw $\overline{DE} \perp \overline{BC}$

Proof :

$\because$ m ($\angle$ C) = 45°

$\therefore$ m (∠ CDE) = 180° – (90° + 45°) = 45°

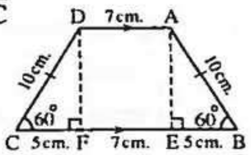
$\therefore$ DE = EC = 5 cm.

$\therefore$ The area of the trapezium $= \frac{1}{2}(7 + 12) \times 5$

$= 47.5 \text{ cm}^2$

2 **Construction :**

Draw $\overline{AE} \perp \overline{BC}$, $\overline{DF} \perp \overline{BC}$



Proof :

$\because$ m (∠ CDF)

= 180° – (90° + 60°) = 30°

$\because$ In the right - angled triangle, the length of the side opposite to the angle of measure 30°

$= \frac{1}{2}$ the length of the hypotenuse.

$\therefore$ FC = 5 cm.

$\therefore (DF)^2 = (DC)^2 - (FC)^2 = 100 - 25 = 75$

$\therefore$ DF $= 5\sqrt{3}$ cm. similarly BE = 5 cm.

$\therefore$ BC = 5 + 7 + 5 = 17 cm.

$\therefore$ The area of the trapezium

$= \frac{1}{2}(7 + 17) \times 5\sqrt{3} = \frac{1}{2} \times 24 \times 5\sqrt{3} = 60\sqrt{3} \text{ cm}^2$

12

Let the length of the smallest diagonal be $3x$ cm.

$\because$ The length of the greatest diagnoal $= 4x$ cm.

$\therefore 3x = 9$ $\therefore x = 3$

$\therefore$ The length of the greatest diagonal = 4 × 3 = 12 cm.

$\therefore$ The area of the rhombus $= \frac{1}{2} \times 9 \times 12 = 54 \text{ cm}^2$

13

Let the length of the smallest diagonal be $5x$ cm.

$\because$ The length of the greatest diagonal $= 8x$ cm.

$\therefore$ The area $= \frac{1}{2} \times 5x \times 8x = 20x^2$

$\therefore 20x^2 = 2000$ $\therefore x^2 = 100$ $\therefore x = 10$ cm.

$\therefore$ The lengths of the two diagonals are 50 cm. and 80 cm.

14

Let the lengths of the two bases be $2x$ cm. and $3x$ cm.

$\because \frac{1}{2}(2x + 3x) = 30$ $\therefore 5x = 60$ $\therefore x = 12$

$\therefore$ The lengths of the two bases are 24 cm. and 36 cm.

The area of the trapezium $= 30 \times 24 = 720 \text{ cm}^2$

15

Let the lengths of the two parallel bases be $3x$ cm. and $2x$ cm.

$\because$ The area $= \frac{1}{2}(3x + 2x) \times 12$

$\therefore \frac{1}{2}(3x + 2x) \times 12 = 180$

$\therefore 30x = 180$ $\therefore x = 6$ cm.

$\therefore$ The lengths of the two bases are 18 cm. and 12 cm.

16

Let the lengths of the two parallel bases and the height be $3x$ cm., $2x$ cm. and $4x$ cm.

$\because$ The area $= \frac{1}{2}(3x + 2x) \times 4x$

$\therefore 10x^2 = 4000$ $\therefore x^2 = 400$ $\therefore x = 20$ cm.

$\therefore$ The lengths of the two parallel bases are 60 cm, 40 cm.

$\therefore$ The length of the middle base $= \frac{60 + 40}{2} = 50$ cm.

17

The area of the piece of land which is in the shape of a trapezium $= \frac{1}{2}(76 + 64) \times 45 = 3150 \text{ m}^2$ and the area of the other piece of land which is in the shape of a rhombus $= \frac{1}{2} \times 74 \times 90 = 3330 \text{ m}^2$

$\therefore$ The area of the rectangular piece of land

$= 3150 + 3330 = 6480 \text{ m}^2$

Let the length of the rectangular piece be $5x$

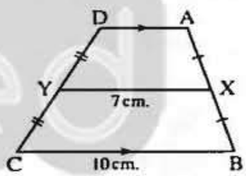
$\therefore$ Its width $= 4x$ m. $\therefore 5x \times 4x = 6480$

$\therefore x^2 = 324$ $\therefore x = \sqrt{324} = 18$ m.

$\therefore$ The length = 5 × 18 = 90 m.

The width = 4 × 18 = 72 m.

18



$\because \frac{AD + BC}{2} = XY$

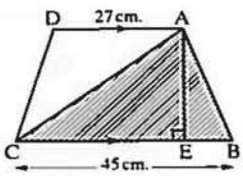
$\therefore \frac{AD + 10}{2} = 7$

$\therefore$ AD = 4 cm.

$\therefore$ The area = the length of the middle base × the perpendicular distance between the two parallel bases $\overline{AD}$ and $\overline{BC}$

$\therefore$ The perpendicular distance between the two parallel bases $\overline{AD}$ and $\overline{BC} = \frac{35}{7} = 5$ cm.

19



$\because$ The area of Δ ABC

$= \frac{1}{2}$ BC × AE

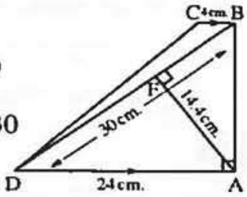
$\therefore 225 = \frac{1}{2} \times 45 \times$ AE

$\therefore$ AE = 10 cm.

The area of the trapezium

$= \frac{1}{2}(27 + 45) \times 10 = 360 \text{ cm}^2$

20



∵ The area of Δ ABD

$= \frac{1}{2} AB \times AD = \frac{1}{2} AF \times BD$

$= \frac{1}{2} AB \times 24 = \frac{1}{2} \times 14.4 \times 30$

∴ AB = 18 cm.

The area of the trapezium ABCD

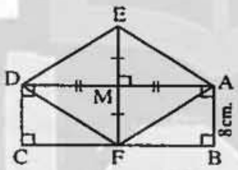
$= \frac{4+24}{2} \times 18 = 252 \text{ cm}^2$

21

The area of the figure ABCDE = the area of Δ ALE + the area of Δ ELD + the area of Δ AMB + the area of Δ CDN + the area of the trapezium BCNM $= \frac{1}{2} \times 2 \times 3 + \frac{1}{2} \times 8 \times 3 + \frac{1}{2} \times 5.6 \times 4 + \frac{1}{2} \times 3 \times 4 + \frac{5.6+4}{2} \times 3$

$= 3 + 12 + 11.2 + 6 + 14.4 = 46.6 \text{ cm}^2$

22



∵ ABCD is a rectangle , $\overline{MF} \perp \overline{AD}$

∴ AB = MF = 8 cm.

∵ M is the midpoint of $\overline{EF}$

∴ EF = 2 × 8 = 16 cm. (1)

∴ the area of the rectangle ABCD = AB × BC

∴ 144 = 8 × BC

∴ $BC = \frac{144}{8} = 18$ cm.

∴ AD = 18 cm. (2)

∴ In the figure AFDE : the two diagonals $\overline{AD}$ and $\overline{EF}$ bisect each the other, $\overline{FE} \perp \overline{AD}$

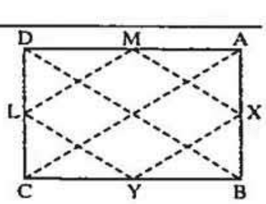
∴ The figure AFDE is a rhombus.

From (1) and (2) we find that :

The area of the figure AFDE $= \frac{1}{2}$ EF × AD

$= \frac{1}{2} \times 16 \times 18 = 144 \text{ cm}^2$

23



1 ∵ X and Y are the midpoints of $\overline{AB}$ and $\overline{BC}$

∴ $\overline{XY} // \overline{AC}$

∴ $XY = \frac{1}{2}$ AC (1)

∴ L and M are the midpoints of $\overline{DC}$ and $\overline{DA}$

∴ $\overline{LM} // \overline{AC}$, $LM = \frac{1}{2}$ AC (2)

From (1) and (2) :

∴ $\overline{LM} // \overline{XY}$, LM = XY

∴ XYLM is a parallelogram. (3)

∵ $XY = \frac{1}{2}$ AC , $XM = \frac{1}{2}$ BD

, but AC = DB (two diagonals in the rectangle ABCD)

∴ XY = XM (4)

From (3) and (4) :

∴ The figure XYLM is a rhombus.

∴ The area of the rhombus

$= \frac{1}{2}$ the product of the lengths of its two diagonals

$= \frac{1}{2} XL \times YM = \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$

2 ∴ $(AC)^2 = (AB)^2 + (BC)^2$

∴ $(AC)^2 = 36 + 64 = 100$

∴ AC = 10 cm. ∵ $XY = \frac{1}{2}$ AC

∴ $XY = \frac{1}{2} \times 10 = 5$ cm.

∴ the height of the rhombus XYLM $= \frac{24}{5} = 4.8$ cm.

24

∵ The area of the figure EOCD

= three times of the area of the figure ABOE

∴ $\frac{4+8}{2} \times$ height $= 3 \times \frac{x+1}{2} \times$ height

∴ $6 = 3 \times \frac{x+1}{2}$ ∴ $\frac{x+1}{2} = 6 \div 3 = 2$

∴ $x + 1 = 4$ ∴ $x = 3$

25

∵ The area of the trapezium = 120 cm² and the length of the middle base = 20 cm.

∴ Its height $= \frac{120}{20} = 6$ cm.

∴ The perimeter of the trapezium = 60 cm. and the length of the middle base = 20 cm.

∴ AD + BC = 2 × 20 = 40 cm.

∴ AB + DC = 60 – 40 = 20 cm.

∴ $AB = DC = \frac{20}{2} = 10$ cm.

From the figure :

∴ $(BE)^2 = (AB)^2 - (AE)^2 = 100 - 36 = 64$

∴ BE = 8 cm., similarly FC = 8 cm.

∵ AD = FE , ∵ AD + BC = 40 cm.

∴ AD + FE + 8 + 8 = 40 ∴ 2 AD = 40 – 16

∴ 2 AD = 24 ∴ AD = 12 cm.

∴ BC = 12 + 8 + 8 = 28 cm.

26

Let BD = 5 x cm. , AC = 6 x cm.

$\because 5x + 6x = 33$ $\therefore 11x = 33$ $\therefore x = 3$

$\therefore$ BD = 15 cm. , AC = 18 cm.

$\therefore ME = \frac{2}{3} MA$ $\therefore ME = \frac{2}{3} \times 9 = 6$ cm.

the area of Δ EBD $= \frac{1}{2}$ BD × EM

$= \frac{1}{2} \times 15 \times 6 = 45 \text{ cm}^2$ (1)

the area of the rhombus ABCD $= \frac{1}{2}$ BD × AC

$= \frac{1}{2} \times 15 \times 18 = 135 \text{ cm}^2$ (2)

From (1) and (2) :

$\therefore$ The area of the shaded part $= 135 - 45 = 90 \text{ cm}^2$

Answers of exams on unit four

Model 1

1

1 b 2 c 3 c
4 b 5 d 6 c

2

1 6 cm.
2 one is carring this base , are equal in area.
3 10
4 their vertices lie on a straight line parallel to this base.
5 equal

3

[a] Prove by yourself
[b] Prove by yourself

4

[a] Prove by yourself
[b] Prove by yourself

5

[a] 12 cm. [b] Prove by yourself

Model 2

1

1 c 2 b 3 d
4 a 5 b 6 d

2

1 half
2 two triangular surfaces equal in area
3 equal in length
4 4 cm.
5 equal in area

3

[a] Prove by yourself.
[b] Prove by yourself.

4

[a] 1 The area of the figure ABCD = 288 cm^2.
2 The length of the perpendicular from b to $\overline{AD}$ = 9.6 cm.

[b] Prove by yourself.

5

[a] The area of rhombus = 96 cm^2.
[b] Prove by yourself.

Answers of Unit Five

Answers of Exercise 6

1

1 angles
2 side lengths
3 similar
4 side lengths.
5 similar
6 equal in measure , proportional
7 congruent
8 3 : 4
9 similar

2

1 b
2 d
3 d
4 b
5 d
6 b
7 c

3

$\because \Delta ABC \sim \Delta XYZ$ $\quad \therefore \frac{AC}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$

$\therefore \frac{6}{XY} = \frac{10}{5} = \frac{AC}{7}$ $\quad \therefore AC = \frac{7 \times 10}{5} = 14$ cm.

, $XY = \frac{5 \times 6}{10} = 3$ cm. (the req.)

4

In $\Delta\Delta$ ABC and ZXY :

$\because m(\angle A) = m(\angle Z)$, $m(\angle C) = m(\angle Y)$

$\therefore m(\angle B) = m(\angle X)$ $\quad \therefore \Delta XYZ \sim \Delta BCA$

$\therefore \frac{XY}{BC} = \frac{YZ}{CA}$ $\quad \therefore \frac{XY}{16} = \frac{3}{12}$

$\therefore XY = \frac{3 \times 16}{12} = 4$ cm.

$\therefore$ The perimeter of $\Delta XYZ = 2 + 3 + 4 = 9$ cm (the req.)

5

$\because \frac{AB}{XY} = \frac{3}{6} = \frac{1}{2}$, $\frac{AC}{XZ} = \frac{5}{10} = \frac{1}{2}$, $\frac{BC}{YZ} = \frac{7}{14} = \frac{1}{2}$

$\therefore \frac{AB}{XY} = \frac{AC}{XZ} = \frac{BC}{YZ}$

$\therefore \Delta ABC \sim \Delta XYZ$ (the first req.)

$\because m(\angle B) + m(\angle C) = 60°$

$\therefore m(\angle A) = 120°$

$\therefore m(\angle A) = m(\angle X)$

$\therefore m(\angle X) = 120°$ (the second req.)

6

$\because \Delta AXY \sim \Delta ABC$

$\therefore m(\angle AXY) = m(\angle ABC)$

but they are corresponding.

$\therefore \overline{XY} \parallel \overline{BC}$ (Q.E.D. 1)

$\because \Delta AXY \sim \Delta ABC$

$\therefore \frac{AY}{AC} = \frac{XY}{BC}$ $\quad \therefore \frac{AY}{AC} = \frac{1}{2}$

$\therefore$ Y is the midpoint of $\overline{AC}$ (Q.E.D. 2)

7

1 $\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{DB}$ is a transversal to them

$\therefore m(\angle B) = m(\angle ADE)$ (corresponding angles)

Similarly $m(\angle C) = m(\angle AED)$ (corresponding angles)

$\angle A$ is a common angle.

$\therefore \Delta ABC \sim \Delta ADE$ $\quad \therefore \frac{DE}{BC} = \frac{AE}{AC}$

$\therefore \frac{8}{x} = \frac{9}{21}$ $\quad \therefore x = \frac{8 \times 21}{9} = 18\frac{2}{3}$ cm.

2 $\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{DB}$ is a transversal to them.

$\therefore m(\angle B) = m(\angle ADE)$ (corresponding angles)

Similarly $m(\angle C) = m(\angle AED)$ (corresponding angles)

$\angle A$ is a common angle.

$\therefore \Delta ABC \sim \Delta ADE$

$\therefore \frac{AB}{AD} = \frac{AC}{AE}$ $\quad \therefore \frac{7}{5} = \frac{AC}{8}$

$\therefore AC = \frac{7 \times 8}{5} = 11.2$ cm.

$\therefore x = 11.2 - 8 = 3.2$ cm.

3 $m(\angle B) = m(\angle ADE)$ (corresponding angles)

, $m(\angle C) = m(\angle AED)$ (corresponding angles)

, $\angle A$ is common in $\Delta\Delta$ ADE and ABC

$\therefore \Delta ADE \sim \Delta ABC$

$\therefore \frac{AD}{AB} = \frac{DE}{BC}$ $\quad \therefore \frac{4}{x+4} = \frac{5}{15}$ $\quad \therefore \frac{4}{x+4} = \frac{1}{3}$

$\therefore x + 4 = 12$ $\quad \therefore x = 8$ cm.

4 $m(\angle DEB) = 180° - 110° = 70°$

$\therefore$ In $\Delta\Delta$ DEB , ACB :

$m(\angle DEB) = m(\angle C)$, $\angle B$ is common

$\therefore m(\angle BDE) = m(\angle A)$ $\quad \therefore \Delta DEB \sim \Delta ACB$

$\therefore \frac{DE}{AC} = \frac{EB}{CB}$ $\quad \therefore \frac{x}{5} = \frac{4}{8}$

$\therefore x = \frac{5 \times 4}{8} = 2\frac{1}{2}$ cm.

5 In $\Delta\Delta$ ABE and DCE

$m(\angle A) = m(\angle D)$, $m(\angle AEB) = m(\angle DEC)$ (V.O.A)

$\therefore m(\angle B) = m(\angle C)$ $\quad \therefore \Delta ABE \sim \Delta DCE$

$\therefore \frac{AB}{DC} = \frac{AE}{DE}$ $\quad \therefore \frac{14}{34} = \frac{x}{22}$

$\therefore x = \frac{14 \times 22}{34} = 9\frac{1}{17}$ cm.

6 In ∆∆ ABC and EDF :

∵ m (∠ A) = m (∠ E) , m (∠ C) = m (∠ F)

∴ m (∠ B) = m (∠ D)

∴ ∆ ABC ~ ∆ EDF ∴ $\frac{AC}{EF} = \frac{BC}{DF}$

∴ $\frac{6}{x} = \frac{8}{12}$ ∴ $x = \frac{6 \times 12}{8} = 9$ cm.

8

∵ The polygon ABCD ~ the polygon XYZL

∴ $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZL} = \frac{DA}{LX}$ ∴ $\frac{10.5}{XY} = \frac{8}{YZ} = \frac{5}{ZL} = \frac{6}{3}$

∴ $XY = \frac{3 \times 10.5}{6} = 5.25$ cm. (the first req.)

, $YZ = \frac{8 \times 3}{6} = 4$ cm. (the second req.)

, $ZL = \frac{5 \times 3}{6} = 2.5$ cm. (the third req.)

9

∵ The figure ABCD ~ the figure XYZL

∴ m (∠ D) = m (∠ L) ∴ m (∠ D) = 80°

m (∠ BCD) = 360° − (125° + 70° + 80°) = 85° (the first req.)

∵ $\frac{AD}{XL} = \frac{BC}{YZ}$ ∴ $\frac{6}{XL} = \frac{8}{2.4}$

∴ $XL = \frac{2 \times 2.4}{4} = 1.8$ cm. (the second req.)

∴ The ratio of enlargement $= \frac{8}{2.4} = \frac{10}{3}$ (the third req.)

∴ $\frac{\text{The perimeter of the figure ABCD}}{\text{The perimeter of the figure XYZL}}$ = the ratio of enlargement

∴ $\frac{26}{\text{The perimeter of the figure XYZL}} = \frac{10}{3}$

∴ The perimeter of the figure XYZL $= \frac{26 \times 3}{10} = 7.8$ cm. (the fourth req.)

10

∵ $\overline{AC} \,//\, \overline{ED}$, $\overleftrightarrow{AD}$ is a transversal to them.

∴ m (∠ A) = m (∠ D) (alternate angles) (1)

∵ $\overline{AC} \,//\, \overline{ED}$, $\overleftrightarrow{CE}$ is a transversal to them.

∴ m (∠ C) = m (∠ E) (alternate angles) (2)

∵ m (∠ ABC) = m (∠ EBD) (V.O.A) (3)

From (1) , (2) , (3) :

∴ ∆ ABC ~ ∆ DBE (the first req.)

∴ $\frac{AB}{DB} = \frac{BC}{BE} = \frac{CA}{ED}$ ∴ $\frac{3}{6} = \frac{BC}{8} = \frac{5}{ED}$

∴ BC = 4 cm. , DE = 10 cm. (the second req.)

the ratio of enlargement $= \frac{DB}{AB} = \frac{6}{3} = 2$ (the third req.)

11

∵ $\overline{AF} \,//\, \overline{DC}$, $\overleftrightarrow{BC}$ is a transversal to them.

∴ m (∠ FBC) = m (∠ C) (alternate angles)

∵ $\overline{AF} \,//\, \overline{DC}$, $\overleftrightarrow{FE}$ is a transversal to them.

∴ m (∠ F) = m (∠ XEC) (alternate angles)

∵ m (∠ BXF) = m (∠ CXE) (V.O.A)

∴ ∆ ECX ~ ∆ FBX (the first req.)

∵ AB = DC (properties of ▱) ∴ AB = 6 cm.

∵ B is the midpoint of $\overline{AF}$

∴ AB = BF = 6 cm.

∵ $\frac{EC}{FB} = \frac{CX}{BX}$ ∴ $\frac{2}{6} = \frac{3}{BX}$ ∴ BX = 9 cm.

∴ BC = 12 cm. ∵ BC = AD (properties of ▱)

∴ AD = 12 cm. (the second req.)

12

In ∆ ∆ ABC , AED :

∵ m (∠ B) = m (∠ AED) , ∠ A is a common angle

∴ m (∠ C) = m (∠ ADE)

∴ ∆ ABC ~ ∆ AED (the first req.)

∴ $\frac{AD}{AC} = \frac{AE}{AB}$ ∴ $\frac{3}{AC} = \frac{4.5}{9}$

∴ $AC = \frac{3 \times 9}{4.5} = 6$ cm. ∴ EC = 6 − 4.5 = 1.5 cm. (the second req.)

13

∵ $\overline{AB} \,//\, \overline{XY}$, $\overleftrightarrow{BC}$ is a transversal to them.

∴ m (∠ ABC) = m (∠ XYZ) (corresponding angles) (1)

∵ $\overline{AC} \,//\, \overline{XZ}$, $\overleftrightarrow{BC}$ is a trarsversal to them.

∴ m (∠ ACB) = m (∠ XZY) (corresponding angles) (2)

∴ In ∆ ∆ ABC , XYZ :

m (∠ A) = m (∠ X) (3)

From (1) , (2) and (3) : ∴ ∆ ABC ~ ∆ XYZ

∴ $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CA}{ZX}$ ∴ $\frac{3}{XY} = \frac{BC}{10} = \frac{7}{14}$ (XZ = 2 AC)

∴ XY = 6 cm. , BC = 5 cm. (the req.)

14

∵ ∆ ADE ~ ∆ ABC ∴ $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$

∵ $\frac{AD}{DB} = \frac{1}{3}$ ∴ $\frac{AD}{AB} = \frac{1}{4}$

$\therefore \frac{AE}{AC} = \frac{1}{4}$ (the first req.)

$\therefore \frac{DE}{BC} = \frac{AE}{AC}$ $\therefore \frac{4}{BC} = \frac{1}{4}$

$\therefore BC = 16$ cm. (the second req.)

15

In $\Delta\Delta$ AED , ABC :

$\because m(\angle AED) = m(\angle B)$, $\angle A$ is a common angle.

$\therefore m(\angle ADE) = m(\angle C)$

$\therefore \Delta AED \sim \Delta ABC$ (the first req.)

$\therefore \frac{AE}{AB} = \frac{AD}{AC}$ $\therefore \frac{4}{AB} = \frac{3}{9}$ $\therefore AB = 12$ cm.

$\therefore BD = 12 - 3 = 9$ cm.

$m(\angle ADE) = m(\angle C) = 93°$ (the second req.)

16

$\because \Delta ABD \sim \Delta ACB$

$\therefore m(\angle DBA) = m(\angle BCA)$

$\therefore m(\angle DBA) = 35°$ (1)

In Δ ABC :

$\because m(\angle ABC) = 180° - (70° + 35°) = 75°$ (2)

From (1) , (2) :

$\therefore m(\angle DBC) = 75° - 35° = 40°$ (the first req.)

$\frac{AB}{AC} = \frac{BD}{CB} = \frac{AD}{AB}$ $\therefore \frac{6}{9} = \frac{AD}{6}$ $\therefore AD = 4$ cm.

$\therefore CD = 9 - 4 = 5$ cm. (the second req.)

17

In $\Delta\Delta$ ABC , CDB :

$m(\angle A) = m(\angle BCD)$ (1)

In Δ ABC : $\because AB = AC$

$\therefore m(\angle DBC) = m(\angle ACB)$ (2)

In Δ BCD : $\because CB = CD$

$\therefore m(\angle BDC) = m(\angle CBA)$ (3)

From (1) , (2) and (3) :

$\therefore \Delta ABC \sim \Delta CDB$ (the first req.)

$\therefore \frac{AB}{CD} = \frac{BC}{DB}$ $\therefore \frac{10}{5} = \frac{5}{DB}$ $\therefore DB = 2.5$ cm.

$\therefore AD = 10 - 2.5 = 7.5$ cm. (the second req.)

18

In Δ ABC : $\because m(\angle B) = 90°$

$\because (AC)^2 = (AB)^2 + (BC)^2 = 64 + 36 = 100$

$\therefore AC = 10$ cm.

, $\because$ D is the midpoint of $\overline{AB}$

$\therefore AD = DB = 4$ cm.

In $\Delta\Delta$ AED , ABC :

$m(\angle AED) = m(\angle B) = 90°$ (given)

, $\angle A$ is common

$\therefore m(\angle ADE) = m(\angle ACB)$

$\therefore \Delta AED \sim \Delta ABC$

$\therefore \frac{DE}{CB} = \frac{AD}{AC}$ $\therefore \frac{DE}{6} = \frac{4}{10}$

$\therefore DE = \frac{6 \times 4}{10} = 2.4$ cm. (The req.)

19

In $\Delta\Delta$ BAC , DAB :

$m(\angle ABC) = m(\angle ADB) = 90°$

, $\angle A$ is a common angle. $\therefore m(\angle C) = m(\angle ABD)$

$\therefore \Delta BAC \sim \Delta DAB$ (the first req.)

$\because \Delta$ ABC is right-angled at B

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 9 + 16 = 25$

$\therefore AC = 5$ cm.

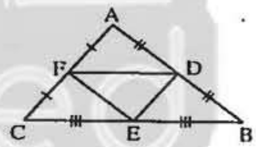
$\therefore \frac{AD}{AB} = \frac{AB}{AC} = \frac{BD}{CB}$ $\therefore \frac{AD}{3} = \frac{3}{5} = \frac{BD}{4}$

$\therefore AD = \frac{3 \times 3}{5} = 1.8$ cm. (the second req.)

$\because DC = AC - AD$

$\therefore DC = 5 - 1.8 = 3.2$ cm. (the third req.)

20



$\because$ D is the midpoint of $\overline{AB}$

F is the midpoint of $\overline{AC}$

$\therefore DF = \frac{1}{2} BC$

$\therefore \frac{DF}{BC} = \frac{1}{2}$

similarly : $\frac{EF}{AB} = \frac{1}{2}$, $\frac{DE}{AC} = \frac{1}{2}$

$\therefore \frac{DF}{BC} = \frac{EF}{AB} = \frac{DE}{AC}$

$\therefore \Delta ABC \sim \Delta EFD$ (Q.E.D.)

21

Assuming that the triangle whose side lengths are given be ABC and the other is XYZ

$\because$ The two triangles are similar.

$\therefore \frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ} = \frac{\text{The perimeter of } \Delta ABC}{\text{The perimeter of } \Delta XYZ}$

$\therefore \frac{4.5}{XY} = \frac{6}{YZ} = \frac{8}{XZ} = \frac{18.5}{74}$

$\therefore$ The longest side is $\overline{XZ}$

, $XZ = 32$ cm. (The req.)

22

$\because \Delta AED \sim \Delta ADB$

$\therefore m(\angle ADE) = m(\angle ABD)$

$\therefore x + 20° = 2x + 5°$ $\therefore x = 15°$

$\therefore m(\angle ADE) = 15° + 20° = 35°$ (the req.)

23

$\because \overline{XY} // \overline{BN}$, $\overline{YZ}$ is a transversal to them.

$\therefore m(\angle Y) = m(\angle NBZ)$ (corresponding angles) (1)

$\because \overline{XY} // \overline{BN}$, $\overleftrightarrow{XZ}$ is a transversal to them.

$\therefore m(\angle X) = m(\angle BNZ)$ (corresponding angles) (2)

In $\Delta\Delta$ XYZ, NBZ:

$\because \angle Z$ is a common angle.

From (1) and (2):

$\therefore \Delta XYZ \sim \Delta NBZ$ (4)

Similarly we can prove that:

$\Delta ABC \sim \Delta NBZ$ (5)

From (4) and (5):

$\therefore \Delta XYZ \sim \Delta NBZ \sim \Delta ABC$ (the first req.)

From (5) we find that: $\frac{BZ}{BC} = \frac{NZ}{AC}$

$\therefore \frac{4}{BC} = \frac{6}{12}$ $\therefore BC = 8$ cm.

$\therefore$ Z is the midpoint of $\overline{BC}$ (the second req.)

Then ZC = 4 cm.

From (4) we find that: $\frac{XZ}{NZ} = \frac{YZ}{BZ}$

$\therefore \frac{8}{6} = \frac{YZ}{4}$ $\therefore YZ = \frac{32}{6} = 5\frac{1}{3}$ cm.

$\therefore YC = YZ + ZC = 5\frac{1}{3} + 4 = 9\frac{1}{3}$ cm. (the third req.)

24

$\because \overline{XY} // \overline{AB}$, $\overline{AX} // \overline{BY}$

$\therefore$ ABYX is a parallelogram.

$\because m(\angle B) = 90°$ $\therefore$ ABYX is a rectangle

$\therefore BY = AX = 4$ cm.

$\because BC = AD = 12$ cm. $\therefore YC = 12 - 4 = 8$ cm.

$\because \Delta AXM$ is right-angled at X

$\therefore (AM)^2 = (AX)^2 + (XM)^2 = 16 + 9 = 25$

$\therefore AM = 5$ cm.

In $\Delta\Delta$ AMX, CMY:

$m(\angle AXM) = m(\angle MYC) = 90°$

$m(\angle AMX) = m(\angle CMY)$ (V.O.A)

$\therefore m(\angle XAM) = m(\angle MCY)$

$\therefore \Delta AMX \sim \Delta CMY$ (the first req.)

$\therefore \frac{AX}{CY} = \frac{XM}{YM} = \frac{AM}{CM}$ $\therefore \frac{4}{8} = \frac{3}{YM} = \frac{5}{CM}$

$\therefore YM = \frac{3 \times 8}{4} = 6$ cm $CM = \frac{5 \times 8}{4} = 10$ cm.

$\therefore$ The perimeter of Δ CYM = 6 + 10 + 8 = 24 cm. (the second req.)

$\because AB = CD$ $\therefore \frac{AB}{CD} = 1$

$\because \frac{YM}{XM} = \frac{6}{3} = 2$ $\therefore \frac{AB}{CD} \neq \frac{YM}{XM}$

$\therefore$ The figure ABYM is not similar to the figure CDXM (the third req.)

25

$\because \overline{DE} // \overline{BC}$, $\overleftrightarrow{EC}$ is a transversal to them.

$\therefore m(\angle C) = m(\angle AED)$ (corresponding angles)

Similarly we can prove that:

$m(\angle B) = m(\angle ADE)$ (corresponding angles)

In $\Delta\Delta$ ADE, ABC:

$\angle A$ is a common angle, $m(\angle C) = m(\angle AED)$, $m(\angle B) = m(\angle ADE)$

$\therefore \Delta ABC \sim \Delta ADE$

$\therefore \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$

$\therefore \frac{3}{(x+1)+3} = \frac{2}{(x-1)+2} = \frac{y}{12}$ (1)

$\therefore \frac{3}{x+4} = \frac{2}{x+1} = \frac{y}{12}$ $\therefore 3(x+1) = 2(x+4)$

$\therefore 3x + 3 = 2x + 8$

$\therefore x = 5$ $\therefore AB = 9$ cm. (the first req.)

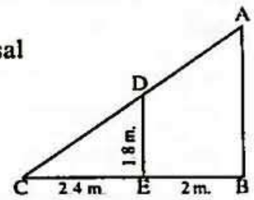
EC = 4 cm. (the second req.)

Substituting in (1):

$\therefore \frac{3}{9} = \frac{y}{12}$ $\therefore y = \frac{3 \times 12}{9} = 4$ cm.

$\therefore DE = 4$ cm. (the third req.)

26



$\because \overline{DE} // \overline{AB}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle A) = m(\angle CDE)$ (corresponding angles)

, $\because \overline{DE} // \overline{AB}$, $\overleftrightarrow{BC}$ is a transversal

$\therefore m(\angle B) = m(\angle CED)$ (corresponding angles)

, $\because \angle C$ is common

$\therefore \Delta ABC \sim \Delta DEC$ $\therefore \frac{AB}{DE} = \frac{BC}{EC} = \frac{AC}{DC}$

$\therefore \frac{AB}{1.8} = \frac{4.4}{2.4}$

$\therefore AB = \frac{1.8 \times 4.4}{2.4} = 3.3$ m. (The req.)

27

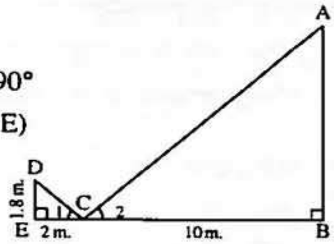
In ΔΔ ABC , DEC :

$\because m(\angle B) = m(\angle E) = 90^\circ$

$, m(\angle ACB) = m(\angle DCE)$

$\therefore m(\angle A) = m(\angle D)$

$\therefore \Delta ABC \sim \Delta DEC$



$\therefore \frac{AB}{DE} = \frac{BC}{CE}$

$\therefore \frac{AB}{1.8} = \frac{10}{2}$ $\therefore AB = \frac{1.8 \times 10}{2} = 9$ m. (The req.)

28

$\because \frac{AB}{BC} = \frac{DE}{EF}$ $\therefore \frac{AB}{DE} = \frac{BC}{EF}$

$\because AB = AC , DE = DF$ $\therefore \frac{AC}{DF} = \frac{AB}{DE} = \frac{BC}{EF}$

$\therefore \Delta ABC \sim \Delta DEF$ (Q.E.D.)

29

In Δ ABC :

$\because x + 2x + 3x = 180^\circ$

$\therefore 6x = 180^\circ$ $\therefore x = 30^\circ$

$\therefore m(\angle C) = 30^\circ , m(\angle B) = 60^\circ , m(\angle A) = 90^\circ$

$\therefore AB = \frac{1}{2} BC$

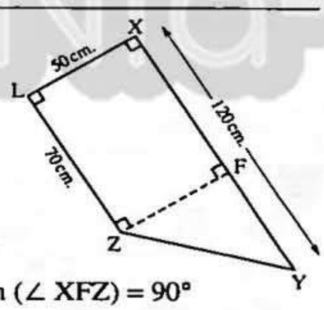
$\therefore AB = \frac{1}{2} \times 18 = 9$ cm.

$\because \Delta ABC \sim \Delta DEF$

$\therefore \frac{AB}{DE} = \frac{BC}{EF}$ $\therefore \frac{9}{3} = \frac{18}{EF}$

$\therefore EF = 6$ cm. (Q.E.D.)

30



Constr. :

Draw :

$\overline{CE} \perp \overline{AB} , \overline{FZ} \perp \overline{XY}$

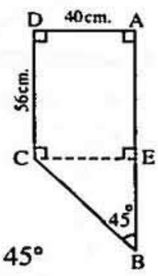
Proof :

In the quadrilateral XLZF

$\because m(\angle X) = m(\angle L) = m(\angle XFZ) = 90^\circ$

$\therefore m(\angle LZF) = 90^\circ$

$\therefore$ The figure XLZF is a rectangle.



$\therefore FZ = XL = 50 , XF = LZ = 70$ cm.

$\therefore FY = XY - XF = 120 - 70 = 50$ cm.

In Δ ZFY which is right-angled at F :

$\because FY = FZ$

$\therefore m(\angle Y) = m(\angle FZY) = \frac{180^\circ - 90^\circ}{2} = 45^\circ$

$\because (YZ)^2 = (FY)^2 + (FZ)^2$

$\therefore (YZ)^2 = 2500 + 2500 = 5000$

$\therefore YZ = 50\sqrt{2}$ cm.

In the quadrilateral ADCE :

$\because m(\angle A) = m(\angle D) = m(\angle AEC) = 90^\circ$

$\therefore m(\angle DCE) = 90^\circ$

$\therefore$ The figure ADCE is a rectangle.

$\therefore EC = AD = 40$ cm. , $AE = DC = 56$ cm.

In Δ BEC which is right-angled at E :

$\because m(\angle B) = 45^\circ$

$\therefore m(\angle ECB) = 180^\circ - (90^\circ + 45^\circ) = 45^\circ$

$\therefore m(\angle B) = m(\angle ECB)$ $\therefore EB = EC = 40$ cm.

$\because (BC)^2 = (BE)^2 + (EC)^2$

$\therefore (BC)^2 = 1600 + 1600 = 3200$

$\therefore BC = 40\sqrt{2}$ cm.

$\because AB = AE + EB$ $\therefore AB = 56 + 40 = 96$ cm.

In the two figures ABCD , XYZL :

$\because m(\angle A) = m(\angle X) = 90^\circ , m(\angle B) = m(\angle Y) = 45^\circ$

$m(\angle C) = m(\angle Z) = 135^\circ$

$m(\angle D) = m(\angle L) = 90^\circ$

$\frac{AB}{XY} = \frac{96}{120} = \frac{4}{5} , \frac{BC}{YZ} = \frac{40\sqrt{2}}{50\sqrt{2}} = \frac{4}{5}$

$\frac{DC}{LZ} = \frac{56}{70} = \frac{4}{5} , \frac{AD}{LX} = \frac{40}{50} = \frac{4}{5}$

$\therefore \frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{LZ} = \frac{DA}{LX} = \frac{4}{5}$

$\therefore$ The figure ABCD ~ the figure XYZL (The req.)

Answers of Exercise 7

1

Fig. (1) :

$\because (AB)^2 = 25 , (BC)^2 = 144 , (AC)^2 = 169$

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$\therefore m(\angle B) = 90^\circ$

Fig. (2) :

$\because (AB)^2 = 225 , (BC)^2 = 400 , (AC)^2 = 625$

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$\therefore m(\angle B) = 90^\circ$

Fig. (3) :

$\because (AB)^2 = 324 , (BC)^2 = 576 , (AC)^2 = 900$

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$\therefore m(\angle B) = 90^\circ$

2

Fig. (1) :

$(DF)^2 = 49$, $(DE)^2 + (EF)^2 = 61$

$\therefore$ Δ DEF is not right-angled.

Fig. (2) :

$(MN)^2 = 169$, $(ML)^2 + (NL)^2 = 169$

$\therefore$ Δ NLM is right-angled at L

Fig. (3) :

$(XY)^2 = (\sqrt{34})^2 = 34$, $(YZ)^2 + (ZX)^2 = 34$

$\therefore$ Δ XZY is right-angled at Z

Fig. (4) :

$(AC)^2 = 49$, $(AB)^2 + (BC)^2 = 34$

$\therefore$ Δ ABC is not right-angled.

3

In Δ ABC :

$\because$ $(AB)^2 = 20.25$, $(AC)^2 = 36$, $(BC)^2 = 56.25$

$\therefore$ $(BC)^2 = (AB)^2 + (AC)^2$

$\therefore$ $m(\angle A) = 90°$

$\therefore$ Δ ABC is right-angled at A (Q.E.D.)

4

In Δ ABC :

$\because$ $m(\angle B) = 90°$

$\therefore$ $(AC)^2 = (7)^2 + (24)^2 = 49 + 576 = 625$

$\therefore$ AC = 25 cm.

In Δ DAC :

$\because$ $(DA)^2 + (DC)^2 = (15)^2 + (20)^2 = 225 + 400 = 625$

$\therefore$ $(DA)^2 + (DC)^2 = (AC)^2$

$\therefore$ $m(\angle ADC) = 90°$ (Q.E.D.)

5

In Δ ABC :

$\because$ $m(\angle B) = 90°$

$\therefore$ $(AC)^2 = (AB)^2 + (BC)^2 = 144 + 256 = 400$

$\therefore$ AC = 20 cm. (The first req.)

In Δ ACD :

$(AD)^2 = 625$, $(AC)^2 = 400$, $(CD)^2 = 225$

$\therefore$ $(AD)^2 = (AC)^2 + (CD)^2$

$\therefore$ $m(\angle ACD) = 90°$ (The second req.)

6

In Δ ABD :

$\therefore$ $m(\angle A) = 90°$, $m(\angle ADB) = 30°$

$\therefore$ $AB = \frac{1}{2}$ BD

$\therefore$ BD = 15 cm.

In Δ BDC :

$(BD)^2 = (15)^2 = 225$, $(CD)^2 = (8)^2 = 64$

, $(BC)^2 = (17)^2 = 289$

$\because$ $(BD)^2 + (DC)^2 = 225 + 64 = 289 = (BC)^2$

$\therefore$ $m(\angle BDC) = 90°$ (Q.E.D.)

7

In Δ ABC :

$\because$ $m(\angle B) = 90°$

$\therefore$ $(AC)^2 = (AB)^2 + (BC)^2 = 81 + 144 = 225$

$\therefore$ AC = 15 cm.

In Δ DAC :

$(AC)^2 = 225$, $(AD)^2 = 64$, $(DC)^2 = 289$

$\therefore$ $(DC)^2 = (AD)^2 + (AC)^2$

$\therefore$ $m(\angle DAC) = 90°$ (The first req.)

$\because$ the area of the figure ABCD

= the area of Δ ABC + the area of Δ DAC

$\therefore$ the area of the figure ABCD

$= \frac{1}{2} \times 9 \times 12 + \frac{1}{2} \times 8 \times 15$

$= 54 + 60 = 114$ cm². (The second req.)

8

In Δ XLY :

$\because$ $m(\angle XLY) = 90°$

$\therefore$ $(XY)^2 = (XL)^2 + (LY)^2 = 36 + 9 = 45$ (1)

In Δ XLZ :

$\because$ $m(\angle XLZ) = 90°$

$\therefore$ $(XZ)^2 = (XL)^2 + (LZ)^2 = 36 + 144 = 180$ (2)

From (1) and (2) :

$\therefore$ $(XY)^2 + (XZ)^2 = 45 + 180 = 225$

but YZ = 3 + 12 = 15

$\therefore$ $(YZ)^2 = (15)^2 = 225$

i.e. $(XY)^2 + (XZ)^2 = (YZ)^2$

$\therefore$ $m(\angle YXZ) = 90°$ (Q.E.D.)

9

In Δ ABC :

$\because m(\angle B) = 90°$

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 36 + 64 = 100$

$\therefore$ AC = 10 cm. (The first req.)

$\because$ D is the midpoint of $\overline{AC}$

$\therefore$ AD = 5 cm.

In Δ ADE :

$\because (ED)^2 = 169$, $(AE)^2 = 144$

$(AD)^2 = 25$

$\therefore (ED)^2 = (AE)^2 + (AD)^2$

$\therefore m(\angle EAD) = 90°$ (The second req.)

10

$\because \Delta$ ABC is right-angled at B

$\therefore (BC)^2 = (AC)^2 - (AB)^2 = 100 - 36 = 64$

$\therefore$ BC = 8 cm. (The first req.)

$\therefore$ DC = 12 – 8 = 4 cm.

From Δ DEC : $(EC)^2 = 25$, $(DE)^2 = 9$, $(DC)^2 = 16$

$\therefore (EC)^2 = (DE)^2 + (DC)^2$

$\therefore m(\angle D) = 90°$ (The second req.)

11

$\because$ ABCD is a rectangle.

$\therefore \Delta$ ABE is right-angled at A

$\therefore (EB)^2 = (AE)^2 + (AB)^2 = 81 + 144 = 225$

$\therefore$ EB = 15 cm.

$\therefore \Delta$ DEC is right-angled at D

$\therefore (EC)^2 = (ED)^2 + (DC)^2 = 256 + 144 = 400$

$\therefore$ EC = 20 cm.

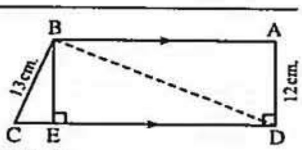
In Δ EBC :

$(BC)^2 = 625$, $(BE)^2 = 225$, $(EC)^2 = 400$

$\therefore (BC)^2 = (BE)^2 + (EC)^2$

$\therefore \overline{BE} \perp \overline{EC}$ (Q.E.D.)

12



$\because \overline{AD} \perp \overline{DC}$, $\overline{BE} \perp \overline{DC}$

$\therefore \overline{AD} // \overline{BE}$

$\therefore$ The figure ADEB is a rectangle.

$\therefore$ AD = BE $\therefore$ BE = 12 cm.

$\therefore (CE)^2 = (BC)^2 - (BE)^2 = 169 - 144 = 25$

$\therefore$ EC = 5 cm. (The first req.)

$\because$ DE = DC – CE

$\therefore$ DE = 33.8 – 5 = 28.8 cm.

$\therefore$ DE = AB

$\therefore$ AB = 28.8 cm. (The second req.)

In Δ DBE :

$\because m(\angle BED) = 90°$

$\therefore (DB)^2 = (DE)^2 + (BE)^2 = 829.44 + 144 = 973.44$

$\therefore$ DB = 31.2 cm. (The third req.)

The area of the trapezium ABCD

$= \frac{1}{2}(28.8 + 33.8) \times 12 = 375.6 \text{ cm}^2$ (The fourth req.)

In Δ DBC :

$\because (DC)^2 = 1142.44$, $(DB)^2 = 973.44$, $(BC)^2 = 169$

$\therefore (DC)^2 = (DB)^2 + (BC)^2$

$\therefore m(\angle DBC) = 90°$ (The fifth req.)

13

In Δ DEC :

$\because (DC)^2 = 225$, $(DE)^2 = 144$, $(EC)^2 = 81$

$\therefore (DC)^2 = (DE)^2 + (EC)^2$ $\therefore m(\angle DEC) = 90°$

$\therefore$ the area of ▱ ABCD = AD × DE = 20 × 12 = 240 cm^2

(The req.)

14

In Δ XYZ :

$\because (YZ)^2 = 25$, $(XY)^2 = 16$, $(ZX)^2 = 9$

$\therefore (YZ)^2 = (XY)^2 + (ZX)^2$

$\therefore m(\angle YXZ) = 90°$

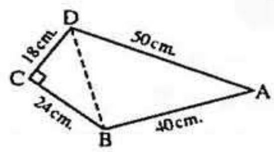
$\therefore$ the area of Δ XYZ $= \frac{1}{2} \times 4 \times 3 = 6$ cm.

(The first req.)

$\because \overline{XE} \perp \overline{ZY}$

$\therefore XE = \frac{2 \times 6}{5} = 2.4$ cm. (The second req.)

15



Construction :

Draw $\overline{BD}$

Proof : In Δ DBC

$\because m(\angle C) = 90°$

$\therefore (DB)^2 = (BC)^2 + (CD)^2 = 576 + 324 = 900$

$\therefore$ DB = 30 cm.

In Δ ABD :

$\because (AD)^2 = 2500$, $(AB)^2 = 1600$, $(BD)^2 = 900$

$\therefore (AD)^2 = (AB)^2 + (BD)^2$

$\therefore m(\angle ABD) = 90°$

$\therefore$ the area of $\Delta ABD = \frac{1}{2} BD \times AB$

$= \frac{1}{2} \times 30 \times 40 = 600 \text{ cm}^2$ (1)

$\therefore$ the area of $\Delta DBC = \frac{1}{2} BC \times CD$

$= \frac{1}{2} \times 24 \times 18 = 216 \text{ cm}^2$ (2)

Adding (1) and (2) :

$\therefore$ the area of the figure ABCD $= 216 + 600 = 816 \text{ cm}^2$

(The req.)

16

$\because \Delta ABC$ is right-angled at B

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (4)^2 + (3)^2 = 25$

$\therefore AC = 5$ cm.

In ΔACD :

$\because (AD)^2 = 169$, $(AC)^2 = 25$, $(CD)^2 = 144$

$\therefore (AD)^2 = (AC)^2 + (CD)^2$ $\quad \therefore m(\angle C) = 90°$

$\therefore$ the area of $\Delta ACD = \frac{1}{2} AC \times CD$

$= \frac{1}{2} \times 5 \times 12 = 30 \text{ cm}^2$

$\because$ the area of $\Delta ABC = \frac{1}{2} AB \times BC$

$= \frac{1}{2} \times 4 \times 3 = 6 \text{ cm}^2$

$\therefore$ the area of the figure ABCD $= 30 - 6 = 24 \text{ cm}^2$

(The req.)

17

In ΔABC :

$\because AB = AC$, $\overleftrightarrow{AD} \perp \overline{BC}$

$\therefore$ D is the midpoint of $\overline{BC}$

$\therefore BD = DC = 4.5$ cm.

In ΔADE :

$\therefore m(\angle ADE) = 90°$

$\therefore (DE)^2 = (AE)^2 - (AD)^2$

$= (10)^2 - (6)^2 = 100 - 36 = 64$

$\therefore DE = \sqrt{64} = 8$ cm.

In ΔADB :

$\therefore m(\angle ADB) = 90°$

$\therefore (AB)^2 = (AD)^2 + (BD)^2$

$= (6)^2 + (4.5)^2 = 36 + 20.25 = 56.25$

$\therefore (AB)^2 + (AE)^2 = 56.25 + (10)^2 = 156.25$ (1)

$\because BE = BD + DE$

$\therefore BE = 4.5 + 8 = 12.5$ cm.

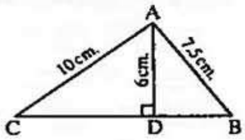
$\therefore (BE)^2 = (12.5)^2 = 156.25$ (2)

From (1) and (2) :

$\therefore (AB)^2 + (AE)^2 = (BE)^2$

$\therefore m(\angle BAE) = 90°$ (Q.E.D)

18



$\because \Delta ABD$ is right-angled at D

$\therefore (BD)^2 = (AB)^2 - (AD)^2$

$= (7.5)^2 - (6)^2 = 20.25$

$\therefore BD = 4.5$ cm.

$\because \Delta ADC$ is right-angled at D

$\therefore (CD)^2 = (AC)^2 - (AD)^2 = (10)^2 - (6)^2 = 64$

$\therefore CD = 8$ cm.

$\therefore BC = BD + CD = 4.5 + 8 = 12.5$ cm. (The first req.)

In ΔABC :

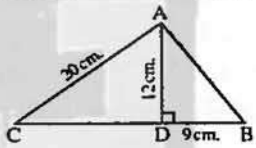
$\because (AB)^2 + (AC)^2 = (7.5)^2 + (10)^2 = 156.25$

$(BC)^2 = (12.5)^2 = 156.25$

$\therefore (AB) + (AC)^2 = (BC)^2$

$\therefore \Delta ABC$ is right-angled at A (The second req.)

19



$\because \Delta ABD$ is right-angled at D

$\therefore (AB)^2 = (BD)^2 + (AD)^2$

$= (9)^2 + (12)^2 = 225$

$\therefore AB = 15$ cm. $\quad \because \Delta ACD$ is right-angled at D

$\therefore (CD)^2 = (AC)^2 - (AD)^2 = (20)^2 - (12)^2 = 256$

$\therefore CD = 16$ cm. $\quad \therefore BC = 9 + 16 = 25$ cm.

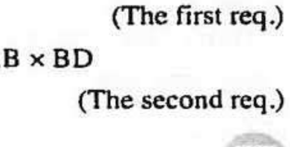
$\because$ In ΔABC :

$(AB)^2 + (AC)^2 = (15)^2 + (20)^2 = 625$, $(BC)^2 = 625$

$\therefore (AB)^2 + (AC)^2 = (BC)^2$

$\therefore \Delta ABC$ is right-angled at A (Q.E.D.)

20



$\therefore$ In ΔABM :

$\because (AB)^2 + (BM)^2$

$= (8)^2 + (6)^2 = 100$

$(AM)^2 = (10)^2 = 100$

$\therefore (AB)^2 + (BM)^2 = (AM)^2$

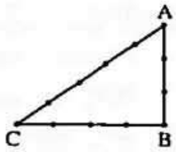
$\therefore \Delta ABM$ is right-angled at B

$\therefore m(\angle ABD) = 90°$ (The first req.)

$\therefore$ the area of $\square$ ABCD $= AB \times BD$

$= 8 \times 12 = 96 \text{ cm}^2$ (The second req.)

21



In Δ ABC :

AB = 3 equal distances

, BC = 4 equal distances

, AC = 5 equal distances

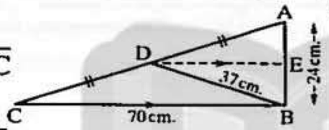
, $\because (AB)^2 + (BC)^2 = 3^2 + 4^2 = 25$

, $(AC)^2 = 5^2 = 25$

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$\therefore m(\angle B) = 90°$ (Q.E.D.)

22



$\because$ D is the midpoint of $\overline{AC}$

, $\overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$

$\therefore$ E is the midpoint of $\overline{AB}$

$\therefore BE = \frac{24}{2} = 12$ cm.

$\because$ D is the midpoint of $\overline{AC}$,

E is the midpoint of $\overline{AB}$

$\therefore DE = \frac{1}{2} BC = \frac{1}{2} \times 70 = 35$ cm.

In Δ BED :

$(BE)^2 + (ED)^2 = (12)^2 + (35)^2 = 1369$

$(BD)^2 = (37)^2 = 1369$

$\therefore (BE)^2 + (ED)^2 = (BD)^2$

$\therefore m(\angle BED) = 90°$

$\because \overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$

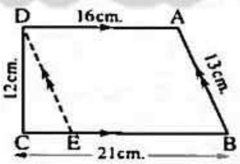
$\therefore m(\angle ABC) = 90°$ (The first req.)

In Δ ABC : which is right-angled at B

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (24)^2 + (70)^2 = 5476$

$\therefore$ AC = 74 cm. (The second req.)

23



Construction :

Draw $\overleftrightarrow{DE}$ such that $\overleftrightarrow{DE} \parallel \overline{AB}$

, and $\overleftrightarrow{DE} \cap \overline{BC} = \{E\}$

Proof : $\because \overline{AD} \parallel \overline{BE}$, $\overline{AB} \parallel \overline{DE}$

$\therefore$ ABED is a parallelogram.

$\therefore$ DE = AB = 13 cm. , BE = AD = 16 cm.

$\therefore$ EC = 21 – 16 = 5 cm.

In Δ DCE :

$\because (DC)^2 + (EC)^2 = (12)^2 + (5)^2 = 169$

$(DE)^2 = (13)^2 = 169$

$\therefore (DE)^2 = (DC)^2 + (EC)^2$

$\therefore m(\angle C) = 90°$ (Q.E.D.)

24

$\because$ the diagonals of the parallelogram bisects each other.

$\therefore$ AC = 2 AM

$\therefore (AC)^2 = 4(AM)^2$ and BD = 2 BM

$\therefore (BD)^2 = 4(BM)^2$

$\because (AC)^2 = 4(AB)^2 + (BD)^2$ (given)

$\therefore 4(AM)^2 = 4(AB)^2 + 4(BM)^2$

Dividing by 4 :

$\therefore (AM)^2 = (AB)^2 + (BM)^2$

$\therefore \Delta$ ABM is right-angled at B

$\therefore m(\angle ABD) = 90°$ (Q.E.D.)

Answers of Exercise 8

1

1 a 2 c 3 a 4 c

5 a 6 d 7 c

2

Fig. (1) : • The projection of A on $\overleftrightarrow{BC}$ is D

• The projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is $\overline{DB}$

Fig. (2) : • The projection of A on $\overleftrightarrow{BC}$ is B

• The projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is B

Fig. (3) : • The projection of A on $\overleftrightarrow{BC}$ is B

• The projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is B

3

Fig. (1) : $\overline{BC}$, the point B , $\overline{AB}$ and the point B

Fig. (2) : $\overline{DC}$, $\overline{DB}$, the point A and $\overline{BA}$

Fig. (3) : $\overline{XC}$, $\overline{XB}$, $\overline{AY}$ and $\overline{BY}$

4

1 $\overline{CD}$ 2 the point C 3 $\overline{AB}$

5

1 $\overline{BD}$ 2 $\overline{AE}$ 3 the point E

4 the point B 5 the point B 6 the point E

7 the point B

6

1 $\overline{DB}$, $\overline{BF}$ 2 $\overline{DC}$, $\overline{EC}$ 3 $\overline{AF}$, $\overline{AE}$

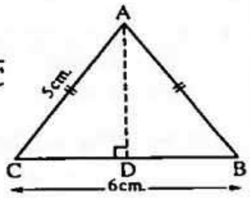
4 $\overline{AF}$, $\overline{BD}$ 5 the point F , the point E

7

1 the point X 2 the point B

3 the point B 4 the point A

8



Construction : Draw $\overline{AD} \perp \overline{BC}$

Proof :

$\overline{BD}$ is the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$

, $\because$ Δ ABC is an isosceles triangle.

$\therefore$ $\overline{AD}$ is a median in Δ ABC

$\therefore$ BD $= \frac{1}{2}$ BC

$\therefore$ BD = 3 cm. (First req.)

In Δ ABD which is right-angled at D

$\therefore$ $(AD)^2 = (AB)^2 - (BD)^2 = 25 - 9 = 16$

$\therefore$ AD = 4 cm.

$\therefore$ The area of Δ ABC $= \frac{1}{2} \times 4 \times 6 = 12$ cm^2. (Second req.)

9

1 $\overline{EM}$ is the projection of $\overline{FM}$ on $\overleftrightarrow{EM}$

In Δ FEM which is right-angled at E

$\therefore$ $(EM)^2 = (FM)^2 - (FE)^2 = 9 - 1 = 8$

$\therefore$ EM $= \sqrt{8} = 2\sqrt{2}$ cm. (First req.)

2 $\overline{AM}$ is the projection of $\overline{BM}$ on $\overleftrightarrow{AM}$

In the right-angled triangle EDM at D

$\therefore$ $(DM)^2 = (EM)^2 - (ED)^2 = 8 - 1 = 7$

$\therefore$ DM $= \sqrt{7}$ cm.

In the right-angled triangle DCM at C

$\therefore$ $(CM)^2 = (DM)^2 - (DC)^2 = 7 - 1 = 6$

$\therefore$ CM $= \sqrt{6}$ cm.

In the right-angled triangle CBM at B

$\therefore$ $(BM)^2 = (CM)^2 - (BC)^2 = 6 - 1 = 5$

$\therefore$ BM $= \sqrt{5}$ cm.

In the right-angled triangle BAM at A

$\therefore$ $(AM)^2 = (BM)^2 - (AB)^2 = 5 - 1 = 4$

$\therefore$ AM = 2 cm. (Second req.)

10

1 $\overline{AD}$ is the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$

In the right-angled triangle BDC at D

we find : m ($\angle$ C) = 30° , m ($\angle$ DBC) = 60°

In the right-angled triangle ABD at D

m ($\angle$ ABD) = 30°

$\therefore$ AD $= \frac{1}{2}$ AB $\therefore$ AD = 3 cm. (First req.)

2 $\overline{DC}$ is the projection of $\overline{BC}$ on $\overleftrightarrow{AC}$

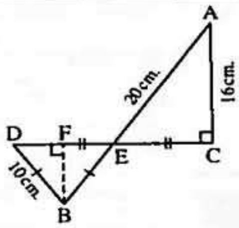
In the right-angled triangle ABC at B

m ($\angle$ C) = 30° $\therefore$ AC = 2 AB

$\therefore$ AC = 12 cm. $\therefore$ DC = AC – AD

$\therefore$ DC = 12 – 3 = 9 cm. (Second req.)

11



Construction : Draw $\overline{BF} \perp \overline{ED}$

Proof :

1 $\overline{FD}$ is the projection of $\overline{BD}$ on $\overleftrightarrow{CD}$

In the right-angled triangle ACE at C

$(CE)^2 = (AE)^2 - (AC)^2 = 400 - 256 = 144$

$\therefore$ CE = 12 cm. $\because$ CE = ED

$\therefore$ ED = 12 cm.

$\therefore$ Δ EBD is an isosceles triangle , $\overline{BF} \perp \overline{ED}$

$\therefore$ $\overline{BF}$ is a median in Δ EBD

$\therefore$ FD $= \frac{1}{2}$ ED $\therefore$ FD = 6 cm. (First req.)

2 $\overline{CF}$ is the projection of $\overline{AB}$ on $\overleftrightarrow{CD}$

$\therefore$ CF = CE + EF

$\therefore$ CF = 12 + 6 = 18 cm. (Second req.)

12

1 $\overline{EC}$ is the projection of $\overline{BC}$ on $\overleftrightarrow{DC}$

$\because$ $\overline{AD} \perp \overline{DC}$, $\overline{BE} \perp \overline{DC}$ $\therefore$ $\overline{AD}$ // $\overline{BE}$

$\therefore$ The figure ADEB is a rectangle.

$\therefore$ AD = BE = 12 cm. $\therefore$ BE = 12 cm.

In the right-angled triangle BEC at E

$(EC)^2 = (BC)^2 - (BE)^2 = 169 - 144 = 25$

$\therefore$ EC = 5 cm. (First req.)

2 $\overline{DE}$ is the projection of $\overline{AB}$ on $\overleftrightarrow{DC}$

$\because$ DE = DC – EC

$\therefore$ DE = 25 – 5 = 20 cm. (Second req.)

3 $\because$ $\overline{DC}$ // $\overleftrightarrow{AB}$

$\therefore$ The length of the projection of $\overline{DC}$ on $\overleftrightarrow{AB}$ = length of $\overline{DC}$ = 25 cm. (Third req.)

4 $\because$ DE = AB $\therefore$ AB = 20 cm.

$\therefore$ the area of the trapezium ABCD

$= \frac{1}{2}(25 + 20) \times 12 = 270$ cm^2. (Fourth req.)

13

1. $\overline{AC}$ is the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$

In the right-angled triangle ACB at C

$(AC)^2 = (AB)^2 - (BC)^2 = 169 - 25 = 144$

$\therefore$ AC = 12 cm. (First req.)

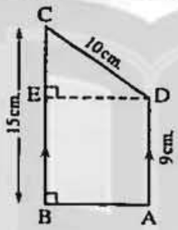
2. $\overline{AD}$ is the projection of $\overline{CD}$ on $\overleftrightarrow{AD}$

In the right-angled triangle DAC at A

$\therefore (AD)^2 = (CD)^2 - (AC)^2 = 225 - 144 = 81$

$\therefore$ AD = 9 cm. (Second req.)

14



Construction : Draw $\overline{DE} \perp \overline{BE}$

Proof :

1. $\overline{EC}$ is the projection of $\overline{DC}$ on $\overleftrightarrow{BC}$

$\because \overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{BC}$

$\therefore \overline{AB} \,//\, \overline{DE}$

$\therefore$ The figure ABED is a rectangle.

$\therefore$ AD = BE = 9 cm

$\therefore$ EC = BC − BE = 15 − 9 = 6 cm. (First req.)

2. $\overline{AB}$ is the projection of $\overline{DC}$ on $\overleftrightarrow{AB}$

In the right-angled triangle DEC at E

$\therefore (DE)^2 = (DC)^2 - (EC)^2 = 100 - 36 = 64$

$\therefore$ DE = 8 cm.

$\because$ DE = AB

$\therefore$ AB = 8 cm. (Second req.)

15

$\therefore \overline{EC}$ is the projection of $\overline{DC}$ on $\overleftrightarrow{BC}$

$\because$ ABCD is a parallelogram. $\therefore$ AB = DC

$\therefore$ DC = 13 cm., DE = $\frac{192}{16}$ = 12 cm.

In the right-angled triangle DEC at E

$(EC)^2 = (DC)^2 - (DE)^2 = 169 - 144 = 25$

$\therefore$ EC = 5 cm. (The req.)

16

1. (a) the point D (b) the point E

(c) $\overline{AE}$ (d) $\overline{AD}$

2. $\overline{DB}$ is the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$

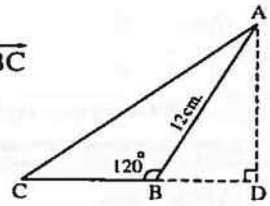
$\because AD = \frac{2 \times 336}{28} = 24$ cm.

In the right-angled triangle ADB at D

$\therefore (BD)^2 = (AB)^2 - (AD)^2 = 900 - 576 = 324$

$\therefore$ BD = 18 cm. (The req.)

17



Construction : Draw $\overline{AD} \perp \overleftrightarrow{BC}$

Proof : $\overline{DB}$ is the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$

$\because$ m ($\angle$ ABC) = 120°

$\therefore$ In the right-angled triangle ADB at D ,

m ($\angle$ ABD) = 60° , m ($\angle$ DAB) = 30°

$\therefore BD = \frac{1}{2} AB$

$\therefore$ BD = 6 cm. (The req.)

Answers of Exercise 9

1

1. $(AD)^2 + (DC)^2$
2. $(BC)^2 - (AB)^2$
3. CD × CB
4. BD × DC
5. BC × AD
6. DBA , DAC

2

1. 3
2. 3.2
3. 2.4
4. 2.16

3

$\because \Delta$ XYL is right-angled at L

$\therefore (XY)^2 = (XL)^2 + (YL)^2 = 81 + 144 = 225$

$\therefore$ XY = 15 cm. (First req.)

$\because \Delta$ XYZ is right-angled at Y , $\overline{YL} \perp \overline{XZ}$

$\therefore (YL)^2 = LZ \times LX$ $\therefore 144 = LZ \times 9$

$\therefore LZ = \frac{144}{9} = 16$ cm. (Second req.)

$\because (ZY)^2 = LZ \times ZX = 16 \times 25 = 400$

$\therefore$ ZY = 20 cm. (Third req.)

4

$\because$ ABCD is a rectangle. $\therefore$ m ($\angle$ ABC) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 36 + 64 = 100$

$\therefore$ AC = 10 cm.

$\because \overline{BE} \perp \overline{AC}$

$\therefore BE = \frac{AB \times BC}{AC} = \frac{6 \times 8}{10} = 4.8$ cm. (First req.)

$(BC)^2 = CE \times AC$ $\therefore 64 = CE \times 10$

$\therefore CE = \frac{64}{10} = 6.4$ cm. (Second req.)

5

$\because \Delta ABC$ is right-angled at A , $\overline{AD} \perp \overline{BC}$

$\therefore (BC)^2 = (AB)^2 + (AC)^2 = 64 + 36 = 100$

$\therefore BC = 10$ cm.

$(AB)^2 = BD \times BC$

$\therefore 64 = BD \times 10$

$\therefore BD = 6.4$ cm. (First req.)

$(AC)^2 = CD \times BC$

$\therefore 36 = CD \times 10$

$\therefore CD = 3.6$ cm. (Second req.)

$(AD)^2 = BD \times CD = 6.4 \times 3.6 = 23.04$

$\therefore AD = 4.8$ cm. (Third req.)

6

$\because \Delta ABC$ is right-angled at B , $\overline{BD} \perp \overline{AC}$

$\therefore (AB)^2 = AD \times AC = 4.5 \times 12.5 = 56.25$

$\therefore AB = 7.5$ cm. (First req.)

$\because (BC)^2 = CD \times CA = 8 \times 12.5 = 100$

$\therefore BC = 10$ cm.

$\because (BD)^2 = DA \times DC = 4.5 \times 8 = 36$ $\therefore BD = 6$ cm.

7

$\because \Delta BCD$ is right-angled at C

$\therefore (BD)^2 = (BC)^2 + (CD)^2 = 49 + 576 = 625$

$\therefore BD = 25$ cm. (First req.)

$\because \Delta ABD$ is right-angled at A

$\therefore (AD)^2 = (BD)^2 - (AB)^2 = 625 - 225 = 400$

$\therefore AD = 20$ cm. (Second req.)

$\because \overline{BE}$ is the projection of $\overline{AB}$ on $\overleftrightarrow{BD}$

, $(AB)^2 = BE \times BD$ $\therefore 225 = BE \times 25$

$\therefore BE = 9$ cm. (Third req.)

$\because \overline{AE}$ is the projection of $\overline{AD}$ on $\overleftrightarrow{AE}$

$\therefore AE = \frac{AB \times AD}{BD} = \frac{15 \times 20}{25} = 12$ cm. (Fourth req.)

8

$\therefore \overline{XY}$ is the projection of $\overline{AY}$ on $\overleftrightarrow{XE}$

$\because \Delta AXY$ is right-angled at X

$\therefore (XY)^2 = (AY)^2 - (AX)^2 = 100 - 64 = 36$

$\therefore XY = 6$ cm. (First req.)

$\because \overline{XF} \perp \overline{AY}$

$\therefore XF = \frac{AX \times XY}{AY} = \frac{6 \times 8}{10} = 4.8$ cm. (Second req.)

$(AX)^2 = AF \times AY$ $\therefore 64 = AF \times 10$

$\therefore AF = 6.4$ cm. (Third req.)

$\because \Delta AXE$ is right angled at X

$\therefore (EX)^2 = (AE)^2 - (AX)^2 = 289 - 64 = 225$

$\therefore EX = 15$ cm.

$\therefore$ The area of $\Delta AXE = \frac{1}{2} \times 8 \times 15 = 60$ cm^2.

(Fourth req.)

9

$\because \overline{LZ}$ is the projection of $\overline{YZ}$ on $\overleftrightarrow{XZ}$

ΔYLZ is right-angled at L

$\therefore (LZ)^2 = (YZ)^2 - (YL)^2 = 144 - 92.16 = 51.84$

$\therefore LZ = 7.2$ cm. (First req.)

$\because \overline{XL}$ is the projection of $\overline{XY}$ on $\overleftrightarrow{XZ}$

, ΔXYZ is right-angled at Y , $\overline{YL} \perp \overline{XZ}$

$\therefore (YL)^2 = ZL \times XL$ $\therefore (9.6)^2 = 7.2 \times XL$

$\therefore XL = \frac{(9.6)^2}{7.2} = 12.8$ cm. (Second req.)

$\because \overline{XY}$ is the projection of $\overline{XZ}$ on $\overleftrightarrow{XY}$

, ΔXYZ is right-angled at Y , $\overline{YL} \perp \overline{XZ}$

$\therefore (YX)^2 = XL \times XZ = 12.8 \times (7.2 + 12.8)$

$(YX)^2 = 256$ $\therefore YX = 16$ cm. (Third req.)

10

$\because ABCD$ is a rectangle

$\therefore AB = DC = 30$ cm.

$\because \Delta ADC$ is right-angled at D , $\overline{DF} \perp \overline{AC}$

$\therefore (AC)^2 = (AD)^2 + (DC)^2 = 1600 + 900 = 2500$

$\therefore AC = 50$ cm.

$\because (AD)^2 = AF \times AC$

$\therefore 1600 = AF \times 50$

$\therefore AF = \frac{1600}{50} = 32$ cm. (First req.)

$DF = \frac{AD \times DC}{AC} = \frac{40 \times 30}{50} = 24$ cm. (Second req.)

$\because \Delta DCE$ is right-angled at C , $\overline{CF} \perp \overline{DE}$

$\therefore (DC)^2 = DF \times DE$ $\therefore 900 = 24 \times DE$

$\therefore DE = \frac{900}{24} = 37.5$ cm.

$\because FE = DE - DF$

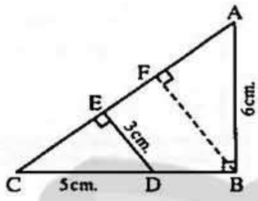
$\therefore FE = 37.5 - 24 = 13.5$ cm.

$\because (CE)^2 = FE \times DE$

$\therefore (CE)^2 = 13.5 \times 37.5 = 506.25$

$\therefore CE = 22.5$ cm. (Third req.)

11



In $\Delta\Delta$ CBA , CED which are right-angled at B and E respectively.

$\because \angle C$ is a common angle.

$\therefore \Delta CED \sim \Delta CBA$ (First req.)

$\therefore \frac{AB}{DE} = \frac{AC}{DC}$

$\therefore AC = \frac{5 \times 6}{3} = 10$ cm. (Second req.)

$\because \Delta ABC$ is right-angled at B

$\therefore (BC)^2 = (AC)^2 - (AB)^2 = 100 - 36 = 64$

$\therefore BC = 8$ cm.

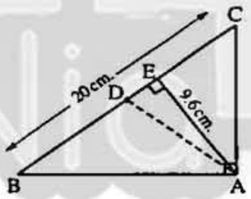
$\overline{AF}$ is the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$, $\overline{BF} \perp \overline{AC}$

$(AB)^2 = AF \times AC$

$\therefore 36 = AF \times 10$

$AF = \frac{36}{10} = 3.6$ cm. (Third req.)

12



Construction : Draw $\overline{AD}$

Proof :

$\because \overline{AD}$ is a median in the right-angled triangle ABC drawn from the vertex of the right angle.

$\therefore AD = \frac{1}{2} BC = 10$ cm.

$\because \Delta AED$ is right-angled at E

$\therefore (ED)^2 = (AD)^2 - (AE)^2 = 100 - 92.16 = 7.84$

$\therefore ED = 2.8$ cm.

$\therefore BE = BD + DE = 10 + 2.8 = 12.8$ cm.

, $CE = CD - ED = 10 - 2.8 = 7.2$ cm.

$\because \Delta ABC$ is right-angled at A , $\overline{AE} \perp \overline{BC}$

$\therefore (AB)^2 = BE \times CB = 12.8 \times 20 = 256$

$\therefore AB = 16$ cm. (First req.)

$(AC)^2 = CE \times CB = 7.2 \times 20 = 144$

$\therefore AC = 12$ cm. (Second req.)

13

$\because \Delta ABD$ is right-angled at B

$\therefore (BD)^2 = (AD)^2 - (AB)^2 = 100 - 36 = 64$

$\therefore BD = 8$ cm.

$\therefore$ the area of $\square$ ABCD $= AB \times BD = 6 \times 8 = 48$ cm^2 (First req.)

$\because \overline{AB} \parallel \overline{DC}$ (Properties of parallelogram)

, $\overline{BD}$ is a transversal.

$\therefore m(\angle ABD) = m(\angle CDB)$ (alternate angles)

$\therefore m(\angle CDB) = 90°$

$\because AB = DC$, $AD = BC$ (properties of parallelogram)

$\therefore DC = 6$ cm. , $BC = 10$ cm.

$\because \Delta DBC$ is right-angled at D , $\overline{DE} \perp \overline{BC}$

$\therefore \overline{BE}$ is the projection of $\overline{DB}$ on $\overleftrightarrow{BC}$

$\therefore (BD)^2 = BE \times BC$ $\therefore 64 = BE \times 10$

$\therefore BE = \frac{64}{10} = 6.4$ cm. (Second req.)

, $DE = \frac{DB \times DC}{BC} = \frac{8 \times 6}{10} = 4.8$ cm. (Third req.)

14

$\because \overline{DB} \perp \overline{AB}$

$\therefore$ The area of $\square$ ABCD $= AB \times DB$

$\therefore 192 = AB \times 16$ $\therefore AB = 12$ cm.

$\because \Delta ABD$ is right-angled at B

$\therefore (AD)^2 = (AB)^2 + (BD)^2 = 144 + 256 = 400$

$\therefore AD = 20$ cm.

$\because \Delta ABD$ is right-angled at B , $\overline{BF} \perp \overline{AD}$

$\therefore BF = \frac{AB \times BD}{AD} = \frac{12 \times 16}{20} = 9.6$ cm.

$\because (DB)^2 = DF \times DA$

$\therefore 256 = DF \times 20$ $\therefore DF = 12.8$ cm.

$\therefore$ the area of the rectangle BEDF $= FD \times BF$

$= 12.8 \times 9.6 = 122.88$ cm^2 (The req.)

15

$\because \overline{AD} \parallel \overline{EC}$, $\overline{AE} \parallel \overline{DC}$

$\therefore$ The figure AECD is a parallelogram.

$\therefore AD = EC = 6$ cm.

$\because$ E is the midpoint of $\overline{BC}$

$\therefore BC = 2 \times 6 = 12$ cm.

$\because$ The area of the trapezium ABCD $= 72$ cm^2

$\therefore 72 = \frac{1}{2}(AD + BC) \times AB$

$\therefore AB = \frac{72 \times 2}{18} = 8$ cm.

$\because \Delta$ ABE is right-angled at B

$\therefore (AE)^2 = (AB)^2 + (BE)^2 = 64 + 36 = 100$

$\therefore AE = 10$ cm.

$\therefore BF = \frac{AB \times BE}{AE} = \frac{8 \times 6}{10} = 4.8$ cm. (The req.)

16

$\because \overline{AB} \parallel \overline{DC}$, $\overline{BC}$ is a transversal to them.

$\therefore m(\angle DCB) + m(\angle ABC) = 180°$

(Two interior angles in one side of the transversal)

$\therefore m(\angle DCB) = 180° - 90° = 90°$

$\because \Delta$ DCE is right-angled at C

$\therefore (DE)^2 = (DC)^2 + (CE)^2 = 81 + (CE)^2$ (1)

, Δ ABE is right-angled at B

$\therefore (AE)^2 = (AB)^2 + (BE)^2 = 256 + (BE)^2$ (2)

$\because \Delta$ DEA is right-angled at E

$\therefore (DA)^2 = (DE)^2 + (AE)^2$

$\therefore 625 = (DE)^2 + (AE)^2$ (3)

From (1) , (2) and (3) :

$\therefore 625 = 81 + 256 + (CE)^2 + (BE)^2$

$\because BE = EC$ $\therefore 625 = 337 + 2(BE)^2$

$\therefore 2(BE)^2 = 288$ $\therefore (BE)^2 = 144$

$\therefore BE = 12$ cm. , $CE = 12$ cm. , $BC = 24$ cm.

The area of the trapezium ABCD

$= \frac{1}{2}(DC + AB) \times BC$

$= \frac{1}{2}(9 + 16) \times 24 = 300\ cm^2$ (First req.)

From (2) : $\therefore (AE)^2 = 256 + 144 = 400$

$\therefore AE = 20$ cm.

$\because \Delta$ DEA is right-angled at E , $\overline{EF} \perp \overline{DA}$

$\therefore \overline{AF}$ is the projection of $\overline{AE}$ on $\overline{AD}$

$\therefore (AE)^2 = AF \times AD$ $\therefore 400 = AF \times 25$

$\therefore AF = 16$ cm. (Second req.)

17

$\because \Delta$ ABC is right-angled at B

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 225 + 400 = 625$

$\therefore AC = 25$ cm.

$\because \Delta$ ABC is right-angled at B , $\overline{BD} \perp \overline{AC}$

$\therefore (BA)^2 = AD \times AC$ $\therefore (15)^2 = AD \times 25$

$\therefore AD = \frac{225}{25} = 9$ cm. $\therefore DC = 16$

$\because (BD)^2 = DA \times DC$ $\therefore (BD)^2 = 9 \times 16 = 144$

$\therefore BD = 12$ cm.

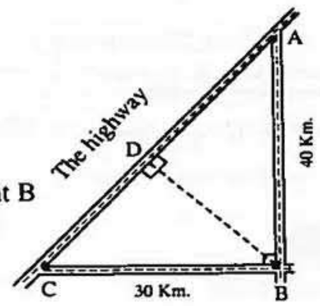
$\because \Delta$ DBC is right-angled at D , $\overline{DF} \perp \overline{BC}$

$\therefore DF = \frac{DB \times DC}{BC} = \frac{12 \times 16}{20} = 9.6$ cm. (First req.)

$\because \Delta$ DAB is right-angled at D , $\overline{DE} \perp \overline{AB}$

$\therefore DE = \frac{AD \times BD}{AB} = \frac{9 \times 12}{15} = 7.2$ cm. (Second req.)

18



Construction :

Draw $\overline{BD} \perp \overline{AC}$

Proof :

$\because \Delta$ ABC is right angled at B

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$= 1600 + 900 = 2500$

$\therefore AC = 50$ km.

, $\because \overline{BD} \perp \overline{AC}$

$\therefore BD = \frac{AB \times BC}{AC} = \frac{40 \times 30}{50} = 24$ km. (The req.)

19

The first method :

$\because \Delta$ ABC is right-angled at A , $\overline{AD} \perp \overline{BC}$

$\therefore (DA)^2 = DB \times DC$

$\therefore (4.8)^2 = 3.6 \times DC$

$\therefore DC = \frac{(4.8)^2}{3.6} = 6.4$ km. (the req.)

The second method :

$\because \Delta$ ABD is right-angled at D

$\therefore (AB)^2 = (AD)^2 + (BD)^2 = 23.04 + 12.96 = 36$

$\therefore AB = 6$ km.

$\therefore$ In $\Delta\Delta$ ABC , DBA :

$m(\angle BAC) = m(\angle BDA) = 90°$

, $\angle B$ is common

$\therefore m(\angle C) = m(\angle BAD)$

$\therefore \Delta ABC \sim \Delta DBA$

$\therefore \frac{AB}{DB} = \frac{BC}{BA}$ $\therefore \frac{6}{3.6} = \frac{BC}{6}$

$\therefore BC = \frac{6 \times 6}{3.6} = 10$ km.

$\therefore DC = BC - BD = 10 - 3.6 = 6.4$ km. (The req.)

20

Let $CD = x$ cm. , $\therefore BD = (25 - x)$ cm.

In Δ ABC $\because m(\angle A) = 90°$, $\overline{AD} \perp \overline{BC}$

$\therefore (AD)^2 = CD \times DB$ $\therefore 144 = x(25 - x)$

$\therefore 144 = 25x - x^2$ $\therefore x^2 - 25x + 144 = 0$

$(x - 9)(x - 16) = 0$ $\therefore x = 9$ cm.

or $x = 16$ (refused because CD < BD)

$\therefore (AB)^2 = BD \times BC = 16 \times 25 = 400$

$\therefore AB = 20$ cm.

$\overline{BD}$ is the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$, BD = 16 cm. (First req.)

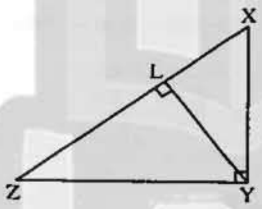
$(AC)^2 = CD \times CB = 9 \times 25 = 225$

$\therefore AC = 15$ cm.

$\overline{DC}$ is the projection of $\overline{AC}$ on $\overleftrightarrow{BC}$

$\therefore DC = 9$ cm. (Second req.)

21



In Δ XYZ ,

$\because m(\angle Y) = 90°$

, $\overline{YL} \perp \overline{XZ}$

$\therefore (XY)^2 = XL \times XZ$

But $XL = \frac{9}{25} XZ$

$\therefore (XY)^2 = \frac{9}{25} XZ \times XZ$ $\therefore XY = \frac{3}{5} XZ$ (1)

$\because (YZ)^2 = ZL \times XZ$ $\because XL = \frac{9}{25} XZ$

$\therefore ZL = \frac{16}{25} XZ$

$\therefore (YZ)^2 = \frac{16}{25} XZ \times XZ$ $\therefore YZ = \frac{4}{5} XZ$ (2)

From (1) and (2) :

$\therefore XY : YZ = \frac{3}{5} XZ : \frac{4}{5} XZ = 3 : 4$ (Q.E.D.)

Answers of Exercise 10

1

1 $\because$ The longest side is $\overline{AC}$, $(AC)^2 = (15)^2 = 225$,

, $\because (AB)^2 + (BC)^2 = (12)^2 + (14)^2$
$= 144 + 196 = 340$

$\therefore (AC)^2 < (AB)^2 + (BC)^2$

$\therefore \Delta$ ABC is an acute-angled triangle

2 $\because$ The longest side is $\overline{AB}$, $(AB)^2 = (8)^2 = 64$

, $\because (AC)^2 + (BC)^2 = (3)^2 + (7)^2 = 9 + 49 = 58$

$\therefore (AB)^2 > (AC)^2 + (BC)^2$

$\therefore \Delta$ ABC is an obtuse-angled triangle at C

3 $\because$ The longest side is $\overline{AB}$

, $(AB)^2 = (25)^2 = 625$

, $\because (AC)^2 + (BC)^2 = (20)^2 + (15)^2$
$= 400 + 225 = 625$

$\therefore (AB)^2 = (AC)^2 + (BC)^2$

$\therefore \Delta$ ABC is right-angled at C

2

$\because (XZ)^2 = (7)^2 = 49$,

$\because (XY)^2 + (YZ)^2 = (4)^2 + (5)^2 = 41$

$\therefore (XZ)^2 > (XY)^2 + (YZ)^2$ $\therefore$ Y is obtuse.

3

$\because (BC)^2 = (10)^2 = 100$

, $\because (AB)^2 + (AC)^2 = (6)^2 + (8)^2 = 36 + 64 = 100$

$\therefore (BC)^2 = (AB)^2 + (AC)^2$ $\therefore \angle A$ is right.

4

$\because (AC)^2 = (15)^2 = 225$

$\because (AB)^2 + (BC)^2 = (10)^2 + (12)^2 = 100 + 144 = 244$

$\therefore (AC)^2 < (AB)^2 + (BC)^2$

$\therefore \angle B$ is acute.

5

1 $\because$ The longest side is $\overline{AC}$

$\therefore \angle B$ which is opposite to $\overline{AC}$ is the greatest in measure.

$\because (AC)^2 = (12)^2 = 144$,

, $(AB)^2 + (BC)^2 = (9)^2 + (10)^2 = 81 + 100 = 181$

$\therefore (AC)^2 < (AB)^2 + (BC)^2$

$\therefore \angle B$ is acute angle.

$\therefore \Delta$ ABC is an acute-angled triangle.

2 $\because$ The longest side is $\overline{AC}$

$\therefore \angle B$ which is opposite to $\overline{AC}$ is the greatest in measure

$\because (AC)^2 = (13)^2 = 169$

, $(AB)^2 + (BC)^2 = (5)^2 + (12)^2 = 25 + 144 = 169$

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$\therefore \angle B$ is a right-angle.

$\therefore \Delta$ ABC is right-angled triangle.

3 $\because$ The longest side is $\overline{BC}$

$\therefore \angle A$ which is opposite to $\overline{BC}$ is the greatest in measure.

$\because (BC)^2 = (16)^2 = 256$

$, (AB)^2 + (AC)^2 = (7)^2 + (14)^2 = 49 + 196 = 245$

$\therefore (BC)^2 > (AB)^2 + (AC)^2$

$\therefore \angle A$ is an obtuse angle.

$\therefore \Delta ABC$ is an obtuse-angled triangle.

6 $\overline{BD}$ is the projection of $\overline{AD}$ on $\overleftrightarrow{BD}$

$\because \Delta ABD$ is right-angled at B

$\therefore (BD)^2 = (AD)^2 - (AB)^2 = 289 - 64 = 225$

$\therefore BD = 15$ cm. (First req.)

In ΔBCD : the longest side is $\overline{BD}$

$, \because (BD)^2 = 225 , (BC)^2 + (CD)^2 = 81 + 144 = 225$

$\therefore (BD)^2 = (BC)^2 + (CD)^2$

$\therefore \Delta BCD$ is right-angled at C (Second req.)

7

$\because AB = DC$ (properties of parallelogram)

$\therefore AB = 8$ cm.

In ΔABC :

$\because (AC)^2 = 361$

$(AB)^2 + (BC)^2 = 64 + 225 = 289$

$\therefore (AC)^2 > (AB)^2 + (BC)^2$

$\therefore \angle ABC$ is obtuse angle. (Q.E.D.)

8

In ΔABC :

$\because m(\angle B) = 90°$

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 1024 + 576 = 1600$

$\therefore AC = 40$ cm.

In ΔACD :

$\because$ The longest side is $\overline{AD}$

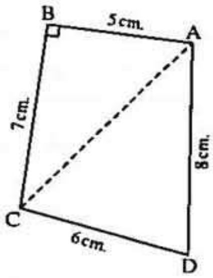
$, \because (AD)^2 = 2025$

$, (AC)^2 + (CD)^2 = 1600 + 81 = 1681$

$\therefore (AD)^2 > (AC)^2 + (CD)^2$

$\therefore \Delta ACD$ is an obtuse-angled at C (Q.E.D.)

9



Construction : Draw $\overline{AC}$

Proof : In ΔABC

$\because m(\angle B) = 90°$

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$= 25 + 49 = 74$

In ΔADC :

$\because (AC)^2 = 74 , (AD)^2 + (DC)^2 = 64 + 36 = 100$

$\therefore (AC)^2 < (AD)^2 + (DC)^2$

$\therefore \angle D$ is an acute-angle. (Q.E.D.)

10

In ΔABC :

$\because (AB)^2 + (BC)^2 = 196 + 2304 = 2500$

$, (AC)^2 = 2500$ $\therefore (AB)^2 + (BC)^2 = (AC)^2$

$\therefore \Delta ABC$ is right-angled at B

$\because \overline{BD}$ is a median drawn from the vertex of the right angle.

$\therefore BD = \frac{1}{2} AC = 25$ cm.

In ΔDBC :

$\because (BC)^2 = 2304$

$, (BD)^2 + (DC)^2 = 625 + 625 = 1250$

$\therefore (BC)^2 > (BD)^2 + (DC)^2$

$\therefore \angle BDC$ is obtues-angle. (Q.E.D.)

11

$\because$ ABCD is a rectangle.

$\therefore AD = BC = 24 , DC = AB = 16$ cm.

$, \because$ F is the midpoint of $\overline{AD}$

$\therefore AF = FD = 12$ cm. $\because DC = 16$ cm.

$, DE = 9$ cm. $\therefore EC = 7$ cm.

$\because \Delta FAB$ is right-angled at A

$\therefore (FB)^2 = (AB)^2 + (AF)^2 = 256 + 144 = 400$

$, \Delta FDE$ is right-angled at D

$\therefore (FE)^2 = (FD)^2 + (DE)^2 = 144 + 81 = 225$

$\because \Delta BCE$ is right-angled at C

$\therefore (BE)^2 = (BC)^2 + (EC)^2 = 576 + 49 = 625$

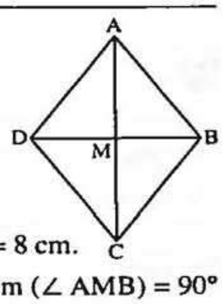
In ΔBFE : $\because (BE)^2 = 625$

$\because (BF)^2 + (FE)^2 = 400 + 225 = 625$

$\therefore (BE)^2 = (BF)^2 + (FE)^2$

$\therefore \Delta BFE$ is right-angled at F (Q.E.D.)

12



Let $\overline{AC} \cap \overline{BD} = \{M\}$

$\because$ The two diagonals of the rhombus are perpendicular and each of them bisects the other.

$\therefore MB = MD = 6$ cm. $, AM = MC = 8$ cm.

$, \overline{AM} \perp \overline{BD}$ $\therefore m(\angle AMB) = 90°$

In Δ ABM :

$(AB)^2 = (AM)^2 + (MB)^2 = 64 + 36 = 100$

$\therefore$ AB = 10 cm.

$\because$ ABCD is a rhombus $\qquad \therefore$ AD = AB = 10 cm.

In Δ ABD :

$\because \overline{BD}$ is the longest side , BD = 12 cm.

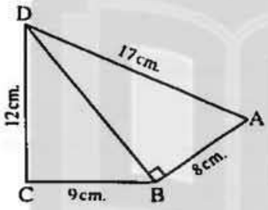
, $\therefore (BD)^2 = 144$

, $\because (AB)^2 + (AD)^2 = 100 + 100 = 200$

$\therefore (BD)^2 < (AB)^2 + (AD)^2$

$\therefore \Delta$ ABD is acute-angled. (Q.E.D.)

13



$\overline{BD}$ is the projection of $\overline{AD}$ on $\overleftrightarrow{BD}$

$\because \Delta$ ABD is right-angled at B

$\therefore (BD)^2 = (AD)^2 - (AB)^2$

$= 289 - 64 = 225$

$\therefore$ BD = 15 cm. (First req.)

In Δ BCD :

$\because \overline{BD}$ is the longest side.

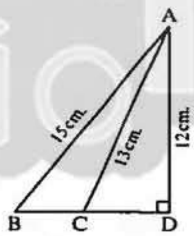
$\because (BD)^2 = 225$

$(BC)^2 + (DC)^2 = 81 + 144 = 225$

$\therefore (BD)^2 = (BC)^2 + (DC)^2$

$\therefore \Delta$ BCD is right-angled at C (Q.E.D.)

14



$\because \Delta$ ADB is right-angled at D

$\therefore (DB)^2 = (AB)^2 - (AD)^2$

$= 225 - 144 = 81$

$\therefore$ DB = 9 cm.

$\because \Delta$ ADC is right-angled at D

$\therefore (DC)^2 = (AC)^2 - (AD)^2 = 169 - 144 = 25$

$\therefore$ DC = 5 $\qquad \therefore$ CB = DB − DC = 9 − 5 = 4 cm.

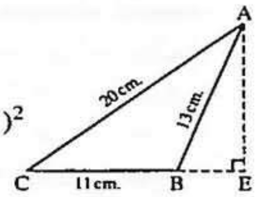
15

1 a	2 a	3 c
4 b	5 b	

16

1 C	2 acute	3 obtuse
4 right	5 acute	6 acute
7 acute	8 obtuse	9 >
10 =	11 2 cm. , 8 cm.	12 obtuse-angled
13 acute		

17



In Δ ABC

$\because (AC)^2 = 400$

, $\because (AB)^2 + (BC)^2 = (13)^2 + (11)^2$

$= 169 + 121 = 290$

$\therefore (AC)^2 > (AB)^2 + (BC)^2$

$\therefore \Delta$ ABC is obtuse-angled at B (First req.)

Draw $\overleftrightarrow{AE} \perp \overleftrightarrow{BC}$ such that

$\overleftrightarrow{AE} \cap \overleftrightarrow{BC} = \{E\}$

, $\because \overline{AE} \perp \overleftrightarrow{BC}$

$\therefore \overline{EB}$ is the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$

$\therefore$ From Δ AEB $\qquad \because m(\angle E) = 90°$

$\therefore (AE)^2 = (AB)^2 - (EB)^2 = 169 - (EB)^2$ (1)

From Δ AEC :

$\because m(\angle E) = 90°$

$(AE)^2 = (AC)^2 - (EC)^2 = 400 - (EB + 11)^2$ (2)

From (1) and (2) :

$\therefore 169 - (EB)^2 = 400 - (EB + 11)^2$

$\therefore (EB + 11)^2 - (EB)^2 = 400 - 169$

$\therefore (EB)^2 + 22\ EB + 121 - (EB)^2 = 231$

$\therefore$ 22 EB = 231 − 121 $\qquad \therefore$ 22 EB = 110

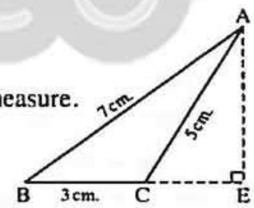
$\therefore EB = \frac{110}{22} = 5$ cm. (Second req.)

$(AE)^2 = 169 - (5)^2 = 169 - 25 = 144$

$\therefore$ AE = 12 cm.

$\therefore$ the area of $\Delta ABC = \frac{1}{2}$ BC × AE $= \frac{11 \times 12}{2} = 66\ cm^2$

18



$\because$ The longest side is $\overline{AB}$

$\therefore \angle$ ACB is the greatest angle in measure.

and it is an obtuse angle for :

$(AB)^2 > (AC)^2 + (CB)^2$

Construction : Draw $\overleftrightarrow{AE} \perp \overleftrightarrow{BC}$ where $\overline{AE} \cap \overleftrightarrow{BC} = \{E\}$

Proof : $\because \Delta$ AEC is right-angled at E

$\therefore (AE)^2 = (AC)^2 - (EC)^2$

$\therefore (AE)^2 = 25 - (EC)^2$ (1)

$\because \Delta$ AEB is right-angled at E

$\therefore (AE)^2 = (AB)^2 - (EB)^2$

$\therefore (AE)^2 = 49 - (EC + 3)^2$ (2)

From (1) and (2) :

$25 - (EC)^2 = 49 - [(EC)^2 + 6\ EC + 9]$

$\therefore 25 - (EC)^2 = 49 - (EC)^2 - 6\ EC - 9$

$\therefore 25 = 40 - 6\ EC$ $\therefore 6\ EC = 40 - 25$

$\therefore 6\ EC = 15$ $\therefore EC = 2.5$ cm.

$, \because EC = \frac{1}{2} AC$ $\therefore m(\angle EAC) = 30°$

$\therefore m(\angle ACE) = 60°$

$\therefore m(\angle ACB) = 180° - 60° = 120°$ (The req.)

Answers of exams on unit five

Model 1

1
1. a
2. c
3. b
4. b
5. b
6. a

2
1. similar
2. $\overline{BC}$
3. acute
4. proportional
5. the same point

3 [a] 1. Prove by yourself 2. 18 cm.

[b] Prove by yourself

4 [a] acute-angled triangle

[b] Prove by yourself , AB = 8 cm.

5 [a] AB = 15 cm. , BC = 20 cm. , BD = 12 cm.

[b] Prove by yourself , BD = 6.4 cm.

Model 2

1
1. d
2. c
3. c
4. b
5. a
6. a

2
1. BC × AD
2. equal in measure
3. 60°
4. 30°
5. CD , CB

3 [a] Prove by yourself [b] BD = 4 cm.

4 [a] BD = 9 cm. , DC = 16 cm. , AD = 12 cm.

[b] $\angle B$, acute

5

[a] 1. Prove by yourself 2. DE = 7.2 cm.

3. The length of the projection of $\overline{AD}$ on $\overleftrightarrow{AB}$ = AE = 9.6 cm.

[b] AB = 8 cm.

Answers of accumulative basic skills

1

1 b	2 d	3 c	4 c	5 d
6 d	7 c	8 b	9 b	10 b
11 a	12 c	13 d	14 a	15 c

2

1 16	2 120°	3 12	4 $20 + 20\sqrt{3}$
5 60°	6 14	7 120	8 56
9 42	10 $10\sqrt{2}$	11 118°	12 115°

Guide Answers

of The Notebook

- Quizzes.
- Final Examinations.

Answers of the quizzes on Algebra and Statistics

Quiz 1

1

[1] c [2] a [3] d

2

[a] [1] $(x+3y)(x-8y)$

[2] $3(x-2)(x-3)$

[b] [1] 1 [2] 12

«Try by yourself to get other values of k»

Quiz 2

1

[1] b [2] a [3] b

2

[a] The dimensions are : $(2x+5)$ cm. , $(x+7)$ cm.

The perimeter = 30 cm.

[b] [1] $(x-6)(x-1)$

[2] $(2x+1)(x-4)$

Quiz 3

1

[1] c [2] d [3] a

2

[a] [1] $(x-5)(x-2)$

[2] $2x(x-5)^2$

[b] 100

Quiz 4

1

[1] a [2] b [3] d

2

[a] [1] $(2x-3)(x+2)$

[2] $3(x-3)(x+3)$

[3] $(3b-4)(3b+4)(a-5b)$

[b] 9800

Quiz 5

1

[1] b [2] a [3] d

2

[a] [1] $(x-2)(x+8)$

[2] $(x-3)(x^2+3x+9)$

[3] $(2a-3b)(a-2b)$

[b] $(x-2)(x^2+2x+4)(x+2)(x^2-2x+4)$

Quiz 6

1

[1] d [2] b [3] b

2

[a] [1] $\frac{1}{2}(x-2)(x^2+2x+4)$

[2] $(2x+y+3)(2x+y-3)$

[3] $(x+y)(x^2-xy+y^2-1)$

[b] $(x-y)(x^2+xy+y^2)(x+y)(x^2-xy+y^2)$

Quiz 7

1

[1] b [2] c [3] a

2

[1] $5(2x^2+y^2-2xy)(2x^2+y^2+2xy)$

[2] $(x-y)(x+y)(x+2y)(x-2y)$

[3] $3x(x+5)(x-5)$

[4] $(x-6)(y-5)$

Quiz 8

1

[1] b [2] c [3] d

2

[a] [1] $(5x+6)(x-2)$

[2] $(a+1)(a+3)(a-3)$

[b] [1] The S.S. = $\{-3, 10\}$

[2] The S.S. = $\{-1, 3\}$

Quiz (9)

1

1 c   2 b   3 a

2

[a] 1 or $-\frac{1}{2}$

[b] 25 cm.

Quiz (10)

1

1 c   2 c   3 a

2

[a] Prove by yourself.

[b] The dimensions of the rectangle are : 7 cm. and 12 cm.

The perimeter of the rectangle = 38 cm.

Quiz (11)

1

1 d   2 b   3 a

2

[a] 1 $(2x-3)(x+2)$

2 $(2x-3)(4x^2+6x+9)$

[b] $n=2$

Quiz (12)

1

1 d   2 a   3 a

2

[a] 1 The S.S. $=\{1, -1\}$

2 The S.S. $=\{6\}$

[b] $\frac{1}{64}$

Quiz (13)

1

1 d   2 c   3 c

2

[a] 1 $(3x+2y)(9x^2-6xy+4y^2)$

2 $(3a+b)(2a-7b)$

[b] 1 9 games   2 3 games

Answers of school book examinations on algebra and statistics

Model 1

1

1. -3
2. 8
3. $\{3\}$
4. $\frac{1}{27}$
5. 25

2

1 (a) 2 (b) 3 (c) 4 (c) 5 (b) 6 (b)

3

1. $(x+3)(x+5)$
2. $(2x+1)(x+3)$
3. $(x-1)(x^2+x+1)$
4. $a(x-7)+3(x-7)=(x-7)(a+3)$

4

[a] $\frac{(2)^{2n}\times(2\times3)^{2n}}{2^{4n}\times3^{2n}}=\frac{2^{2n}\times2^{2n}\times3^{2n}}{2^{4n}\times3^{2n}}=\frac{2^{4n}}{2^{4n}}=1$

[b] $\because x^2-8x+12=0$ $\therefore (x-2)(x-6)=0$

$\therefore x=2$ or $x=6$ $\therefore$ The S.S. $=\{2, 6\}$

5

[a] Let the number of all balls $=x$

, the probability of drawing a white ball

$=1-\frac{2}{3}=\frac{1}{3}$ $\therefore \frac{1}{3}=\frac{5}{x}$

$\therefore x=15$ $\therefore$ The number of balls $=15$

[b] $\because 3^x=27$ $\therefore 3^x=3^3$ $\therefore x=3$

$\therefore 4^{3+y}=1$ $\therefore 4^{3+y}=4^0$ $\therefore y=-3$

Model 2

1

1. 1
2. $8, x^2$
3. $125x^3-8y^3$
4. 15
5. $\frac{5}{9}$

2

1 (c) 2 (b) 3 (c) 4 (a) 5 (a) 6 (c)

3

1. $(2x-3)(2x+3)$
2. $(x+2)(x^2-2x+4)$
3. $x(x-5)$
4. $(x-3)(x+2)$

4

[a] $\because x^2-x-6=0$ $\therefore (x-3)(x+2)=0$

$\therefore x=3$ or $x=-2$ $\therefore$ The S.S. $=\{3, -2\}$

[b] $\frac{(\sqrt{2})^5\times(3)^{-2}}{3\times(\sqrt{2})^9}=(\sqrt{2})^{5-9}\times3^{-2-1}$

$=(\sqrt{2})^{-4}\times3^{-3}$

$=\frac{1}{(\sqrt{2})^4}\times\frac{1}{3^3}=\frac{1}{4\times27}=\frac{1}{108}$

5

[a] $\because \frac{2^x\times3^x}{2^{2x}\times3^x}=2^{-1}$

$\therefore 2^{x-2x}=2^{-1}$ $\therefore 2^{-x}=2^{-1}$

$\therefore -x=-1$ $\therefore x=1$

[b] Let the number of all balls $=x$

$\therefore \frac{1}{6}=\frac{2}{x}$ $\therefore x=12$

$\therefore$ The number of red balls $=12-(2+4)=6$

Model examination for the merge students

1

1. d
2. b
3. c
4. a
5. c

2

1. 5
2. $\frac{2}{5}$
3. 6
4. 4^4
5. 0

3

1. $(x-y)(x+y)$
2. $(x-2)(x^2+2x+4)$
3. $(x-2)(x-3)$
4. $(a+b)(x+y)$

4

1 ✘ 2 ✘ 3 ✔ 4 ✔ 5 ✔

5

$\frac{(2^2)^n\times(2\times3)^{2n}}{2^{4n}\times3^{2n}}=\frac{2^{2n}\times2^{2n}\times3^{2n}}{2^{4n}\times3^{2n}}$

$=2^{2n+2n-4n}\times3^{2n-2n}$

$=2^0\times3^0=1$

Answers of schools examinations on algebra and statistics

1 Cairo

1 1 8 2 2 balls 3 5
4 $\frac{1}{25}$ 5 $(x-2)$

2 1 (d) 2 (a) 3 (b)
4 (b) 5 (d) 6 (a)

3

1 $(x+2)(x^2-2x+4)$
2 $(x-3)(x-2)$
3 $3(x-7)+a(x-7)=(x-7)(3+a)$
4 $(3x-2)(3x+2)$
5 $x^4+16x^2+64-16x^2$
$=(x^2+8)^2-16x^2$
$=(x^2+8+4x)(x^2+8-4x)$

4

[a] $\because x^3-8x^2+12x=0$
$\therefore x(x^2-8x+12)=0$
$\therefore x(x-6)(x-2)=0$
$\therefore x=0$ or $x=6$ or $x=2$
$\therefore$ The S.S. $=\{0, 6, 2\}$

[b] 1 let the number be x:
$\therefore x^2+3x=28$
$\therefore x^2+3x-28=0$
$\therefore (x-4)(x+7)=0$
$\therefore x=4$ or $x=-7$ (refused)
$\therefore$ The number is : 4

2 $\because \left(\frac{2}{5}\right)^{2x+1}=\left(\frac{2}{5}\right)^3$
$\therefore 2x+1=3$ $\therefore 2x=2$ $\therefore x=1$

5

[a] 1 $\frac{(2\times3)^{2n}\times(2^2)^n}{2^{4n}\times3^{2n}}=\frac{2^{2n}\times3^{2n}\times2^{2n}}{2^{4n}\times3^{2n}}=\frac{2^{4n}}{2^{4n}}=1$

2 $\left(\frac{x}{y}\right)^{-2}=\left(\frac{3}{\sqrt{2}}\right)^{-2}=\left(\frac{\sqrt{2}}{3}\right)^2=\frac{2}{9}$

[b] 1 The probability of getting a card carries a perfect square number $=\frac{4}{20}=\frac{1}{5}$

2 The probability of getting a card carries a prime number $=\frac{8}{20}=\frac{2}{5}$

2 Cairo

1 1 (c) 2 (b) 3 (c)
4 (b) 5 (d) 6 (a)

2 1 $\varnothing$ 2 $\frac{1}{9}$ 3 $(x-1)$
4 $[0, 1]$ 5 1

3

[a] 1 $x(a+b)+5(a+b)=(a+b)(x+5)$
2 $(x-1)(x^2+x+1)$
3 $x^4+4x^2+4-4x^2$
$=(x^2+2)^2-4x^2$
$=(x^2+2+2x)(x^2+2-2x)$

[b] $\because x^2+9x+18=0$ $\therefore (x+6)(x+3)=0$
$\therefore x=-6$ or $x=-3$
$\therefore$ The S.S. $=\{-6, -3\}$

4

[a] $\because 3^{x-1}=27$ $\therefore 3^{x-1}=3^3$
$\therefore x-1=3$ $\therefore x=4$

[b] $\frac{(\sqrt{5})^7\times(\sqrt{5})^3}{(\sqrt{5})^9\times(\sqrt{2})^{-3}}=(\sqrt{5})^{7+3-9}\times(\sqrt{2})^3$
$=\sqrt{5}\times2\sqrt{2}=2\sqrt{10}$

5

[a] $\left(\frac{y}{x}\right)^{-2}=\left(\frac{\sqrt{3}}{3}\right)^{-2}=\left(\frac{3}{\sqrt{3}}\right)^2$
$=\frac{(3)^2}{(\sqrt{3})^2}=\frac{9}{3}=3$

[b] $\frac{4^x\times6^{2x}}{2^{4x}\times3^{2x}}=\frac{(2^2)^x\times(2\times3)^{2x}}{2^{4x}\times3^{2x}}$
$=\frac{2^{2x}\times2^{2x}\times3^{2x}}{2^{4x}\times3^{2x}}$
$=2^{2x+2x-4x}=2^0=1$

3 Cairo

1 1 (c) 2 (d) 3 (a)
4 (c) 5 (d) 6 (c)

2 1 1　2 2　3 7
4 $\frac{y}{x}$　5 $\varnothing$

3

1 $(3x-4)(3x+4)$
2 $(x-5)(x^2+5x+25)$
3 $(2x-1)(x+4)$
4 $x^2(x+1)+(x+1)=(x+1)(x^2+1)$

4

[a] $\because 2x^3=18x$　$\therefore 2x^3-18x=0$
$\therefore 2x(x^2-9)=0$　$\therefore 2x(x-3)(x+3)=0$
$\therefore x=0$ or $x=3$ or $x=-3$
$\therefore$ The S.S. $=\{0, 3, -3\}$

[b] $\frac{(2^2)^{x+1}\times(3^2)^{2-x}}{(2\times3)^{2x}}=\frac{2^{2x+2}\times3^{4-2x}}{2^{2x}\times3^{2x}}$
$=2^{2x+2-2x}\times3^{4-2x-2x}$
$=2^2\times3^{4-4x}$

When $x=1$
$\therefore 2^2\times3^{4-4}=4\times3^0=4\times1=4$

5

[a] $\because 3^{x-4}=27$　$\therefore 3^{x-4}=3^3$
$\therefore x-4=3$　$\therefore x=7$

[b] Let the width of the rectangle be x
$\therefore$ The length of the rectangle $=2x$
$\therefore 2x\times x=32$　$\therefore 2x^2=32$
$\therefore 2x^2-32=0$
$\therefore 2(x^2-16)=0$　$\therefore (x-4)(x+4)=0$
$\therefore x=4$ or $x=-4$ (refused)
$\therefore$ The width = 4 cm.
and the length = 8 cm.
$\therefore$ The perimeter $=(4+8)\times2=12\times2=24$ cm.

4 Giza

1 1 (c)　2 (b)　3 (a)
4 (b)　5 (a)　6 (c)

2 1 75　2 $\{3\}$　3 1
4 1　5 8

3

1 $(x-1)(x^2+x+1)$
2 $(2x-3)(2x+3)$
3 $a(x-7)+3(x-7)=(x-7)(a+3)$
4 $(2x-1)(x-3)$
5 $x^4+4x^2y^2+4y^4-4x^2y^2$
$=(x^2+2y^2)^2-4x^2y^2$
$=(x^2+2y^2+2xy)(x^2+2y^2-2xy)$

4

[a] $\frac{(\sqrt{2})^5\times(3)^{-2}}{3\times(\sqrt{2})^9}=(\sqrt{2})^{5-9}\times(3)^{-2-1}$
$=(\sqrt{2})^{-4}\times(3)^{-3}$
$=\frac{1}{(\sqrt{2})^4}\times\frac{1}{3^3}=\frac{1}{4}\times\frac{1}{27}=\frac{1}{108}$

[b] $\because x^2-8x+12=0$
$(x-6)(x-2)=0$
$\therefore x=6$ or $x=2$　$\therefore$ The S.S. $=\{6, 2\}$

5

[a] $\because 2^x=16$　$\therefore 2^x=2^4$　$\therefore x=4$
, $\because 3^{x+y}=1$　$\therefore 3^{x+y}=3^0$　$\therefore x+y=0$
, $\because x=4$　$\therefore 4+y=0$　$\therefore y=-4$

[b] Let the number of all balls be x
$\therefore \frac{1}{6}=\frac{2}{x}$　$\therefore x=12$
$\therefore$ The number of red balls $=12-(2+4)=6$

5 Giza

1 1 (d)　2 (b)　3 (d)
4 (d)　5 (a)　6 (c)

2 1 $\frac{1}{2}$　2 $\frac{5}{9}$　3 $\varnothing$
4 10　5 $2a^2$, 6

3

1 $3(x^2-16)=3(x-4)(x+4)$
2 $(x-2)(x-5)$
3 $x^2(x+2)-4(x+2)=(x+2)(x^2-4)$
$=(x+2)(x+2)(x-2)$
$=(x+2)^2(x-2)$
4 $2(x^3-8y^3)=2(x-2y)(x^2+2xy+4y^2)$

4

[a] 1 $\because 3^{2n-5} = 1$ $\therefore 3^{2n-5} = 3^0$

$\therefore 2n - 5 = 0$ $\therefore 2n = 5$

$\therefore n = \frac{5}{2}$

$\therefore$ The S.S. $= \{\frac{5}{2}\}$

2 $\because \left(\frac{2}{3}\right)^{2n} = \frac{81}{16}$ $\therefore \left(\frac{2}{3}\right)^{2n} = \left(\frac{3}{2}\right)^4$

$\therefore \left(\frac{2}{3}\right)^{2n} = \left(\frac{2}{3}\right)^{-4}$

$\therefore 2n = -4$ $\therefore n = -2$

$\therefore$ The S.S. $= \{-2\}$

[b] 1 The probability of getting a number divisible by $4 = \frac{5}{20} = \frac{1}{4}$

2 The probability of getting a number multiple of $7 = \frac{2}{20} = \frac{1}{10}$

5

[a] Let the number be x

$\therefore x^2 + 3x = 28$ $\therefore x^2 + 3x - 28 = 0$

$\therefore (x-4)(x+7) = 0$

$\therefore x = 4$ or $x = -7$

$\therefore$ The real number is : 4 or -7

[b] $\frac{4^n \times 6^{2n}}{2^{4n} \times 3^{2n}} = \frac{(2^2)^n \times (2 \times 3)^{2n}}{2^{4n} \times 3^{2n}}$

$= \frac{2^{2n} \times 2^{2n} \times 3^{2n}}{2^{4n} \times 3^{2n}} = \frac{2^{4n}}{2^{4n}} = 1$

6 Alexandria

1

1 81 2 15 3 6
4 8 , 2 y 5 1

2

1 (d) 2 (c) 3 (b)
4 (c) 5 (b) 6 (b)

3

[a] 1 $(3x-2)(3x+2)$

2 $(5y+4)(2y-3)$

3 $4x^4 + 4x^2 + 1 - 4x^2$

$= (2x^2+1)^2 - 4x^2$

$= (2x^2 + 1 + 2x)(2x^2 + 1 - 2x)$

[b] $\because 2x^2 - 2x - 12 = 0$ $\therefore 2(x^2 - x - 6) = 0$

$\therefore 2(x-3)(x+2) = 0$ $\therefore x = 3$ or $x = -2$

$\therefore$ The S.S. $= \{3, -2\}$

4

[a] $\frac{x^6 \times x^2}{x^3} = x^{6+2-3} = x^5$

[b] $3(x-7) + a(x-7) = (x-7)(3+a)$

5

[a] 1 The probability of getting a card carries prime number $= \frac{6}{15} = \frac{2}{5}$

2 The probability of getting a card carries a number divisible by $3 = \frac{5}{15} = \frac{1}{3}$

[b] $\because 2^{x-2} = 32$ $\therefore 2^{x-2} = 2^5$

$\therefore x - 2 = 5$ $\therefore x = 7$

7 Alexandria

1

1 14 2 3 3 0
4 5 , 3 x 5 25

2

1 (c) 2 (b) 3 (c)
4 (b) 5 (d) 6 (a)

3

[a] Let the number of all balls be x

$\therefore \frac{1}{6} = \frac{2}{x}$ $\therefore x = 12$

$\therefore$ The number of red balls $= 12 - (2+4) = 6$

[b] 1 $(2x+1)(x+3)$

2 $x(x-5)$

4

[a] $\because x^2 - 8x + 12 = 0$

$\therefore (x-6)(x-2) = 0$

$\therefore x = 6$ or $x = 2$

$\therefore$ The S.S. $= \{6, 2\}$

[b] $\frac{(\sqrt{2})^5 \times 3^{-2}}{3 \times (\sqrt{2})^9} = (\sqrt{2})^{5-9} \times 3^{-2-1}$

$= (\sqrt{2})^{-4} \times 3^{-3}$

$= \frac{1}{(\sqrt{2})^4} \times \frac{1}{3^3} = \frac{1}{4} \times \frac{1}{27} = \frac{1}{108}$

5

[a] Let the width of the rectangle be x cm.

$\therefore$ The length of the rectangle $= (x + 4)$ cm.

$\therefore$ The area $= x(x + 4)$

$\therefore x(x+4) = 21$ $\therefore x^2 + 4x = 21$

$\therefore x^2 + 4x - 21 = 0$ $\therefore (x-3)(x+7) = 0$

$\therefore x = 3$ or $x = -7$ (refused)

$\therefore$ The width $= 3$ cm. and the length $= 7$ cm.

[b] $\because \frac{8^x \times 9^x}{18^x} = 64$ $\therefore \frac{(2^3)^x \times (3^2)^x}{(2 \times 3^2)^x} = 2^6$

$\therefore \frac{2^{3x} \times \cancel{3^{2x}}}{2^x \times \cancel{3^{2x}}} = 2^6$ $\therefore 2^{3x - x} = 2^6$

$\therefore 2^{2x} = 2^6$ $\therefore 2x = 6$ $\therefore x = 3$

$\therefore 4^{-x} = 4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

8 El-Kalyoubia

1 1 (b) 2 (c) 3 (b) 4 (c) 5 (c) 6 (a)

2 1 1 2 $\frac{5}{9}$ 3 $1, a^4$ 4 $\frac{1}{4}$ 5 4

3

[a] 1 $(x - y)(x + y)$

2 $(y + 2)(y^2 - 2y + 4)$

[b] $\because x^2 - x - 6 = 0$ $\therefore (x-3)(x+2) = 0$

$\therefore x = 3$ or $x = -2$ $\therefore$ The S.S. $= \{3, -2\}$

4

[a] 1 $a(x-7) + 3(x-7) = (x-7)(a+3)$

2 $x(x-5)$

[b] $\because 3^x = 27$ $\therefore 3^x = 3^3$ $\therefore x = 3$

$, \because 4^{x+y} = 1$ $\therefore 4^{x+y} = 4^0$

$\therefore x + y = 0$ $\therefore 3 + y = 0$ $\therefore y = -3$

5

[a] $\frac{(2^2)^{x+1} \times (3^2)^{2-x}}{(2 \times 3)^{2x}} = \frac{2^{2x+2} \times 3^{4-2x}}{2^{2x} \times 3^{2x}}$

$= 2^{2x+2-2x} \times 3^{4-2x-2x}$

$= 2^2 \times 3^{4-4x} = 4 \times 3^{4-4x}$

When $x = 1$

$\therefore 4 \times 3^{4-4} = 4 \times 3^0 = 4$

[b] The probability that the chosen digit is an even number $= \frac{2}{5}$

9 El-Monofia

1 1 (b) 2 (b) 3 (c) 4 (a) 5 (b) 6 (b)

2 1 1 2 1 3 1 4 $(x + L)$ 5 3

3

[a] Let the number be x

$\therefore$ its multiplicative inverse $= \frac{1}{x}$

$\therefore x + \frac{1}{x} = 2$

(multiplying both sides of the equation by x)

$\therefore x^2 + 1 = 2x$

$\therefore x^2 - 2x + 1 = 0$ $\therefore (x-1)^2 = 0$

$\therefore x = 1$ $\therefore$ The number $= 1$

[b] 1 $3(x^2 - 5x + 4) = 3(x-4)(x-1)$

2 $\frac{1}{3}(x^3 - 27) = \frac{1}{3}(x-3)(x^2 + 3x + 9)$

3 $x^4 + 18x^2 + 81 + 9x^2 - 18x^2$

$= (x^2 + 9)^2 - 9x^2$

$= (x^2 + 9 + 3x)(x^2 + 9 - 3x)$

4

[a] $\frac{(2^2)^{x+1} \times (3^2)^{2-x}}{(2 \times 3)^{2x}} = \frac{2^{2x+2} \times 3^{4-2x}}{2^{2x} \times 3^{2x}}$

$= 2^{2x+2-2x} \times 3^{4-2x-2x}$

$= 2^2 \times 3^{4-4x} = 4 \times 3^{4-4x}$

when $x = 1$

$\therefore 4 \times 3^{4-4x} = 4 \times 3^{4-4} = 4 \times 3^0 = 4 \times 1 = 4$

[b] $x^2 + (xz)^2 \times y^2 = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}\right)^2 \times \left(\frac{1}{\sqrt{3}}\right)^2$

$= \frac{3}{4} + \left(\frac{\sqrt{6}}{4}\right)^2 \times \frac{1}{3} = \frac{3}{4} + \frac{6}{16} \times \frac{1}{3}$

$= \frac{3}{4} + \frac{1}{8} = \frac{6+1}{8} = \frac{7}{8}$

5

[a] 1 The probability of drawing a card carries an odd number $= \frac{5}{11}$

2 The probability of drawing a card carries a number divisible by $5 = \frac{3}{11}$

[b] 1 $a(x-7) + 3(x-7) = (x-7)(a+3)$

2 $(3x - 5)(3x + 5)$

10 El-Gharbia

1 1 (c) 2 (d) 3 (d) 4 (d) 5 (c) 6 (c)

2 1 $\{10, -10\}$ 2 1 3 8 4 $(5x-7)$ 5 75

3

1 $(x+5)(x+3)$

2 $(x-3)(x^2+3x+9)$

3 $(2x+1)(x+3)$

4 $a(x-7)+3(x-7)=(x-7)(a+3)$

4

[a] $\because \frac{(2^3)^x \times (3^2)^x}{(2\times 3^2)^x}=64$ $\therefore \frac{2^{3x}\times 3^{2x}}{2^x \times 3^{2x}}=2^6$

$\therefore 2^{3x-x}=2^6$ $\therefore 2^{2x}=2^6$

$\therefore 2x=6$ $\therefore x=3$

$\therefore 4^{-x}=4^{-3}=\frac{1}{4^3}=\frac{1}{64}$

[b] $\because x^2-8x+12=0$ $\therefore (x-2)(x-6)=0$

$\therefore x=2$ or $x=6$ $\therefore$ The S.S. $=\{2, 6\}$

5

[a] 1 $\frac{(2^2)^n \times (2\times 3)^{2n}}{2^{4n}\times 3^{2n}}=\frac{2^{2n}\times 2^{2n}\times 3^{2n}}{2^{4n}\times 3^{2n}}=\frac{2^{4n}}{2^{4n}}=1$

2 Let the rational number be x

$\therefore 4x^2=81$ $\therefore x^2-\frac{81}{4}=0$

$\therefore \left(x+\frac{9}{2}\right)\left(x-\frac{9}{2}\right)=0$

$\therefore x=\frac{-9}{2}$ or $x=\frac{9}{2}$

$\therefore$ The S.S. $=\left\{\frac{-9}{2}, \frac{9}{2}\right\}$

[b] 1 The probability of the drawn ball is yellow $=\frac{4}{12}=\frac{1}{3}$

2 The probability of the drawn ball is Green $=\frac{5}{12}$

3 The probability of the drawn ball is not red $=\frac{4+5}{12}=\frac{9}{12}=\frac{3}{4}$

11 El-Dakahlia

1 1 (b) 2 (a) 3 (c) 4 (c) 5 (d) 6 (a)

2 1 -5 2 60 3 $8, 2x$ 4 $1, x^2$ 5 1

3

1 $(2x-3)(x-1)$

2 $x^2(x+1)+3(x+1)=(x+1)(x^2+3)$

3 $6(x^2-4)=6(x-2)(x+2)$

4 $(2x+1)(4x^2-2x+1)$

4

[a] $\because x^2-9x+14=0$

$\therefore (x-7)(x-2)=0$

$\therefore x=7$ or $x=2$

$\therefore$ The S.S. $=\{2, 7\}$

[b] $\because \frac{(3^2)^{n+1}\times (2^3)^n}{(2\times 3)^{2n+2}}=16$ $\therefore \frac{3^{2n+2}\times 2^{3n}}{2^{2n+2}\times 3^{2n+2}}=2^4$

$\therefore 2^{3n-2n-2}=2^4$

$\therefore 2^{n-2}=2^4$ $\therefore n-2=4$ $\therefore n=6$

5

[a] $\because 3^x=81$ $\therefore 3^x=3^4$ $\therefore x=4$

$, \because 4^{x+y}=1$ $\therefore 4^{x+y}=4^0$

$\therefore x+y=0$ $\therefore 4+y=0$ $\therefore y=-4$

[b] Let the number of all balls be x

$\therefore \frac{8}{x}=\frac{2}{3}$ $\therefore 2x=24$ $\therefore x=12$

$\therefore$ The number of white balls $=12-8=4$

12 Ismailia

1 1 (b) 2 (c) 3 (d) 4 (a) 5 (c) 6 (b)

2 1 28 2 10 3 3 4 $\varnothing$ 5 $(x-y)$

3

[a] 1 $(x-9)(x-2)$

2 $(x+3)(x^2-3x+9)$

3 $4x^4+4x^2y^2+y^4-4x^2y^2$

$=(2x^2+y^2)^2-4x^2y^2$

$=(2x^2+y^2+2xy)(2x^2+y^2-2xy)$

[b] $\because 2^{x} = 2^{5}$ $\therefore x = 5$

$, 3^{y+1} = 27$ $\therefore 3^{y+1} = 3^{3}$

$\therefore y + 1 = 3$ $\therefore y = 2$

$\therefore x - y = 5 - 2 = 3$

4

[a] $\because \frac{(2^3)^x \times (3^2)^x}{(2 \times 3^2)^x} = 64$ $\therefore \frac{2^{3x} \times 3^{2x}}{2^x \times 3^{2x}} = 2^6$

$\therefore 2^{3x - x} = 2^6$ $\therefore 2^{2x} = 2^6$

$\therefore 2x = 6$ $\therefore x = 3$

$\therefore 2^x = 2^3 = 8$

[b] $\because x^2 + 4x - 12 = 0$ $\therefore (x + 6)(x - 2) = 0$

$\therefore x = -6$ or $x = 2$

$\therefore$ The S.S. $= \{-6, 2\}$

5

[a] 1 $(2x - 3)(2x + 3)$

2 $5x + 5y + ax + ay = 5(x + y) + a(x + y)$

$= (x + y)(5 + a)$

[b] 1 The probability of getting a red ball $= \frac{5}{11}$

2 The probability of getting a red or blue ball

$= \frac{5+3}{11} = \frac{8}{11}$

3 The probability of getting not a red ball

$= \frac{3+3}{11} = \frac{6}{11}$

13 El-Beheira

1 1 (d) 2 (b) 3 (b) 4 (b) 5 (a) 6 (b)

2 1 16 2 $(x - 3)$ 3 2 4 3^{19} 5 $\frac{1}{2}$

3

[a] $\frac{(2^2)^{x+1} \times (3^2)^{2-x}}{(2 \times 3)^{2x}} = \frac{2^{2x+2} \times 3^{4-2x}}{2^{2x} \times 3^{2x}}$

$= 2^{2x+2-2x} \times 3^{4-2x-2x}$

$= 2^2 \times 3^{4-4x}$

$= 4 \times 3^{4-4x}$

when $x = 1$

$\therefore 4 \times 3^{4-4x} = 4 \times 3^{4-4} = 4 \times 3^0 = 4$

[b] $\because x^2 - x - 12 = 0$ $\therefore (x - 4)(x + 3) = 0$

$\therefore x = 4$ or $x = -3$ $\therefore$ The S.S. $= \{4, -3\}$

4

[a] 1 $(3x + 1)(x + 2)$

2 $a(x - 7) + 3(x - 7) = (x - 7)(a + 3)$

[b] $\because 3^x = 3^3$ $\therefore x = 3$

$, 4^{x+y} = 1$ $\therefore 4^{x+y} = 4^0$ $x + y = 0$

$\therefore 3 + y = 0$ $\therefore y = -3$

5

[a] Let the width of the rectangle be x cm.

$\therefore$ The length of the rectangle $= (x + 4)$ cm.

$\therefore$ The area $= x(x + 4)$ $\therefore x(x + 4) = 12$

$\therefore x^2 + 4x - 12 = 0$ $\therefore (x - 2)(x + 6) = 0$

$\therefore x = 2$ or $x = -6$ (refused)

$\therefore$ The width = 2 cm. and the length = 6 cm.

[b] 1 The probability that the chosen card is an even number $= \frac{5}{10} = \frac{1}{2}$

2 The probability that the chosen card is a prime number $= \frac{4}{10} = \frac{2}{5}$

14 El-Menia

1 1 $\{3\}$ 2 4 3 $\{5, -5\}$ 4 -1 5 27

2 1 (c) 2 (d) 3 (c) 4 (c) 5 (b) 6 (c)

3

[a] $\because \frac{(2^3)^x \times (3^2)^x}{(2 \times 3^2)^x} = 64$ $\therefore \frac{2^{3x} \times 3^{2x}}{2^x \times 3^{2x}} = 2^6$

$\therefore 2^{3x - x} = 2^6$ $\therefore 2^{2x} = 2^6$

$\therefore 2x = 6$ $\therefore x = 3$

[b] $\because x^2 - 1 = 8$ $\therefore x^2 - 9 = 0$

$\therefore (x - 3)(x + 3) = 0$

$\therefore x = 3$ or $x = -3$

$\therefore$ The S.S. $= \{3, -3\}$

4

1 $a(x - 7) + 3(x - 7) = (x - 7)(a + 3)$

2 $(x + 2)(x^2 - 2x + 4)$

3 $(x - 3)(x + 2)$

4 $(2x - 3)(2x + 3)$

5 $x^4 + 36x^2 + 324 - 36x^2$

$= (x^2 + 18)^2 - 36x^2$

$= (x^2 + 18 + 6x)(x^2 + 18 - 6x)$

5

[a] $\because 3^{x-2} = 9$ $\therefore 3^{x-2} = 3^2$

$\therefore x - 2 = 2$ $\therefore x = 4$

[b]

1 The probability of selecting a white marble $= \frac{18}{50} = \frac{9}{25}$

2 The probability of selecting a yellow marble $= \frac{0}{50} = 0$

3 The probability of selecting a red or blue marble $= \frac{12+20}{50} = \frac{32}{50} = \frac{16}{25}$

4 The probability of selecting a non red marble $= \frac{18+20}{50} = \frac{38}{50} = \frac{19}{25}$

15 Aswan

1

1 $\frac{1}{8}$ 2 20 3 $\frac{1}{6}$

4 $8, x^2$ 5 0.2

2

1 (c) 2 (d) 3 (b)

4 (b) 5 (a) 6 (b)

3

[a] $\because \left(\frac{2}{5}\right)^{2x-1} = \frac{8}{125}$ $\therefore \left(\frac{2}{5}\right)^{2x-1} = \frac{2^3}{5^3}$

$\therefore \left(\frac{2}{5}\right)^{2x-1} = \left(\frac{2}{5}\right)^3$

$\therefore 2x - 1 = 3$ $\therefore 2x = 4$

$\therefore x = 2$

[b] $\because x^2 - 6x = 0$ $\therefore x(x-6) = 0$

$\therefore x = 0$ or $x = 6$

$\therefore$ The S.S. $= \{0, 6\}$

4

[a] $\frac{(\sqrt{3})^{-5} \times (\sqrt{3})^{-4}}{(\sqrt{3})^{-10}} = (\sqrt{3})^{-5-4+10} = \sqrt{3}$

[b] Let the number of all balls be x

$\therefore \frac{1}{6} = \frac{2}{x}$ $\therefore x = 12$

$\therefore$ The number of red balls $= 12 - (2+4) = 6$

5

1 $(x+3)(x+5)$

2 $(5x - y)(5x + y)$

3 $(x-2)(x^2 + 2x + 4)$

4 $2(9y^2 - 6y + 1) = 2(3y - 1)^2$

5 $81x^4 + 36x^2z^2 + 4z^4 - 36x^2z^2$

$= (9x^2 + 2z^2)^2 - 36x^2z^2$

$= (9x^2 + 2z^2 + 6xz)(9x^2 + 2z^2 - 6xz)$

Answers of the quizzes on Geometry

Quiz 1

1

1 equal in area 2 12 3 8 cm.

2

[a] 1 56 cm^2 2 4 cm.

[b] Prove by yourself.

Quiz 2

1

1 8 cm. 2 50 3 2.4

2

20 cm^2

Quiz 3

1

1 equal areas 2 1 : 2 3 $\frac{1}{2}$

2

[a] Prove by yourself. [b] 50 cm^2

Quiz 4

1

1 two triangular surfaces equal in area.

2 $\frac{1}{3}$

3 lie on a straight line parallel to this base.

2

Prove by yourself.

Quiz 5

1

1 24 2 equal in area 3 60

2

[a] Prove by yourself.

[b] 24 cm^2

Quiz 6

1

1 10

2 proportional , equal in measure.

3 10

2

[a] Prove by yourself , AC = 10 cm. , BD = 3 cm.

[b] Prove by yourself.

Quiz 7

1

1 C 2 14

3 27 cm. , 36 cm. , 48 cm.

2

[a] 1 25 cm. 2 Prove by yourself

[b] Prove by yourself.

Quiz 8

1

1 1 2 ≤ 3 point

2

[a] 13 cm.

[b] 1 The length of projection of $\overline{AB}$ on $\overleftrightarrow{AC}$ = AD = 4 cm.

2 The length of projection of $\overline{BC}$ on $\overleftrightarrow{AC}$ = DC = 12 cm.

Quiz 9

1

1 A 2 equals 3 $\frac{1}{2}$

2

[a] 1 Prove by yourself. 2 3.2 cm.

[b] Prove by yourself.

Quiz 10

1

1 10 2 obtuse 3 84

2

[a] 1 15 cm. 2 Prove by yourself. 3 9.6 cm.

[b] acute – angled triangle.

Answers of school book examinations on geometry

Model 1

1

1 AC 2 C 3 The point A

4 154 5 45

2

1 (c) 2 (b) 3 (c) 4 (a) 5 (b) 6 (b)

3

[a] Let the two triangles be Δ ABC , Δ XYZ

, $\because \Delta ABC \sim \Delta XYZ$

$\therefore \frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ} = \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta XYZ}$

$\therefore \frac{3}{XY} = \frac{4}{YZ} = \frac{5}{XZ} = \frac{12}{36}$ $\therefore XY = \frac{3 \times 36}{12} = 9$ cm.

, $YZ = \frac{4 \times 36}{12} = 12$ cm. , $XZ = \frac{5 \times 36}{12} = 15$ cm.

[b] $\Delta\Delta$ CBX , DBX have the same base $\overline{BX}$ and $\overleftrightarrow{DC} // \overleftrightarrow{AB}$

$\therefore$ Area of Δ CBX = Area of Δ DBX (1)

, $\Delta\Delta$ CYD , BYD have the same base $\overline{YD}$ and $\overleftrightarrow{BC} // \overleftrightarrow{YD}$

$\therefore$ Area of Δ CYD = Area of Δ BYD (2)

, $\because$ Area of Δ CBX = Area of Δ CYD (3)

From (1) , (2) and (3) :

$\therefore$ Area of Δ DBX = Area of Δ BYD and they have the common base $\overline{BD}$

$\therefore \overline{XY} // \overline{BD}$ (Q.E.D.)

4

[a] $\because$ Δ ABD is right-angled at D

$\therefore (AB)^2 = (AD)^2 + (DB)^2 = (4)^2 + (2)^2 = 20$

, $\because$ Δ ADC is right-angled at D

$\therefore (AC)^2 = (AD)^2 + (DC)^2 = (4)^2 + (8)^2 = 80$

In Δ ABC : $\because (AB)^2 + (AC)^2 = 20 + 80 = 100$

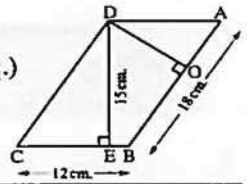
, $(BC)^2 = 100$

$\therefore m(\angle BAC) = 90°$ (Q.E.D.)

[b] Area of ▱ ABCD

$= 12 \times 15 = 180$ cm^2. (First req.)

$\therefore DO = \frac{180}{18} = 10$ cm. (Second req.)



5

[a] In Δ ABC : $m(\angle C) = 180° - (50° + 60°) = 70°$

$\therefore m(\angle C) > m(\angle B) > m(\angle A)$

$\therefore AB > AC > BC$

$\therefore$ The descending order is : AB , AC and BC

[b] $\because$ $\Delta\Delta$ ABD , ACD have the common base $\overline{AD}$, $\overline{AD} // \overline{BC}$

$\therefore$ The area of Δ ABD = the area of Δ ACD subtracting the area of Δ ADE

$\therefore$ The area of Δ ABE = the area of Δ DCE (Q.E.D.)

Model 2

1

1 Proportional in length , equal in measure

2 6 cm. 3 B 4 obtuse-angled triangle

5 base lying on one of two parallel straight lines including them.

2

1 (d) 2 (b) 3 (b) 4 (d) 5 (b) 6 (d)

3

$\because \overline{AO} \perp \overleftrightarrow{BC}$

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times BC \times AO = \frac{1}{2} \times 7 \times 5 = 17.5$ cm^2.

, $\because \overline{BE} \perp \overline{AC}$

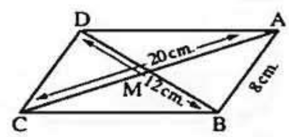
$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times AC \times BE = 17.5$

$\therefore \frac{1}{2} \times 10 \times BE = 17.5$

$\therefore BE = 3.5$ cm.

4

[a] Let $\overline{AC} \cap \overline{BD} = \{M\}$



$\therefore AM = 10$ cm.

, $BM = 6$ cm.

$\therefore$ In Δ ABM : $(AM)^2 = (10)^2 = 100$

, $(AB)^2 + (BM)^2 = (8)^2 + (6)^2 = 64 + 36 = 100$

$\therefore$ Δ ABM is right-angled at B

$\therefore m(\angle ABD) = 90°$ (First req.)

$\therefore$ Area of ▱ ABCD

$= AB \times BD = 8 \times 12 = 96$ cm^2. (Second req.)

[b] $\because \overline{CD}$ is a median in Δ ABC

$\therefore$ The area of Δ DBC = the area of Δ ADC

$\therefore$ The area of Δ DBC = $\frac{1}{2}$ the area of Δ ABC (1)

, $\because \overline{BE}$ is a median in Δ ABC

$\therefore$ The area of Δ EBC = the area of Δ EBA

$\therefore$ The area of Δ EBC = $\frac{1}{2}$ the area of Δ ABC (2)

From (1) and (2) :

$\therefore$ The area of Δ DBC = the area of Δ EBC (Q.E.D.1)

and they have a common base $\overline{BC}$ and on one side of it

$\therefore \overline{DE} // \overline{BC}$ (Q.E.D.2)

5

[a] $\because \Delta ABC \sim \Delta DBA$

$\therefore m(\angle BAC) = m(\angle BDA) = 90°$

$\therefore \overline{AD} \perp \overline{BC}$ (First req.)

$\therefore$ In Δ ABC : $m(\angle BAC) = 90°$, $\overline{AD} \perp \overline{BC}$

$\therefore (BC)^2 = (AB)^2 + (AC)^2 = (8)^2 + (6)^2 = 100$

$\therefore$ BC = 10 cm. $\therefore (AB)^2 = BD \times BC$

$\therefore 64 = BD \times 10$ $\therefore$ BD = 6.4 cm. (Second req.)

[b] In Δ ABD : $\because m(\angle ADB) = 90°$

$\therefore (BD)^2 = (AB)^2 - (AD)^2 = (26)^2 - (24)^2 = 100$

$\therefore$ BD = 10 cm.

, In Δ ACD : $\because m(\angle ADC) = 90°$

$\therefore (CD)^2 = (AC)^2 - (AD)^2 = (30)^2 - (24)^2 = 324$

$\therefore$ CD = 18 cm.

$\therefore$ BC = CD + DB = 18 + 10 = 28 cm. (First req.)

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 28 \times 24 = 336 \text{ cm}^2$ (Second req.)

Model examination for the merge students

1

1 c 2 b 3 b

4 d 5 a

2

1 a point 2 > 3 32

4 are equal in area 5 base length

3

1 2.4 2 BEC 3 ACD

4 congruent 5 3.6

4

Given : **Area of the figure ABYX = area of the figure DCYX**

R.T.P : $\overline{AD} // \overline{BC}$

Proof : $\because \overline{XY}$ is median in Δ XBC

$\therefore$ area of Δ **BYX** = area of Δ **CYX** (1)

$\because$ area of the figure ABYX = area of the figure DCYX (2)

By subtracting (1) from (2) :

$\therefore$ Area of Δ **ABX** = area of Δ **DCX**

By adding area of Δ ADX to both side

$\therefore$ Area of Δ **ABD** = area of Δ **ACD**

$\therefore \overline{AD} // \overline{BC}$

5

$\because \Delta ABC \sim \Delta AED$

$\therefore \frac{AB}{AE} = \frac{BC}{ED} = \frac{CA}{DA}$ $\therefore \frac{8}{4} = \frac{8}{ED} = \frac{CA}{3}$

$\therefore ED = \frac{8 \times 4}{8} = 4$ cm.

, $AC = \frac{8 \times 3}{4} = 6$ cm. , EC = 6 − 4 = 2 cm.

Answers of schools examinations on geometry

1 Cairo

1

1 (c) 2 (b) 3 (a)
4 (b) 5 (b) 6 (c)

2

1 proportional , equal in measure.
2 120° 3 >
4 50 5 30

3

[a] In Δ ABD : ∵ $\overline{DB} \perp \overline{AB}$

∴ m (∠ ABD) = 90°

∴ $(BD)^2 = (AD)^2 - (AB)^2 = (17)^2 - (8)^2 = 225$

∴ BD = 15 cm. (First req.)

In Δ BCD :

∵ $(BD)^2 = (15)^2 = 225$

, $(BC)^2 + (CD)^2 = (9)^2 + (12)^2 = 225$

∴ $(BD)^2 = (BC)^2 + (CD)^2$

∴ m (∠ C) = 90° (Second req.)

[b] ∵ $\overline{AD}$ is a median in Δ ABC

∴ The area of Δ ABD = The area of Δ ACD (1)

∵ $\overline{ED}$ in a median in Δ EBC

∴ The area of Δ EBD = The area of Δ ECD (2)

Subtracting (2) from (1) :

∴ The area of Δ ABE = The area of Δ ACE

(Q.E.D.)

4

[a] ∵ Δ AXY ~ Δ ABC

∴ $\frac{AY}{AC} = \frac{AX}{AB}$ ∴ $\frac{6}{AC} = \frac{7}{14}$

∴ $AC = \frac{6 \times 14}{7} = 12$ cm.

∴ XC = 12 – 7 = 5 cm. (The req.)

[b] ΔΔ ADB , ADC have the same base $\overline{AD}$

, $\overline{BC} // \overline{AD}$

∴ The area of Δ ADB = The area of Δ ADC

Subtracting the area of Δ AMD from both sides

∴ The area of Δ AMB = The area of Δ DMC (1)

, ∵ $\overline{MD}$ is median in Δ EMC

∴ The area of Δ MDE = The area of Δ DMC (2)

From (1) and (2) :

∴ The area of Δ MDE = The area of Δ AMB

(Q. E. D.)

5

In Δ BCD : ∵ m (∠ BCD) = 90°

∴ $(BD)^2 = (BC)^2 + (CD)^2 + (7)^2 + (24)^2 = 625$

∴ BD = 25 cm.

, in Δ ABD : ∵ m (∠ BAD) = 90°

∴ $(AD)^2 = (BD)^2 - (AB)^2 = (25)^2 - (15)^2 = 400$

∴ AD = 20 cm. (First req.)

, ∵ $\overline{EB}$ is the projection $\overline{AB}$ on $\overleftrightarrow{BD}$

, $(AB)^2 = EB \times BD$ ∴ $(15)^2 = EB \times 25$

∴ 225 = EB × 25

∴ $EB = \frac{225}{25} = 9$ cm. (Second req.)

2 Cairo

1

1 (c) 2 (b) 3 (c)
4 (c) 5 (c) 6 (c)

2

1 72 2 similar
3 equal in area 4 10
5 equal in measure

3

[a] In Δ ACD : ∵ m (∠ DAC) = 90°

∴ $(AC)^2 = (CD)^2 - (AD)^2 = (17)^2 - (8)^2 = 225$

∴ AC = 15 cm.

, in Δ ABC : ∵ $(AC)^2 = (15)^2 = 225$

∵ $(AB)^2 + (BC)^2 = (9)^2 + (12)^2 = 225$

∴ $(AC)^2 = (AB)^2 + (BC)^2$

∴ m (∠ B) = 90° (Q.E.D.)

[b] The area of a trapezium = $\frac{1}{2}$ (12 + 7) × 5

= 47.5 cm^2.

4

[a] In ΔΔ ABC , DBE

∵ $\overline{AC} // \overline{ED}$, $\overleftrightarrow{AD}$ is a transversal

$\therefore$ m (∠ A) = m (∠ D) (alternate angles) (1)

, $\because$ $\overline{AC} \parallel \overline{ED}$, $\overleftrightarrow{CE}$ in a transversal

$\therefore$ m (∠ C) = m (∠ E) (alternate angles) (2)

, $\because$ m (∠ ABC) = m (∠ DBE) (V. O. A.) (3)

From (1) , (2) and (3) :

$\therefore$ Δ ABC ~ Δ DBE (Q. E. D.)

[b] In Δ ABC :

$\because$ m (∠ A) = 90°

$\therefore (BC)^2 = (AC)^2 + (AB)^2 = (15)^2 + (20)^2 = 625$

$\therefore$ BC = 25 cm.

, $\because$ $\overline{EB}$ in the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$

, $\because (AB)^2 = EB \times BC$ $\therefore (20)^2 = EB \times 25$

$\therefore EB = \frac{400}{25} = 16$ cm. (First req.)

$\therefore$ EC = BC − EB = 25 − 16 = 9 cm. (Second req.)

5

[a] $\because (AC)^2 = (9)^2 = 81$

, $(AB)^2 + (BC)^2 = (7)^2 + (6)^2 = 85$

, $\because (AC)^2 < (AB)^2 + (BC)^2$

$\therefore$ Δ ABC is a cute-angled triangle (The req.)

[b] $\because$ The area of Δ AEB = the area of Δ DEC

Adding the area of Δ ADE to both sides

$\therefore$ The area of Δ ABD = the area of Δ ACD

but they have a common base $\overline{AD}$

and on one side of it

$\therefore$ $\overline{AD} \parallel \overline{BC}$ (Q. E. D.)

3 Cairo

1

1 (a) 2 (d) 3 (c)

4 (b) 5 (d) 6 (a)

2

1 proportional , equal in measure.

2 6 3 65°

4 Y 5 10

3

[a] 1 BD × BC

2 DB × DC

[b] In ΔΔ AED , ABC

$\because$ $\overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore$ m (∠ ADE) = m (∠ B) (corresponding angles) (1)

, $\because$ $\overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore$ m (∠ AED) = m (∠ C) (corresponding angles) (2)

, $\because$ ∠ A is a common angle (3)

From (1) , (2) and (3) :

$\therefore$ Δ ADE ~ Δ ABC (First req.)

, $\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$ $\therefore \frac{4}{6} = \frac{DE}{7.5}$

$\therefore DE = \frac{4 \times 7.5}{6} = 5$ cm. (Second req.)

4

[a] $\because$ ΔΔ ADB , ADE have a common base $\overline{AD}$

, $\overline{AD} \parallel \overline{BC}$

$\therefore$ The area of Δ ADB = the area of Δ ADE

By subtrating the area of Δ ADM from both sides.

$\therefore$ The area of Δ ABM = the area of Δ DEM

but the area of Δ ABM = the area of Δ EMC

$\therefore$ The ara of Δ EMC = the area of Δ DEM

and they have a common base $\overline{ME}$ and on one side of it.

$\therefore$ $\overline{ME} \parallel \overline{DC}$ (Q. E. D.)

[b] In ΔΔ ABC , DBA

$\because$ m (∠ BAC) = m (∠ BDA) = 90°

, ∠ B is a common angle

$\therefore$ m (∠ C) = m (∠ BAD)

$\therefore$ Δ ABC ~ Δ DBA (1)

, in ΔΔ ABC , DAC

$\because$ m (∠ BAC) = m (∠ ADC) = 90°

, ∠ C in a common angle

$\therefore$ m (∠ B) = m (∠ DAC)

$\therefore$ Δ ABC ~ Δ DAC (2)

From (1) , (2) :

$\therefore$ Δ ABC ~ Δ DBA ~ Δ DAC (Q. E. D.)

5

[a] ∵ ΔΔ ABD , ACD have the same base $\overline{AD}$

, $\overline{BC} // \overline{AD}$

∴ The area of Δ ABD = the area of Δ ACD

Subtrating the area of Δ AMD from both sides.

∴ The area of Δ AMB = the area of Δ DMC (1)

, ∵ ΔΔ MBX , MCY have equal bases in length and on one straight line and they are common in the vertex M

∴ The area of Δ MBX = the area of Δ MCY (2)

Adding (1) and (2) :

∴ The area of the figure ABXM = the area of the figure DCYM (Q.E.D.)

[b] In Δ ABC : ∵ m (∠ B) = 90°

∴ $(AC)^2 = (BC)^2 + (AB)^2 = (4)^2 + (3)^2 = 25$

∴ AC = 5 cm. (Frist req.)

In Δ ACD :

∵ $(AC)^2 + (CD)^2 = (5)^2 + (12)^2 = 169$

, $(AD)^2 = (13)^2 = 169$

∴ $(AD)^2 = (AC)^2 + (CD)^2$

∴ m (∠ ACD) = 90° (Second req.)

4 Giza

1

1 (c) 2 (b) 3 (d)
4 (a) 5 (c) 6 (b)

2

1 equal
2 proportional
3 The point B
4 equal in area
5 18

3

[a] ∵ ΔΔ ABD , ACD have the same base $\overline{AD}$

, $\overline{BC} // \overline{AD}$

∴ The area of Δ ABD = the area of Δ ACD

Subtrating the area of Δ AMD from both sides

∴ The area of Δ AMB = the area of Δ DMC (1)

∵ ΔΔ MBX , MCY have equal bases in length and on one straight line and they are common in the vertex M

∴ The area of Δ MBX = the area of Δ MCY (2)

Adding (1) and (2) :

∴ The area of shape ABXM = The area of shape DCYM (Q.E.D.)

[b] In Δ ABC : ∵ m (∠ A) = 90° , $\overline{AD} \perp \overline{BC}$

∴ $(AC)^2 = CD \times CB = 8 \times 12.5 = 100$

∴ AC = 10 cm.

, $(AB)^2 = BD \times BC = 4.5 \times 12.5 = 56.25$

∴ AB = 7.5 cm.

, $(AD)^2 = BD \times CD = 4.5 \times 8 = 36$

∴ AD = 6 cm. (The req.)

4

[a] In ΔΔ AMB , DMC

∵ $\overline{AB} // \overline{CD}$, $\overleftrightarrow{AD}$ is a transversal

∴ m (∠ A) = m (∠ D) (alternate angles) (1)

, ∵ $\overline{AB} // \overline{CD}$, $\overleftrightarrow{BC}$ is a transversal

∴ m (∠ B) = m (∠ C) (alternate angles) (2)

, ∵ m (∠ AMB) = m (∠ DMC) (V. O. A.) (3)

From (1) , (2) and (3) :

∴ Δ AMB ~ Δ DMC (First req.)

, $\frac{AM}{DM} = \frac{MB}{MC} = \frac{AB}{DC}$

∴ $\frac{6}{12} = \frac{MB}{16} = \frac{10}{DC}$

∴ $DC = \frac{10 \times 12}{6} = 20$ cm.

, $MB = \frac{6 \times 16}{12} = 8$ cm. (Second req.)

[b] ∵ $(AC)^2 = (7)^2 = 49$

, $(AB)^2 + (BC)^2 = (5)^2 + (6)^2 = 61$

, ∵ $(AC)^2 < (AB)^2 + (BC)^2$

∴ Δ ABC is an acute-angled triangle. (The req.)

5

[a] In Δ BCD : ∵ m (∠ BCD) = 90°

∴ $(BD)^2 = (BC)^2 + (CD)^2 = (7)^2 + (24)^2 = 625$

∴ BD = 25 cm.

In Δ BAD : ∵ $(BD)^2 = 625$

, $(AB)^2 + (AD)^2 = (15)^2 + (20)^2 = 625$

, ∵ $(BD)^2 = (AB)^2 + (AD)^2$

∴ m (∠ BAD) = 90° (Q. E. D.)

[b] $\because$ The area of Δ ADC = the area of Δ AEB

Subtracting the area of Δ ADE from both sides

$\therefore$ The area of Δ CED = the area of Δ BDE and they have a common base $\overline{DE}$ and on one side of it

$\therefore \overline{DE} // \overline{BC}$ (Q.E.D.)

5 Giza

1

1. proportional
2. C
3. $5\sqrt{2}$
4. The point B
5. 28

2

1. (a)
2. (b)
3. (c)
4. (c)
5. (c)
6. (b)

3

[a] $\because \overline{AD}$ is a median in Δ ABC

$\therefore$ The area of Δ ABD = the area of Δ ACD (1)

, $\because \overline{ED}$ is a median in Δ EBC

$\therefore$ The area of Δ EBD = the area of Δ ECD (2)

Subtracting (2) from (1) :

$\therefore$ The area of Δ ABE = the area of Δ ACE (Q.E.D.)

[b] $\because (AB)^2 = (7)^2 = 49$

, $(BC)^2 + (AC)^2 = (3)^2 + (5)^2 = 34$

, $\because (AB)^2 > (BC)^2 + (AC)^2$

$\therefore \Delta$ ABC is obtuse-angled triangle at C (The req.)

4

[a] In $\Delta\Delta$ ADE , ABC :

$\because \overline{DE} // \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore m(\angle ADE) = m(\angle B)$ (corresponding angles) (1)

, $\because \overline{DE} // \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle AED) = m(\angle C)$ (corresponding angles) (2)

, $\because \angle A$ is a common angle (3)

From (1) , (2) and (3) :

$\therefore \Delta ADE \sim \Delta ABC$ (First req.)

, $\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$ $\quad \therefore \frac{6}{8} = \frac{DE}{6} = \frac{AE}{4}$

$\therefore AE = \frac{6 \times 4}{8} = 3$ cm.

, $DE = \frac{6 \times 6}{8} = 4.5$ cm. (Second req.)

[b] The area of trapezium

$= \frac{1}{2}(5 + 9) \times 4$

$= 28$ cm^2. (The req.)

5

[a] $\because m(\angle A) = 90°$, $\overline{AD} \perp \overline{BC}$

$\therefore (AC)^2 = CD \times BC = 16 \times 25 = 400$

$\therefore AC = 20$ cm.

, $(AB)^2 = BD \times BC = 9 \times 25 = 225$

$\therefore AB = 15$ cm.

, $AD = \frac{AC \times AB}{BC} = \frac{20 \times 15}{25} = 12$ cm. (The req.)

[b] $\because \Delta\Delta$ XLY , XLZ have the same base $\overline{XL}$, $\overline{XL} // \overline{YZ}$

$\therefore$ The area of Δ XLY = the area of Δ XLZ

Subtracting the area of Δ XML from both sides

$\therefore$ The area of Δ XMY = the area of Δ LMZ (Q. E. D.)

6 Alexandria

1

1. 48
2. 0
3. 13
4. an acute-angled trianlge.
5. 1

2

1. (a)
2. (c)
3. (a)
4. (d)
5. (b)
6. (c)

3

[a] In Δ ABC : $\because \overline{AD}$ is a median in Δ ABC

$\therefore$ The area of Δ ADC = $\frac{1}{2}$ the area of Δ ABC (1)

, $\because$ M is the point of concurrence

$\therefore AM = \frac{2}{3}$ AD

$\therefore$ The area of Δ AMC = $\frac{2}{3}$ the area of Δ ADC (2)

From (1) and (2) :

$\therefore$ The area of Δ AMC = $\frac{2}{3}\left(\frac{1}{2}\right.$ the area of $\left.\Delta ABC\right)$

$\therefore$ The area of Δ AMC = $\frac{1}{3}$ the area of Δ ABC (Q. E. D.)

[b] $\because \Delta$ EBC has the common base $\overline{BC}$ with the ▱ ABCD , E $\in \overline{AD}$

$\therefore$ The area of Δ EBC = $\frac{1}{2}$ the area of ▱ ABCD

but the area of ▱ ABCD = the area of ▱ ABMN (have a common base $\overline{AB}$ and between two parallel straight lines $\overleftrightarrow{AB}$, $\overleftrightarrow{CN}$)

∴ The area of △ EBC = $\frac{1}{2}$ the area of ▱ ABMN

(Q. E. D.)

4

[a] In △ ABD : ∵ m (∠ ABD) = 90°

∴ $(BD)^2 = (AD)^2 - (AB)^2$

∴ $(BD)^2 = (5)^2 - (3)^2 = 16$

∴ BD = 4 cm.

, In △ BCD : ∵ m (∠ BCD) = 90°

, m (∠ BDC) = 30°

∴ BC = $\frac{1}{2}$ BD = 2 cm. (The req.)

[b] In △△ ABC , DEC

∵ m (∠ ABC) = m (∠ DEC) = 90°

∠ C is common

∴ m (∠ BAC) = m (∠ ECD)

∴ △ ABC ~ △ DEC (First req.)

, $\frac{ED}{AB} = \frac{CD}{AC}$ ∴ $\frac{3}{6} = \frac{5}{AC}$

∴ AC = $\frac{5 \times 6}{3}$ = 10 cm. (Second req.)

5

[a] Let the length of the other base be x cm.

∴ $\frac{1}{2}(10 + x) \times 8 = 88$ ∴ $10 + x = 22$

∴ $x = 12$ cm.

∴ the length of the other base = 12 cm. (The req.)

[b] In △ ABD : ∵ m (∠ ABD) = 90°

∴ $(AB)^2 = (AD)^2 - (BD)^2 = (17)^2 - (8)^2 = 225$

∴ AB = 15 cm. (First req.)

, In △ ABC : ∵ $(AB)^2 = 225$

, $(BC)^2 + (AC)^2 = (9)^2 + (12)^2 = 225$

, ∵ $(AB)^2 = (BC)^2 + (AC)^2$

∴ m (∠ ACB) = 90 (Second req.)

, ∵ $\overline{AE}$ is the projection of $\overline{AC}$ on $\overleftrightarrow{AB}$

∴ $(AC)^2 = AE \times AB$

∴ 144 = AE × 15

∴ AE = $\frac{144}{15}$ = 9.6 cm. (Third req.)

7 Alexandria

1

1 10 2 equal in area. 3 zero

4 an acute-angled triangle.

5 equal in measure

2

1 (b) 2 (a) 3 (b)

4 (a) 5 (b) 6 (d)

3

[a] In △△ AMB , DMC

∵ $\overline{AB}$ // $\overline{CD}$, $\overleftrightarrow{AD}$ is a transversal

∴ m (∠ A) = m (∠ D) (alternate angles) (1)

, ∵ $\overline{AB}$ // $\overline{CD}$, $\overleftrightarrow{BC}$ is a transversal

∴ m (∠ B) = m (∠ C) (alternate angles) (2)

, ∵ m (∠ AMB) = m (∠ DMC) (V. O. A.) (3)

From (1) , (2) and (3) :

∴ △ AMB ~ △ DMC (First req.)

, $\frac{AM}{DM} = \frac{AB}{DC}$ ∴ $\frac{6}{12} = \frac{10}{DC}$

∴ DC = $\frac{10 \times 12}{6}$ = 20 cm. (Second req.)

[b] ∵ The area of △ AMB = the area of △ DMC

Adding the area of △ ADM to both sides

∴ The area of △ ABD = the area of △ ACD

but and they have a common base $\overline{AD}$ and on one side of it

∴ $\overline{AD}$ // $\overline{BC}$ (Q. E. D.)

4

[a] ∵ m (∠ A) = 90°

∴ $(BC)^2 = (AB)^2 + (AC)^2 = (8)^2 + (6)^2 = 100$

∴ BC = 10 cm.

, ∵ $\overline{AD} \perp \overline{BC}$

, $(AC)^2 = CD \times BC$

∴ 36 = CD × 10 ∴ CD = 3.6 cm.

, AD = $\frac{AB \times AC}{BC} = \frac{8 \times 6}{10}$ = 4.8 cm. (The req.)

[b] ∵ △ BFC , ▱ ABCD have the common base $\overline{BC}$, F ∈ $\overline{AD}$

∴ The area of △ BFC = $\frac{1}{2}$ the area of ▱ ABCD (1)

∵ $\overline{FB}$ is a median in △ FEC

$\therefore$ The area of Δ BFC $= \frac{1}{2}$ the area of Δ FEC (2)

From (1) and (2) :

$\therefore$ The area of Δ FEC = the area of ▱ ABCD

(Q.E.D.)

5

[a] Let the height of trapezium be X cm.

$\therefore \frac{1}{2}(24 + 12) \times X = 450$

$\therefore 18X = 450$ $\therefore X = 25$ cm.

$\therefore$ The height is : 25 cm. (The req.)

[b] In Δ ABC :

$\because (AC)^2 = (BC)^2 + (AB)^2 = (8)^2 + (6)^2 = 100$

$\therefore$ AC = 10 cm.

In Δ ACD : $\because (AC)^2 = 100$

, $(AD)^2 + (CD)^2 = (7)^2 + (5)^2 = 74$

$\therefore (AC)^2 > (AD)^2 + (CD)^2$

$\therefore \angle$ D is an obtuse angled (The req.)

8 El-Kalyoubia

1

1 (b) 2 (b) 3 (b)

4 (d) 5 (a) 6 (b)

2

1 AC 2 C 3 A

4 The base length 5 equal

3

[a] $\because \Delta\Delta$ ABD , ACD have the same base $\overline{AD}$, $\overline{AD} \parallel \overline{BC}$

$\therefore$ The area of Δ ABD = the area of Δ ACD

Subtracting the area of Δ ADE from both sides

$\therefore$ The area of Δ ABE = the area of Δ DCE

(Q. E. D.)

[b] Assuming that the triangle whose side lengths are given be ABC and the other is XYZ

$\because$ The two triangles are similar.

$\therefore \frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ} = \frac{\text{The perimeter of } \Delta ABC}{\text{The perimeter of } \Delta XYZ}$

$\therefore \frac{3}{XY} = \frac{4}{YZ} = \frac{5}{XZ} = \frac{12}{36}$

$\therefore XY = \frac{3 \times 36}{12} = 9$ cm.

$\therefore YZ = \frac{4 \times 36}{12} = 12$ cm.

$\therefore XZ = \frac{5 \times 36}{12} = 15$ cm. (The req.)

4

[a] $\because \Delta$ BFC , ▱ ABCD have the common base $\overline{BC}$, F $\in \overline{AD}$

$\therefore$ The area of Δ BFC $= \frac{1}{2}$ the area of ▱ ABCD (1)

$\because \overline{FB}$ is a median in Δ FCE

$\therefore$ The area of Δ BFC $= \frac{1}{2}$ the area of Δ FCE (2)

From (1) and (2) :

$\therefore$ The area of Δ FCE = The area of ▱ ABCD

(Q.E.D.)

[b] The area of ▱ ABCD = BC × DE = 12 × 15 = 180 cm^2.

, $\because$ The area of ▱ ABCD = AB × DO

$\therefore 180 = 18 \times DO$

$\therefore DO = \frac{180}{18} = 10$ cm. (The req.)

5

[a] $\because m(\angle A) = 90°$, $\overline{AD} \perp \overline{BC}$

$\therefore (AB)^2 = BD \times BC = 9 \times 25 = 225$

$\therefore$ AB = 15 cm.

, $\because (AD)^2 = BD \times DC = 9 \times 16 = 144$

$\therefore$ AD = 12 cm. (The req.)

[b] $\because (BC)^2 = (7)^2 = 49$

, $(AB)^2 + (AC)^2 = (3)^2 + (5)^2 = 34$

$\therefore (BC)^2 > (AB)^2 + (AC)^2$

$\therefore \Delta$ ABC is an obtuse-angled at A (The req.)

9 El-Sharkia

1

1 Tow triangular surfaces equal in area.

2 40

3 The base length , its corresponding height

4 15 cm^2. 5 half

2

1 (c) 2 (c) 3 (d)

4 (a) 5 (b) 6 (b)

3

[a] In $\Delta\Delta$ AED , CEB

$\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle A) = m(\angle C)$ (alternate angles) (1)

, ∵ $\overline{AD} // \overline{BC}$, $\overleftrightarrow{BD}$ is a transversal

∴ m (∠ B) = m (∠ D) (alternate angles) (2)

, ∵ m (∠ AED) = m (∠ CEB) (V. O. A.) (3)

From (1) , (2) and (3) :

∴ Δ AED ~ Δ CEB (First req.)

, ∵ $\frac{AE}{CE} = \frac{ED}{EB} = \frac{AD}{CB}$ ∴ $\frac{3}{CE} = \frac{2}{EB} = \frac{4}{8}$

∴ CE = $\frac{3 \times 8}{4}$ = 6 cm. , EB = $\frac{2 \times 8}{4}$ = 4 cm.

∴ The perimeter of Δ EBC = 6 + 4 + 8 = 18 cm. (Second req.)

[b] In Δ ABC : ∵ m (∠ B) = 90°

∴ $(AC)^2 = (AB)^2 + (BC)^2 = (3)^2 + (4)^2 = 25$

∴ AC = 5 cm.

, In Δ ACD : ∵ $(AD)^2 = (13)^2 = 169$

, $(AC)^2 + (CD)^2 = (5)^2 + (12)^2 = 169$

∴ $(AD)^2 = (AC)^2 + (CD)^2$

∴ m (∠ ACD) = 90° (Q. E. D.)

4

[a] ∵ The area of Δ ACD = the area of Δ ABE

Subtracting the area of Δ ADE from both sides

∴ The area of Δ CDE = the area of Δ BDE

but they have a common base $\overline{DE}$ and on one side of it

∴ $\overline{ED} // \overline{BC}$ (Q. E. D.)

[b] ∵ $(BC)^2 = (10)^2 = 100$

, $(AB)^2 + (CA)^2 = (8)^2 + (7)^2 = 113$

∴ $(BC)^2 < (AB)^2 + (CA)^2$

∴ Δ ABC is acute-angled triangle (The req.)

5

[a] ∵ m (∠ A) = 90° , $\overline{AD} \perp \overline{BC}$

∴ $(AB)^2$ = BD × BC = 9 × 25 = 225

∴ AB = 15 cm.

, $(AC)^2$ = CD × CB = 16 × 25 = 400

∴ AC = 20 cm.

, $(AD)^2$ = BD × CD = 9 × 16 = 144

∴ AD = 12 cm. (The req.)

[b] ∵ ΔΔ ABC , DBC have a common base $\overline{BC}$

, $\overline{AD} // \overline{BC}$

∴ The area of Δ ABC = the area of Δ DBC

Subtracting the area of Δ MBC from both sides.

∴ The area of Δ AMB = the area of Δ DMC (1)

, ∵ $\overline{MX}$ is a median in Δ BMC

∴ The area of Δ BMX = the area of Δ CMX (2)

adding (1) and (2) :

∴ The area of the figure ABXM = the area of the figure DCXM (Q. E. D.)

10 El-Dakahlia

1

1 (c) 2 (d) 3 (c)

4 (d) 5 (a) 6 (a)

2

1 10 cm. 2 proportional

3 The point A 4 100° 5 25

3

[a] ∵ m (∠ A) = 90° , $\overline{AD} \perp \overline{BC}$

∴ $(AC)^2$ = CD × CB = 9 × 25 = 225

∴ AC = 15 cm.

, $(AD)^2$ = CD × BD = 9 × 16 = 144

∴ AD = 12 cm. (The req.)

[b] ∵ ΔΔ ABC , DBC have a common base $\overline{BC}$

, $\overline{AD} // \overline{BC}$

∴ The area of Δ ABC = the area of Δ DBC

Subtracting the area of Δ OBC from both sides.

∴ The area of Δ AOB = the area of Δ DOC (1)

, ∵ XB = XC

∴ The area of Δ BOX = the area of Δ COX (2)

Adding (1) and (2) :

∴ The area of the figure ABXO = the area of the figure DCXO (Q. E. D.)

4

[a] In Δ ABC : ∵ m (∠ B) = 90°

∴ $(AC)^2 = (AB)^2 + (BC)^2 = (7)^2 + (24)^2 = 625$

∴ AC = 25 cm.

In Δ ACD : $(AC)^2 = (25)^2 = 625$

, $(AD)^2 + (CD)^2 = (15)^2 + (20)^2 = 625$

$\therefore (AC)^2 = (AD)^2 + (CD)^2$

$\therefore m(\angle D) = 90°$ (Q.E.D.)

[b] $\because (AC)^2 = (10)^2 = 100$

, $(AB)^2 + (BC)^2 = (5)^2 + (8)^2 = 89$

$\therefore (AC)^2 > (AB)^2 + (BC)^2$

$\therefore \Delta$ ABC is obtuse-angled triangle. (The req.)

5

[a] $\because$ The length of the middle base $= \frac{1}{2}(4 + 10) = 7$ cm.

$\therefore$ The area of the trapezium $= 7 \times 5 = 35$ cm². (The req.)

[b] In $\Delta\Delta$ AOD , COB

$\because \overline{DA} \parallel \overline{CB}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle A) = m(\angle C)$ (alternate angles) (1)

, $\because \overline{DA} \parallel \overline{CB}$, $\overleftrightarrow{DB}$ is a transversal

$\therefore m(\angle D) = m(\angle B)$ (alternate angles) (2)

, $\because m(\angle AOD) = m(\angle BOC)$ (V.O.A.) (3)

From (1) , (2) and (3) :

$\therefore \Delta AOD \sim \Delta COB$ (First req.)

let length of $\overline{AO} = x$ cm.

$\therefore \frac{AO}{CO} = \frac{AD}{CB}$ $\quad \therefore \frac{x}{12 - x} = \frac{3}{6}$

$\therefore 6x = 36 - 3x$ $\quad \therefore 9x = 36$ $\quad \therefore x = 4$

$\therefore$ AO = 4 cm. (Second req.)

11 Suez

1

1 (b) 2 (c) 3 (d)
4 (b) 5 (d) 6 (c)

2

1 in area 2 C
3 similar 4 6 cm.
5 The point A

3

[a] In Δ ABC : $\because m(\angle CAB) = 90°$

$\therefore (BC)^2 = (AC)^2 + (AB)^2 = (6)^2 + (8)^2 = 100$

$\therefore$ BC = 10 cm.

, $\because \overline{DB}$ is the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$

, $(AB)^2 = DB \times BC$

$\therefore 64 = DB \times 10$

$\therefore DB = \frac{64}{10} = 6.4$ cm. (The req.)

[b] $\because$ The area of Δ AMB = the area of Δ DMC

Adding the area of Δ AMD to both sides

$\therefore$ The area of Δ ABD = the area of Δ ACD

but they have a common base $\overline{AD}$ and on one side of it

$\therefore \overline{AD} \parallel \overline{BC}$ (Q.E.D.)

4

[a] $\because (AC)^2 = (12)^2 = 144$

, $(AB)^2 + (BC)^2 = (9)^2 + (10)^2 = 181$

$\therefore (AC)^2 < (AB)^2 + (BC)^2$

$\therefore \angle B$ is acute. (The req.)

[b] $\because \overline{AD}$ is a median in Δ ABC

$\therefore$ The area of Δ ABD = the area of Δ ACD (1)

, $\because \overline{ED}$ is a medain in Δ EBC

$\therefore$ The area of Δ EBD = the area of Δ ECD (2)

Subtracting (2) from (1) :

$\therefore$ The area of Δ ABE = the area of Δ ACE (Q. E. D.)

5

[a] In Δ ABC : $\because m(\angle B) = 90°$

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (9)^2 + (12)^2 = 225$

$\therefore$ AC = 15 cm.

In Δ ACD : $(CD)^2 = (17)^2 = 289$

, $(AC)^2 + (AD)^2 = (15)^2 + (8)^2 = 289$

$\therefore (CD)^2 = (AC)^2 + (AD)^2$

$\therefore m(\angle DAC) = 90°$ (Q.E.D.)

[b] In $\Delta\Delta$ AED , ABC

$\because \overline{ED} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore m(\angle AED) = m(\angle B)$ (corresponding angles) (1)

, $\because \overline{ED} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore$ m (∠ ADE) = m (∠ C) (corresponding angles) (2)

, $\because$ ∠ A is a common angle (3)

From (1) , (2) and (3) :

$\therefore$ Δ AED ~ Δ ABC

, $\frac{AE}{AB} = \frac{ED}{BC} = \frac{AD}{AC}$ $\therefore \frac{5}{15} = \frac{4}{AC}$

$\therefore AC = \frac{4 \times 15}{5} = 12$ cm.

$\therefore DC = 12 - 4 = 8$ cm. (The req.)

12 Port Said

1

1 (c) 2 (a) 3 (a)
4 (b) 5 (d) 6 (c)

2

1 120° 2 C 3 2 : 5
4 5 5 a right

3

[a] In Δ ABC : $\because$ m (∠ A) = 90° , $\overline{AD} \perp \overline{BC}$

$\therefore (AB)^2 = BD \times BC = 16 \times 25 = 400$

$\therefore AB = 20$ cm.

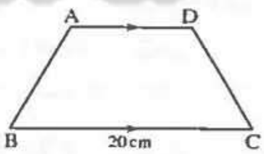
, $(AC)^2 = CD \times BC = 9 \times 25 = 225$

$\therefore AC = 15$ cm.

, $(AD)^2 = CD \times BD = 9 \times 16 = 144$

$\therefore AD = 12$ cm. (The req.)

[b] $\because BC = 2 AD = 20$



$\therefore AD = 10$ cm.

$\therefore$ The area of the trapezium ABCD

$= \frac{1}{2}$ (BC + AD) × The height

$\therefore 180 = \frac{1}{2}(20 + 10) \times$ height

$\therefore$ The height $= \frac{180}{15} = 12$ cm.

(The req.)

4

[a] In ΔΔ ABC , DBE

$\because \overline{AC} \parallel \overline{ED}$, $\overleftrightarrow{AD}$ is a transversal

$\therefore$ m (∠ A) = m (∠ D) (alternate angles) (1)

$\because \overline{AC} \parallel \overline{ED}$, $\overleftrightarrow{CE}$ is a transversal

$\therefore$ m (∠ C) = m (∠ E) (alternate angles) (2)

, $\because$ m (∠ ABC) = m (∠ DBE) (V. O. A.) (3)

From (1) , (2) and (3) :

$\therefore$ Δ ABC ~ Δ DBE (First req.)

, $\frac{AB}{DB} = \frac{BC}{BE} = \frac{AC}{DE}$

$\therefore \frac{3}{6} = \frac{BC}{8} = \frac{5}{DE}$ $\therefore DE = \frac{5 \times 6}{3} = 10$ cm.

, $BC = \frac{3 \times 8}{6} = 4$ cm. (Second req.)

[b] $\because$ ΔΔ AXF , CYF have equal bases in lengths , $\overline{AC} \parallel \overline{XY}$

$\therefore$ The area of Δ AXF = the area of Δ CYF (1)

, $\because \overline{BF}$ is a median in Δ BXY

$\therefore$ The area of Δ BXF = the area of Δ BYF (2)

Adding (1) and (2) :

$\therefore$ The area of Δ ABF = the area of Δ CBF

(Q.E.D.)

5

[a] $\because$ The area of Δ ABE = the area of Δ ACD

Subtracting the area of Δ ADE from both sides

$\therefore$ The area of Δ BDE = the area of Δ CDE but they have a common base $\overline{DE}$ and one side of it

$\therefore \overline{DE} \parallel \overline{BC}$ (Q.E.D.)

[b] In Δ ABC : $\because$ m (∠ B) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (9)^2 + (12)^2 = 225$

$\therefore AC = 15$ cm.

In Δ ACD : $\because (CD)^2 = (17)^2 = 289$

, $(AC)^2 + (AD)^2 = (15)^2 + (8)^2 = 289$

$\therefore (CD)^2 = (AC)^2 + (AD)^2$

$\therefore$ m (∠ DAC) = 90° (Q.E.D.)

13 Damietta

1

1 (b) 2 (c) 3 (a)
4 (d) 5 (c) 6 (b)

2

1 half 2 60° 3 110°
4 equal to 5 30

3

[a] $\because 450 = \frac{1}{2}(24 + 12) \times$ the height

$\therefore$ The height $= \frac{450}{18} = 25$ cm. (Thr req.)

[b] $\because$ ΔΔ ABD , ACD have a common base $\overline{AD}$

, $\overline{AD} // \overline{BC}$

$\therefore$ The area of Δ ABD = the area of Δ ACD

Subtracting the area of Δ MAD from both sides

$\therefore$ The area of Δ AMB = the area of Δ DMC (1) (Q. E. D. 1)

, $\because$ $\overline{MH}$ is a median in Δ BMC

$\therefore$ The area of Δ BMH = the area of Δ CMH (2)

Adding (1) and (2) :

$\therefore$ The area of the figure ABHM = the area of the figure DCHM (Q. E. D. 2)

4

[a] $\because$ The area of Δ EAB = The area of Δ EDC

Subtracting the area of Δ ADE from both sides

$\therefore$ The ara of Δ ABD = The area of Δ ACD

but they have a common base $\overline{AD}$ and on one side of it

$\therefore$ $\overline{AD} // \overline{BC}$ (Q. E. D.)

[b] $\because (AB)^2 = (7)^2 = 49$

$, (BC)^2 + (AC)^2 = (3)^2 + (6)^2 = 45$

$\therefore (AB)^2 > (BC)^2 + (AC)^2$

$\therefore$ Δ ABC is an obtuse-angled triangle (The req.)

5

[a] In Δ ABC : $\because$ m (∠ ABC) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (7)^2 + (24)^2 = 625$

$\therefore$ AC = 25 cm.

In Δ ACD : $\because$ m (∠ ADC) = 90°

$\therefore (CD)^2 = (AC)^2 - (AD)^2 = (25)^2 - (15)^2 = 400$

$\therefore$ CD = 20 cm. (First req.)

$\because$ $\overline{AE}$ is the projection of $\overline{AD}$ on $\overleftrightarrow{AC}$

$, (AD)^2 = AE \times AC$

$\therefore (15)^2 = AE \times 25$

$\therefore AE = \frac{225}{25} = 9$ cm. (Second req.)

[b] In ΔΔ ADE , ABC :

$\because$ $\overline{DE} // \overline{BC}$, $\overleftrightarrow{AB}$ in a transveral

$\therefore$ m (∠ ADE) = m (∠ B) (corresponding angles) (1)

, $\because$ $\overline{DE} // \overline{BC}$, $\overleftrightarrow{AC}$ in a transversal

$\therefore$ m (∠ AED) = m (∠ C) (corresponding angles) (2)

, $\because$ ∠ A is a common angle (3)

From (1) , (2) and (3) :

$\therefore$ Δ ADE ~ Δ ABC (First req.)

$\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$ $\therefore \frac{2}{6} = \frac{DE}{12} = \frac{AE}{9}$

$\therefore DE = \frac{2 \times 12}{6} = 4$ cm.

$, AE = \frac{2 \times 9}{6} = 3$ cm. (Second req.)

14 Beni Suef

1

1 (b) 2 (b) 3 (c)
4 (b) 5 (a) 6 (b)

2

1 lie on a straight line parallel to this base.
2 angles 3 an obtuse-angled.
4 an acute
5 equal in measure.

3

[a] $\because$ ABCD and BECD are two parallelograms having the common base $\overline{DC}$, $\overleftrightarrow{AE} // \overline{DC}$

$\therefore$ The area of ▱ ABCD = the area of ▱ BECD (1)

$\because$ Δ ABD and ▱ ABCD have the common base $\overline{AB}$, D ∈ $\overline{DC}$

$\therefore$ The area of Δ ABD $= \frac{1}{2}$ the area of ▱ ABCD (2)

, $\because$ Δ MEC and ▱ BECD have the common base $\overline{EC}$, M ∈ $\overline{BD}$

$\therefore$ The area of Δ MEC $= \frac{1}{2}$ the area of ▱ BECD (3)

From (1) , (2) and (3) :

$\therefore$ The area of Δ ABD = The area of Δ MEC (Q. E. D.)

[b] $\because$ ΔΔ ABD , ACD have a common base $\overline{AD}$

, $\overline{AD} \parallel \overline{BC}$

$\therefore$ The area of Δ ABD = the area of ACD

Subtracting the area of Δ MAD from both sides

$\therefore$ The area of Δ ABM = the area of Δ DCM

(Q.E.D.)

4

[a] The area of rhombus $= \frac{1}{2}(18 \times 24) = 216 \text{ cm}^2$.

, $\because$ The area of rhombus = the area of trapezium

$\therefore$ 216 = the length of the middle base × 12

$\therefore$ The length of the middle base $= \frac{216}{12} = 18$ cm.

(The req.)

[b] In ΔΔ AED , ABC

$\because m(\angle AED) = m(\angle B)$

, $\angle A$ is a common angle

$\therefore m(\angle ADE) = m(\angle C)$

$\therefore \Delta AED \sim \Delta ABC$ (First req.)

, $\frac{AE}{AB} = \frac{ED}{BC} = \frac{AD}{AC}$ $\therefore \frac{4}{AB} = \frac{3}{9}$

$\therefore AB = \frac{4 \times 9}{3} = 12$ cm.

$\therefore BD = 12 - 3 = 9$ cm. (Second req.)

5

In Δ ABC : $\because m(\angle ABC) = 90°$

$\therefore (AB)^2 = (AC)^2 - (BC)^2 = (17)^2 - (8)^2 = 225$

$\therefore AB = 15$ cm.

In Δ ABD : $\because (AB)^2 = (15)^2 = 225$

, $(AD)^2 + (BD)^2 = (12)^2 + (9)^2 = 225$

$\therefore (AB)^2 = (AD)^2 + (BD)^2$

$\therefore m(\angle ADB) = 90°$ (First req.)

, $DE = \frac{AD \times BD}{AB} = \frac{9 \times 12}{15} = 7.2$ cm. (Second req.)

, $\overline{AE}$ is the projection of $\overline{AD}$ on $\overleftrightarrow{AB}$

, $\because (AD)^2 = AE \times AB$

$\therefore (12)^2 = AE \times 15$

$\therefore AE = \frac{144}{15} = 9.6$ cm. (Third req.)

15 Qena

1

1 (d) 2 (b) 3 (a)

4 (c) 5 (b) 6 (b)

2

1 equal in area. 2 similar. 3 6

4 an obtuse 5 proportional.

3

[a] The area of the rhombus

$= \frac{1}{2} \times 12 \times 10 = 60 \text{ cm}^2$.

$\therefore$ The side length × 8 = 60

$\therefore$ The side length $= \frac{60}{8} = 7.5$ cm.

$\therefore$ The perimeter of the rhombus

$= 4 \times 7.5 = 30$ cm. (The req.)

[b] In ΔΔ ADE , ABC

$\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore m(\angle ADE) = m(\angle B)$ (corresponding angles) (1)

, $\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle AED) = m(\angle C)$ (corresponding angles) (2)

, $\because \angle A$ is a common angle (3)

From (1) , (2) and (3) :

$\therefore \Delta ADE \sim \Delta ABC$ (First req.)

, $\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$ $\therefore \frac{2}{BC} = \frac{3}{9}$

$\therefore BC = \frac{2 \times 9}{3} = 6$ cm. (Second req.)

4

[a] $\because$ The area of Δ AMB = the area of Δ CMD

Adding the area of Δ AMD to both sides

$\therefore$ The area of Δ ABD = the area of Δ ACD

but they have a common base $\overline{AD}$ and on one side of it

$\therefore \overline{AD} \parallel \overline{BC}$ (Q. E. D.)

[b] In Δ ABC : $\because$ m ($\angle$ B) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (3)^2 + (4)^2 = 25$

$\therefore$ AC = 5 cm.

In Δ ACD : $\because (AD)^2 = (13)^2 = 169$

, $(AC)^2 + (CD)^2 = (5)^2 + (12)^2 = 169$

$\therefore (AD)^2 = (AC)^2 + (CD)^2$

$\therefore$ m ($\angle$ ACD) = 90° (First req.)

The area of the figure ABCD = The area of Δ ABC + the area of Δ ACD = $\frac{1}{2} \times 4 \times 3 + \frac{1}{2} \times 12 \times 5$

= 36 cm². (Second req.)

5

[a] $\because \overline{AD}$ is a median in Δ ABC

$\therefore$ The area of Δ ABD = The area of Δ ACD (1)

, $\because \overline{ED}$ is a median in Δ EBC

$\therefore$ The area of Δ EBD = the area of Δ ECD (2)

Subtracting (2) from (1) :

$\therefore$ The area of Δ ABE = the area of Δ ACE

(Q. E. D.)

[b] In Δ ABC : $\because$ m ($\angle$ A) = 90° , $\overline{AD} \perp \overline{BC}$

$\therefore (AB)^2 = BD \times BC = 9 \times 25 = 225$

$\therefore$ AB = 15 cm.

, $(AC)^2 = CD \times CB = 16 \times 25 = 400$

$\therefore$ AC = 20 cm.

$\therefore$ The perimeter of Δ ABC = AB + BC + AC

= 15 + 25 + 20

= 60 cm. (First req.)

, The area of Δ ABC = $\frac{1}{2}$ AB $\times$ AC

= $\frac{1}{2} \times 15 \times 20 = 150$ cm². (Second req.)